DECAYING DC COMPONENT EFFECT ELIMINATION ON PHASOR ESTIMATION USING AN ADAPTIVE FILTERING ALGORITHM

Kleber M. Silva[†]
klebermelo@unb.br

Bernard F. Küsel[†]
bernard_kusel@hotmail.com

[†]University of Brasília
Department of Electrical Engineering
Brasília - DF, CEP: 70910-900, Brazil

Abstract - In this paper a novel adaptive mimic filter is presented to remove the decaying DC component effect on phasor estimation. The key idea is to use an adaptive scheme to obtain the decaying time constant, thereby the digital mimic filter parameters are readjusted. The proposed phasor estimation algorithm combines the adaptive mimic filter with the full cycle discrete Fourier transform. Its performance is compared to those of other algorithms reported in the literature on the subject. A wide variety of EMTP-simulated data is used to carry out a statistical analysis of the performance of the algorithms on decaying DC component removal. The obtained results indicate that the proposed algorithm presents excellent performance.

Keywords - Phasor estimation, decaying DC component, digital mimic filter, discrete Fourier transform.

1 INTRODUCTION

MANY protective relaying functions uses the phasors at the power system fundamental frequency [1]. The phasors are estimated from the voltage and current sampled waveforms using digital filtering algorithms. The discrete Fourier transform (DFT)-based algorithms are the most commonly used. As a fault takes place, besides the fundamental component the fault signals may also include harmonics and a decaying DC component. Harmonics may be easily filtered out, but the decaying DC component is more difficult to deal, because it is a non-periodic signal whose frequency spectrum encompasses a wide range of frequencies. As a result, it may seriously affect the estimated phasor, causing undesirable oscillations about its actual value [2].

The decaying DC component depend on power system configuration and the fault parameters, such as its location, resistance and inception angle. Therefore, the decaying DC component can not be known a priori. As rule of thumb, usually it is assumed that the time constant typically ranges from 0.5 to five cycles [4].

Several algorithms to eliminate the decaying DC component effect in phasor estimation have been reported [2]-[9]. In [3], the least error square (LES) algorithm is used to estimate the phasor at the fundamental frequency. The first two terms of the Taylor series expansion are used to model the decaying DC component. The LES algorithm performs well over a wide range of time constants, but not for small ones typically ranging from 0.1 to 0.5 cycles [8].

In [4], the decaying DC component is removed using a digital mimic filter, which is the digital version of the well known mimic circuit. The digital filter parameters are tuned to a particular time constant. In the case in which it is equal to the actual time constant of the evaluated signal, the mimic filter achieves the best performance. Otherwise, the larger the mismatch, the worse is the performance of the mimic filter. These same drawbacks are observed for algorithms based on the Kalman filter [5]. They perform well only if the time constant of the evaluated signal is the same as the one modeled in the Kalman state transition matrix [4].

Some algorithms for the decaying DC component removal are based on the waveform analysis. In [6], the parameters of the decaying DC component are estimated integrating the evaluated signal over one cycle. The DC value at each sampling instant is subtracted from the original samples and then the full cycle DFT (FCDFT) algorithm is applied to estimate the phasor.

Most efforts have been aimed to propose modifications in DFT-based algorithms [7, 8, 9]. The key idea of these algorithms is to perform the phasor correction based on the error in DFT output caused by the decaying DC component. In [7], the parameters of the decaying DC component are estimated using three successive FCDFT outputs. Modified FCDFT algorithms are proposed in [8], which are based on partial sums of one cycle of samples. These simplified algorithms perform well over a wide range of time constants. In [9], the decaying DC component effect on phasor estimation is suppressed using the difference between the outputs of the FCDFT for odd-sample-set and even-sample-set.

In this paper a novel adaptive mimic filter (AMF) is presented to eliminate the decaying DC component effect on phasor estimation. The key idea is to use an adaptive algorithm to obtain the decaying time constant of the signal, thereby the digital mimic filter parameters are readjusted. As a result, the decaying DC component may be completely filtered out. The proposed phasor estimation algorithm combines the AMF with the FCDFT algorithm. Its performance is compared to those of other algorithms reported in the literature on the subject. A wide variety of EMTP-simulated data is used to carry out a statistical analysis of the performance of the algorithms on decaying DC component removal. The proposed algorithm presents excellent overall results.

2 THE TRADITIONAL DIGITAL MIMIC FILTER

According to the mimic circuit principle, if an exponentially decaying DC current has to pass through a mimic circuit consisting of an impedance of the form

$$H(s) = K\left(1 + s\tau\right) \ , \tag{1}$$

the voltage across the impedance will be only a DC constant, if τ is equal to the decaying time constant of the current waveform. The digital version of this equation is known as the digital mimic filter equation [4]

$$H(z) = K\left[(1 - \tau_d) + \tau_d z^{-1}\right] \ , \tag{2}$$

where τ_d is expressed in number of samples.

Consider a discrete sinusoidal signal $x(k)$

$$x(k) = A_0\Gamma^k + \sum_{n=1}^{N/2} A_n \sin\left(\frac{2\pi n}{N}k + \varphi_n\right) \tag{3}$$

where $\Gamma = e^{-\triangle t/\tau}$ is the exponential term; A_0 and τ are the amplitude and the time constant of the decaying DC component; A_n and φ_n are the amplitude and phase angle of the n-th harmonic component; $\triangle t$ is the sampling interval; and N the number of samples per cycle.

Applying the digital mimic filter to $x(k)$

$$z(k) = K\left[(1 + \tau_d)\,x(k) - \tau_d x(k-1)\right] \ . \tag{4}$$

The gain K must be one at the fundamental frequency

$$K = \sqrt{\frac{1}{\left[(1+\tau_d) - \tau_d \cos\left(\frac{2\pi}{N}\right)\right]^2 + \left[\tau_d \sin\left(\frac{2\pi}{N}\right)\right]^2}} \ . \tag{5}$$

In the case of τ_d is equal to the time constant of the decaying DC component presents in the input signal $x(k)$, the output signal $z(k)$ consists of a DC constant added to the $N/2$ order harmonic components.

The fundamental phasor of the discrete signal $z(k)$ can be estimated using the FCDFT [1]

$$Z(k) = \frac{2}{N}\sum_{n=1}^{N} z(k - N + n)e^{-j\frac{2\pi}{N}n} \ , \tag{6}$$

whereby one can obtain the phasor $Z(k)$ as follows

$$Z(k) = K\left[(1 - \tau_d)\,X(k) + \tau_d X(k-1)\right] \ , \tag{7}$$

where $X(k)$ is the fundamental frequency phasor of $x(k)$ obtained using the FCDFT.

Since the FCDFT has zero gain at the DC and harmonic frequencies, $Z(k)$ is only the fundamental phasor.

3 THE PROPOSED PHASOR ESTIMATION ALGORITHM

The proposed phasor estimation algorithm combines the novel AMF with the FCDFT, in order to obtain the phasor without the harmful effect of the decaying DC component. The key idea is to readjust adaptively the parameters of the digital mimic filter to the actual decaying time constant τ of the DC component of the fault signals. In order to estimate τ, firstly Γ is estimated.

Consider a discrete signal $x(k)$ of the form described in (3), with $n = 1,\ 2,\ 3, \ldots,\ N/2$. The following partial sum terms can be defined [8]:

$$PS_1 = \sum_{k=1}^{N/2} x\left(2k - 1\right) \tag{8}$$

$$PS_2 = \sum_{k=1}^{N/2} x\left(2k\right) \ . \tag{9}$$

From trigonometric relationship and algebraic manipulations in (8) and (9), PS_1 and PS_2 can be solved as

$$PS_1 = A_0\frac{\Gamma\left(\Gamma^N - 1\right)}{\Gamma^2 - 1} \tag{10}$$

$$PS_2 = A_0\frac{\Gamma^2\left(\Gamma^N - 1\right)}{\Gamma^2 - 1} \ . \tag{11}$$

From (10) and (11), Γ can be computed as follows

$$\Gamma = \frac{PS_2}{PS_1} \ . \tag{12}$$

The time constant τ can be obtained approximating Γ using the first two terms of its Taylor series expansion, accordingly

$$\tau = \frac{\triangle t}{1 - \Gamma} \ . \tag{13}$$

Physically, τ does not change during a given transient. However, the estimator (12) may lead to unrealistic values for Γ in the first few samples. To overcome this drawback, τ is forced to stay in between $\tau_{min} = 0.5$ cycles and $\tau_{max} = 5$ cycles, which are in accordance with the typical range of time constant values [4]. Even so, τ_{max} can be set using a grater value, for instance, in medium high voltage level in which τ can reach 300 ms.

Averaging the estimated values of τ over a number of samples leads to further damping of unwanted oscillations. Simulation results has shown that the use of two samples average leads to good results.

For each new value of τ, the readjustment of the digital mimic filter parameter is performed. Since τ_d is expressed in number of samples, it can be computed as follows

$$\tau_d = round\left(\tau/\triangle t\right) \ , \tag{14}$$

where the operator $round(\cdot)$ rounds a float point number to the nearest integers. Then, the gain of the mimic filter for the fundamental frequency is readjusted using (5).

The proposed phasor estimation algorithm can be summarized as shown in Fig. 1. For each new sample of $x(k)$, the phasor $X(k)$ is estimated using the FCDFT described in (6). The time constant τ is initially set to one cycle. By using (7), the phasor $Z(k)$ is obtained. Then, its phase angle is corrected to remove the phase angle displacement

Figure 1: Flowchart of the proposed phasor estimation algorithm.

due to the digital mimic filter, resulting the fundamental phasor $\overline{Z}(k)$. From the fault beginning time, τ is estimated using the adaptive scheme. For each new value of τ, τ_d is computed by (14) and the filter gain K is readjusted using (5). Then, the phasor $Z(k)$ is estimated using (7) with the new values of τ_d and K. Finally, the phase angle correction is performed and the fundamental phasor $\overline{Z}(k)$ is obtained without the decaying DC component harmful effect.

In multifunctional relays, the proposed phasor estimation algorithm is performed for each voltage and current input signal, which may lead to different digital mimic filters, since the decaying DC component may be different for each signal. That is the why the aforementioned phase angle correction is necessary. It is performed by applying a linear transformation of rotation to the phasor $Z(k)$

$$\begin{bmatrix} \overline{Z}_{re}(k) \\ \overline{Z}_{im}(k) \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Z_{re}(k) \\ Z_{im}(k) \end{bmatrix} \tag{15}$$

where the subscripts re and im correspond to the real and imaginary parts of the phasor, respectively, and ϕ is the phase angle displacement caused by the mimic filter

$$\phi = \arctan\left[\frac{\tau_d \sin(2\pi/N)}{(1+\tau_d) - \tau_d \cos(2\pi/N)}\right]. \tag{16}$$

4 PERFORMANCE EVALUATION

The frequency and time response of the proposed phasor estimation algorithm was compared to those of the simplified algorithm 1 (here named as modified FCDFT) [8] and the LES algorithm [3]. The obtained results are discussed next.

4.1 *FREQUENCY RESPONSE ANALYSIS*

Each algorithm uses two orthogonal filters h_c and h_s to estimate, respectively, the real and imaginary parts of the fundamental frequency phasor. Analyzing the frequency response of these filters, their sensitivity to different frequency components can be pointed out.

(a)

(b)

Figure 2: Magnitude of the frequency response of: (a) real-part h_c filters and (b) imaginary-part h_s filters, related to the evaluated algorithms.

In Figs. 2 are shown the frequency response magnitudes of the filters h_c and h_s convoluted with a third order Butterworth low-pass filter with cutoff frequency at 180 Hz. The filters of the proposed algorithm are also convoluted with the digital mimic filter. The overall influence of its time constant τ_d on frequency responses magnitude is negligible, in such way that a value of one cycle was chosen to plot these figures.

In Fig. 2(a), it can be seen that the h_c filter of the proposed algorithm has the largest side lobes among the real part filters, but these lobes are smaller than those observed in frequency responses of the h_s filters related to the modified FCDFT and the LES algorithms. On the other hand, the frequency response of the h_s filter related to the proposed algorithm presents the smallest side lobes. The overall results indicate that the modified FCDFT and the LES algorithms are quite sensitive to high frequencies related to transients signals that appears following any fault or disturbance in an electrical power system.

4.2 *TIME RESPONSE ANALYSIS*

The time responses analysis of phasor estimation algorithms is important to verify their performances during the transient interval in which data windows contains both pre-fault and fault samples. Thus, some important characteristics to digital relaying are pointed out, such as the speed and accuracy.

(a)

(b)

Figure 3: Amplitude of the fundamental phasor for the test signal with τ equal to (a) one cycle and (b) two cycle.

Figure 4: Percentage overshoot in the estimation of the fundamental phasor amplitude for different values of τ.

(a)

(b)

Figure 5: Amplitude of the fundamental phasor for the test signal with τ equal to (a) one cycle and (b) two cycle.

Figure 6: Single line diagram of the power system for testing.

To evaluate time response, test signals cases and ATP simulated data are used. In order to do that, the simulations was carried out using a sampling rate of 160 samples per cycle. The output signal was preprocessed using a third order Butterworth low-pass filter with cutoff frequency at 180 Hz. Then, the output signal was resampled to the sampling rate of 16 samples per cycle. Resampled signals were used to evaluate the performance of phasor estimation algorithms.

4.2.1 *USING TEST SIGNALS CASES*

The sensitivity to variation of the decaying DC component was investigated using the following test signals:

$$x(k) = A\Gamma^k - A\cos\left(\frac{2\pi}{N}k\right) \ , \qquad (17)$$

where $A = 100$, $N = 16$ and $\Gamma = e^{-\triangle t/\tau}$.

In Fig. 3(a) and 3(b) are shown the estimated amplitude of the fundamental phasor considering the test signal with τ equal to one cycle and two cycle, respectively. The proposed algorithm provides the smallest overshoot for these two test signals.

In Fig. 4, it is shown the percentual overshoot for the fundamental phasor amplitude estimation, considering test signals with τ raging from 0.5 to 5 cycles [4]. The proposed algorithm presents the best performance, leading to almost zero overshoot for a wide range of τ. The proposed algorithm presents the worst performance for the lowest values of τ due to the linear approximation used in (13). Even so, its maximum overshoot is 0.14% at τ equal to 0.5 cycles.

In order to evaluate the sensitivity to noise, a white random noise was added to the test signals. The ratio of the maximum noise to the amplitude of the fundamental frequency component was set to 2%. In Fig. 5(a) and 5(b) the estimated amplitude of the fundamental phasor are shown for the same test cases shown in Fig. 3(a) and 3(b), but with the white random noise included. The least sensitive is the proposed algorithm. The modified FCDFT and LES algorithms have the worst performances, presenting many spikes. It can be clarified analyzing the frequency responses of these algorithms shown in Figs. 2. The proposed algorithm presents the smallest side lobes, and the modified FCDFT and LES algorithms present the largest ones, leading to the largest errors.

4.2.2 *USING EMTP-GENERATED DATA*

The basic power system model shown in Fig. 6 is used to generate fault data in ATP (Alternative Transients Program) to test the performance of the algorithms on decaying DC component effect elimination. The sources and components subscripts “0” and “1” correspond to zero sequence and positive sequence values, respectively. There are two 230 kV ideal sources S1 and S2 and one transmission line 180 km long. The current transformer (CT)

(a)

(b)

Figure 7: Fundamental phasor amplitude: (a) phase C current for an BC fault (40 km far away from bus 1, incidence angle of 60^0 and fault resistance of 10 Ω); (b) phase B current for an BCG fault (140 km far away from bus 1, incidence angle of 90^0 and fault resistance of 30 Ω).

Simulation variables	Chosen Values
Fault location (km)	10, 20, 30, . . ., 150,160 and 170
Fault resistance (Ω)	Phase-Phase: 1, 10 and 20 Phase-Ground: 25, 35 and 45
Incidence angle (°)	15, 75 and 135
Source impedance (% of the nominal values)	Source S: 10, 100 and 1000 Source R: 10, 100 and 1000
Fault type	AG-BG-CG-AB-AC-BC ABG-ACG-BCG-ABC

Table 1: Simulation variables used to simulate faults.

shown in the figure is also included in the ATP simulations. Its model and parameters are those ones reported by the IEEE Power System Relay Committee [10].

In Figs. 7 the fundamental phasor amplitude for two different faults simulated in ATP are shown. In Fig. 7(a), it is shown the amplitude of the phase C current for a phase-to-phase fault between phases B and C (40 km far away from bus 1, with incidence angle of 60^0 and fault resistance of 10 Ω). In Fig. 7(b) is shown the amplitude of the phase B current for a phase-to-phase to ground fault between phases B and C (140 km far away from bus 1, with incidence angle of 90^0 and fault resistance of 30 Ω). One can see that the proposed algorithm presents for these cases the best performance on decaying DC component effect elimination, since it provides the smallest overshoot.

To verify the robustness of the proposed algorithm, extensive fault simulations under various fault conditions were carried out. The following simulation variables were taken into account: fault location, fault resistance, fault incidence angle (reference at the voltage in phase A of the source S1), source impedance (or source strength) and fault type. The chosen values for the variables used to generate the fault database are summarized in Tab. 1.

The combination of the variables shown in Tab. 1 results in a total of 13770 faults. A statistical analysis of the performances of the algorithms on decaying DC component elimination was carried out.

Figure 8: The cumulative frequency polygon plotting the percentage of faults against the maximum overshoot in current phasor amplitude estimation.

The performance of the algorithms can also be evaluated by the analysis of the cumulative frequency polygon shown in Fig. 8, which is a plot of the percentage of faults against the maximum overshoot in amplitude estimation of the current phasor. The proposed algorithm presents the best performance, providing the smallest overshoot in the most faults, followed by the modified FCDFT and the LES algorithm, respectively. For example, the proposed algorithm provides an overshoot equal to or smaller than 1,0% for about 82% of the faults, whereas for the modified FCDFT and the LES algorithm this percentage is about 48% and 30%, respectively.

5 CONCLUSIONS

This paper presents a novel AMF to remove the decaying DC component effect on phasor estimation. The key idea is to use an adaptive scheme to tune the digital mimic filter parameters to the time constant of the evaluated signal. The proposed phasor estimation algorithm combines the AMF with the FCDFT algorithm.

The performance of the proposed algorithm was compared with those of other algorithms reported in the literature on the subject. The sensitivity to the value of the decaying time constant τ was evaluated using simulated test signals. The obtained results indicates that the proposed algorithm presents the best performance, leading to almost zero overshoot for a wide range of τ.

A wide variety of ATP-simulated data was also used to analyze the performance of the algorithms. A statistical analysis was carried out. The obtained results indicate that the proposed algorithm leads to the smallest overshoots for most cases.

ACKNOWLEDGMENTS

The authors would like to thanks the Brazilian National Research Council (CNPq) for the financial support.

REFERENCES

[1] A. G. Phadke and J. S. Thorp, “Computer Relaying for Power Systems”. John Wiley & Sons Inc, 2nd ed., West Sussex, UK: 2009.

[2] E. O. Schweitzer III and D. Hou, “Filtering for Protective Relays”. 47th Anual Georgia Tech Protective Relaying Conference, Atlanta, Georgia, 1993.

[3] M. S. Sachdev and M. A. Baribeu “A New Algortihm for Digital Impedance Relays”. IEEE Transactions on Power Apparatus and Systems, PAS-98, no. 6, pp. 2232-2240, nov. 1979.

[4] G. Benmouyal “Removal of DC-Offset in Current Waveforms Using Digital Mimic Filtering”. IEEE Transaction on Power Delivery, vol. 10, no. 2, pp. 621630, apr. 1995.

[5] Adly A. Girgis “A New Kalman Filtering Based Digital Distance Relay”. IEEE Transactions on Power Apparatus and Systems, PAS-101, no. 9, pp. 3471-3480, sep. 1982.

[6] Yoon-Sung Cho and Chul-Kyun Lee and Gilsoo Jang and Heung Jae Lee “An Innovative Decaying DC Component Estimation Algorithm for Digital Relaying”. IEEE Transactions on Power Delivery, vol. 24, no. 1, pp. 73-78, jan. 2009.

[7] J.-C. Gu and S.-L Yu “Removal of DC Offset in Current and Voltage Signals Using a Novel Fourier Algorithm”. IEEE Trans. Power Deliv., vol. 15, no. 1, pp. 73-79, 2000.

[8] Yong Guo and Madlen Kezunovic and Deshu Chen “Simplified Algorithms for Removel of teh Effect of Exponentially Decaying DC-offset on the Fourier Algorithm”. IEEE Transactions on Power Deliv., vol. 18, no. 3, pp. 711-717, jul. 2003.

[9] Sang-Hee Kang and Dong-Gyu Lee and Soon-Ryul Nam and P. A. Crossley and Yong-Cheol Kang “Fourier Transform-Based Modified Phasor Estimation Method Immune to the Effect of the DC Offsets”. IEEE Transactions on Power Delivery, vol. 24, no. 3, pp. 1104-1111, jul. 2009.

[10] IEEE PES/PSRC Special Publication “EMTP Reference Models for Transmission Line Relay Testing”, 2004. Available: “http://www.pes-psrc.org/Reports/D10_Report_rev0.zip”.