

# DIMENSIONING OF RESERVE CAPACITY BY MEANS OF A MULTIDIMENSIONAL METHOD CONSIDERING UNCERTAINTIES

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**Abstract** – One of the most crucial constraints in operation of power systems is the balance between generation and load. Reserve capacity is held ready to be able to keep this balance also in case of the occurrence of unpredictable events like inevitable deviations of power injections from their predicted values. The amount of the required reserve capacity in a control area heavily depends on prediction quality what constitutes the need for high quality predictions.

In this paper, an extension to a convolution-based method is presented to examine the influence of prediction quality on the amount of reserve generation capacity required in a control area.

**Keywords:** Control reserve capacity, convolution-based method, multi-dimensional, prediction quality, renewable energies, parameter influence

## 1 INTRODUCTION

The balance between generation and load is needed to ensure a stable operation of electric power systems. Due to unpredictable events of power infeed, like power plant outages and forecast errors of regenerative energies, or inevitable deviation of the predicted load behaviour reserve capacity is needed and is held ready to keep this required balance.

In the ENTSO-E-region the reserve power is segmented temporally in primary, secondary and tertiary control [1]. Having concrete rules for the dimensioning of the primary control, several methods are proposed for the determination of the needed secondary and tertiary control reserve to face unpredictable events. Although deterministic methods are proposed, they are not up-to-date in opened electricity markets where transport system operators need a cost efficient and demand-oriented calculation of the reserve capacity. Another possibility is the utilisation of probabilistic methods. For the determination of the loss of load probability or the impact of wind in-feed on the generation capacity convolution-based methods are already used [2][3][4][5]. Today's methods are based on variational calculus with predefined parameter settings. The main disadvantage is the computational effort to consider every possible combination of the parameter sets. The values of the needed parameters like prediction errors differ strongly in literature. Nevertheless the control reserve is provided and bought accurately on a few megawatts.

This paper presents an extension of a convolution-based method that eliminates this disadvantage. With an interval-based description of the input parameters, their

influence on the required reserve capacity is traceable and permits the straightforward analysis of the result. Well-chosen graphic presentations demonstrate the benefit of improvements in regard to prediction technologies of the input parameters.

Preconditioning a probability distribution on the intervals of each parameter, the most probable value for the required reserve capacity can be determined. Because of today's size of the intervals, the maximum of the resulting distribution function is flat. On account of this, a resolution of more than 10 – 20 MW for calculating and providing reserve capacity, like it is often done today, is overkill.

Demonstrated on an exemplary electric power system, the proposed extension of the convolution based method clarifies and quantifies the need of high quality predictions for an accurate determination of the reserve capacity. This method points out the most influencing parameters. With the knowledge of the amelioration potential, a systematic improvement of prediction quality can be initialized to reduce the costs of reserve capacity.

## 2 DEFINITION OF INPUT PARAMETERS

The amount of the needed reserve capacity heavily depends on the prediction quality of the influencing parameters. The important input parameters for the determination of secondary and tertiary control reserve are shown in Table 1. Besides these, some parameters are also mentioned in the context of reserve capacity dimensioning, like steps in feed-in schedules of conventional power plants or rules for interchanges between control areas. They are however either control area specific or insufficiently researched and therefore not regarded in this paper.

| Type              | Input parameters   |
|-------------------|--|
| Secondary control | - Random load noise<br>- ¼ h power plant outages   |
| Tertiary control  | - Load prediction error<br>- Regenerative energies prediction error<br>- 1 h power plant outages |

**Table 1:** Important input parameters [6]

The input parameters in Table 1 need an analytic or discrete probabilistic density function description to use them for the probabilistic determination of the reserve capacity.

## 2.1 Load

The short-term prediction of load in a control area is done for the resource scheduling of power stations. Deviations of the real load from the predicted values imply the demands of reserve capacity. To minimize the required reserve capacity, high quality methods for the prediction models are needed which take into account load influencing parameters and the load structure of the regarded region. Due to complex influences, a minimal prediction error exists nevertheless of today about 1.5 - 5 % (e.g. [7],[8]) of the maximal load  $P_{Load}$ . The distribution of the load prediction error (LPE) is adequate to a Gaussian distribution (1) with the standard deviation  $\sigma_{LPE}$  and the expectation  $\mu_{LPE}$  presented in Table 2.

$$h_{LPE}(P) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{LPE}} \cdot e^{-\frac{(P-\mu_{LPE})^2}{2 \cdot \sigma_{LPE}^2}} \quad (1)$$

In addition to the load prediction error, the stochastic behaviour of loads generates random noise, which sweeps around the  $\frac{1}{4}$  h mean value. It has an impact on the amount of secondary control capacity. According to [6], the random load noise (LN) can also be described with a Gaussian distribution (2). The given values for the standard deviation  $\sigma_{LN}$  and the expectation  $\mu_{LN}$  in [6] are based on a 10 GW system. They need to be converted with the equations (3) and (4) for other power systems sizes.

$$h_{LN}(P) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{LN}} \cdot e^{-\frac{(P-\mu_{LN})^2}{2 \cdot \sigma_{LN}^2}} \quad (2)$$

$$\mu_{LN} = \mu_{LN,10GW} \cdot \frac{P_{Load}}{P_{Load,10GW}} \quad (3)$$

$$\sigma_{LN} = \sigma_{LN,10GW} \cdot \sqrt{\frac{P_{Load}}{P_{Load,10GW}}} \quad (4)$$

## 2.2 Regenerative energy sources

### 2.2.1 Wind

The wind energy has a rising part in the European production of electric energy. Due to the volatility of wind and the necessity of a reliable power grid, the prediction of wind energy becomes more and more essential for the transmission system operators. Based on numerical weather prediction, the wind power prediction tool determines a feed-in forecast of the wind turbines of a control area. Because of natural imprecision in the weather prediction and approximation in the up-scaling in the feed-in forecast process, a prediction error of few per cent is induced. The probability density of this prediction error (WPE) is a normal distribution (5), according to [6] and [9], but any other distribution functions are also possible. The range of the standard deviation  $\sigma_{WPE}$  and the expectation  $\mu_{WPE}$  in dependency of  $P_{Wind}$  is shown in Table 2.

$$h_{WPE}(P) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{WPE}} \cdot e^{-\frac{(P-\mu_{WPE})^2}{2 \cdot \sigma_{WPE}^2}} \quad (5)$$

### 2.2.2 Photovoltaic

The feed-in of photovoltaic resembles the behaviour of wind turbines. As a result of the high installation rates of the last years (within Germany 5.250 MW of new photovoltaic in the period 01.2010-09.2010 [10]) a prediction of the solar energy seems expedient. The prediction models are quite similar to the wind energy prediction tools. In contrast, only few studies of the prediction quality are published until today, so the effect of the solar energy on the dimensioning of the reserve capacity is not regarded in this paper.

| Parameter             | Standard deviation                    | Expectation                             |
|-----------------------|---------------------------------------|---|
| Load prediction error | $\sigma_{LPE} = 1.5 - 5 \% P_{Load}$  | $\mu_{LPE} = 0 \text{ MW}$              |
| Load noise            | $\sigma_{LN} = 0.5 - 1.5 \% P_{Load}$ | $\mu_{LN} = 0 -  1.4\% \cdot P_{Load} $ |
| Wind prediction error | $\sigma_{WPE} = 4.0 - 12 \% P_{Wind}$ | $\mu_{LN} = (-0.7) - 0\% P_{Wind}$      |

**Table 2:** Parameter intervals

### 2.3 Power plant outages

Regarding the conventional generation, unpredictable outages of power plants require reserve capacity. This behaviour is describable with a two state Markov process. The probability of an outage of one power plant in  $\frac{1}{4}$  h (7) or 1 h (8) is described through its mean service term  $T_B$  and mean outage term  $T_A$ [2]. Exemplary values for these terms are given in Table 3. Equation (9) determines the probability of no outage of the power of the regarded power plant. For all power plants, the resulting probability function of power deficits is calculated with (10). An extensive description is given in [6].

$$W_{1/4h}(P_i) = \frac{\frac{1}{4}h}{(T_B + T_A)} \quad (7)$$

$$W_{1h}(P_i) = \frac{1h}{(T_B + T_A)} \quad (8)$$

$$\overline{W}(P_i) = 1 - W(P_i) \quad (9)$$

$$W(P_D = \sum_{i=1}^k P_i) = W_0 \prod_{i=1}^k \frac{W(P_i)}{\overline{W}(P_i)} \quad (10)$$

| Type    | Mean service term<br>$T_B$ | Mean outage term<br>$T_A$ |
|---------|----------------------------|---------------------------|
| Nuclear | 1507 h                     | 63 h                      |
| Lignite | 494 h                      | 30 h                      |
| Coal    | 402 h                      | 30 h                      |
| Gas     | 490 h                      | 52 h                      |

**Table 3:** Characteristics of power plants [6]

## 2.4 Assumed parameters

For the demonstration of the presented extended convolution-based method, a 10-GW-system is regarded. The highest load is therefore 10 GW. The installed wind power is about 3 GW. Table 4 shows the assumed conventional power plant fleet. The intervals for the varied input parameters are defined with their lower limit  $\sigma_{LL}$  and their upper limit  $\sigma_{UL}$  in Table 5. A standard value for the deviation is also given ( $\sigma_{std}$ ). The expectation is not varied in this example.

| Type              | Quantity | Total power |
|-------------------|----------|-------------|
| Nuclear at 800 MW | 4        | 3200 MW     |
| Lignite at 500 MW | 4        | 2000 MW     |
| Lignite at 350 MW | 4        | 1400 MW     |
| Coal at 600 MW    | 4        | 2400 MW     |
| Gas at 250 MW     | 4        | 1000 MW     |
| $\Sigma$          | 20       | 10000 MW    |

**Table 4:** Power plant fleet of exemplary 10-GW-system

|     | $\mu$  | $\sigma_{LL}$ | $\sigma_{std}$ | $\sigma_{UL}$ |
|-----|--------|---------------|----------------|---------------|
| LN  | 140 MW | 0.5%          | 1.2%           | 1.5%          |
| LPE | 0 MW   | 1.5%          | 2.5%           | 5.0%          |
| WPE | 0 MW   | 4.0%          | 6.0%           | 12.0%         |

**Table 5:** Input parameters of exemplary 10-GW-system

## 3 DETERMINATION OF RESERVE CAPACITY

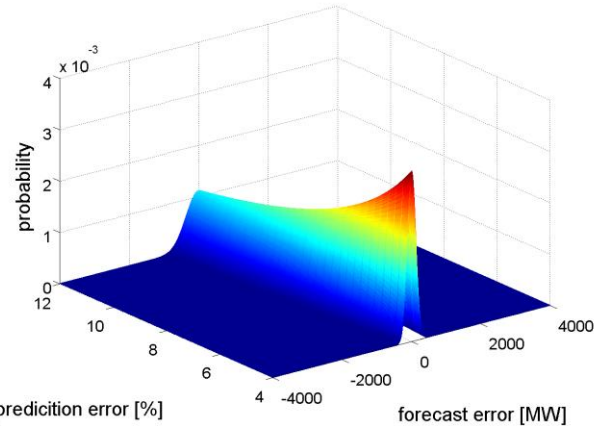
The convolution-based method for the determination is based on probabilistic with a risk-based approach. The dimensioning of the required reserve capacity is done with a certain supposed probability of a deficit. This implies for an assumed deficit probability of 0.1% that the calculated reserve capacity is insufficient in about 9 h a year. The advantages of the convolution-based method are the calculation speed and the uncomplicated mathematical description of the influences and interrelationships. In addition to this, the modelling of the input parameters in intervals is possible without prejudices.

### 3.1 Algorithm

Assuming stochastic independency among the input parameters, the resulting probability distribution can be determined by convolution.

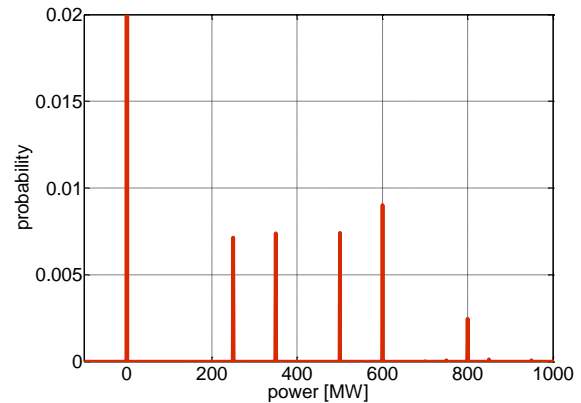
#### 3.1.1 Preparation of the input parameters

The probability distributions of the input parameters are two-dimensional, taking into account the given intervals of their standard deviations of Table 2. Figure 1 represents the two-dimensional probability distribution of the wind prediction error with the variation of the standard deviation.



**Figure 1:** Probability distribution of the wind prediction error

This step is done for every input parameter, except for the power plant outages. The one-dimensional distribution of power losses due to the outages is presented in Figure 2. The probability for the outage of 0 MW, i.e. operation without any disturbance, is 0.966. After this, for every power value of one power plant or the combination of several, a discrete probability of an outage of this specific power is given.



**Figure 2:** Probability of power loss due to power plant outages

For the analysis of the influences of the input parameters on the required reserve capacity their probability distributions need to be n-dimensional. Ensuring the result analysis in consideration of only one input parameter, a new dimension is needed for every input parameter variation. For the exemplary variation of the wind prediction error, the load prediction error and the load noise, four dimensions (the three varied input parameters plus the power dimension) are necessary. The matrices of the input parameters are therefore transformed into 4-dimensional matrices.

#### 3.1.2 Convolution instructions

In respect to Table 1 the probability distribution of the required negative secondary reserve capacity  $h_{SR-}$  is calculated with (11). The positive secondary reserve capacity  $h_{SR+}$  (12) takes additionally into account the probability of the power plant outage in a  $\frac{1}{4}$  h ( $h_{KW4}$ ). The positive and the negative secondary reserve capacity are two-dimensional probability distributions.

$$h_{SR-} = h_{LN} \quad (11)$$

$$h_{SR+} = h_{LN} * h_{KW4} \quad (12)$$

The negative total reserve capacity  $h_{TRC-}$  is determined with equation (13). In addition to this, the power plant outage probability of 1 h ( $h_{KW}$ ) is needed for the positive total reserve capacity  $h_{TRC+}$  (14). The resulting probability distribution for the positive and negative total reserve capacity are four-dimensional.

$$h_{TRC-} = h_{LN} * h_{LPE} * h_{WPE} \quad (13)$$

$$h_{TRC+} = h_{LN} * h_{LPE} * h_{WPE} * h_{KW} \quad (14)$$

### 3.1.3 Determination of the deficit probability

The distribution function of the four types of reserve capacity is calculated next. For this, the probability density functions are integrated, as shown in (15).

$$H(P) = \int_{-\infty}^P h(p) dp \quad (15)$$

The last step towards the determination of the deficit probability is done with equation (16).

$$D(P) = 1 - H(P) \quad (16)$$

The result of this function is the probability of a deficit in dependency of the provided power of reserve capacity. For a chosen acceptable deficit probability the required reserve capacity can be determined now.

### 3.1.4 Determination of the tertiary control reserve

Finally, the tertiary control reserve capacity (TR) is the subtraction of the secondary control reserve capacity (SR) from the total reserve capacity (TRC) in the equations (17) and (18).

$$(TR -) = (TRC -) - (SR -) \quad (17)$$

$$(TR +) = (TRC +) - (SR) \quad (18)$$

## 3.2 Presentation of the results

The results of the convolutions are in general n-dimensional matrices, depending on the number of varied input parameters. Even for the presented example, the 4-dimensional matrices of the deficit probability cannot be shown graphically. On account of this, two ways of illustration are implemented.

### 3.2.1 Classic illustration

This illustration is already established for the classic determination of reserve capacity in [6]. The variation of only one input parameter is used in this version. The other input parameters are set on their standard values (see Table 5). The resulting matrix is reduced to two dimensions and the diagram is shown in Figure 1.

Due to the visibility in the diagram, not every row of the matrices is printed. The important curves depending on the upper and lower limit as well as the standard

value are printed. The intersection points of these curves with the tolerance limit  $D_{TL}$  of 0.1% result in the required reserve capacity.

This version permits an uncomplicated analysis of the results, because the required reserve capacity can directly be read off along the tolerance limit. The resulting range of fluctuation is shown with the curves of the lower and upper limit of the input parameter interval. Regarding only one of the varied parameters is the main disadvantage of this illustration version.

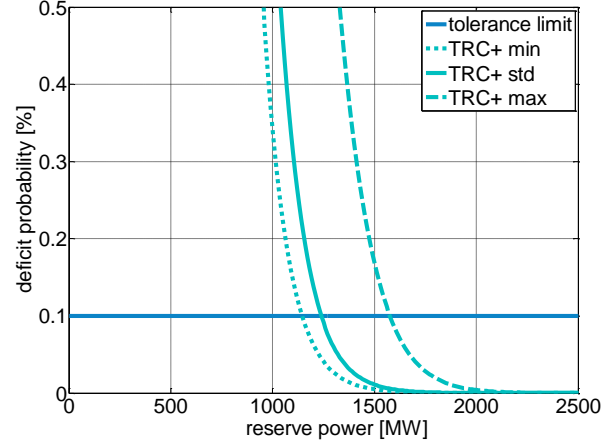


Figure 3: Results with the variation of WPE

### 3.2.2 Illustration for an influence analysis

The assumed tolerance limit of deficit probability is fixed on an arguable percentage, normally 0.1%. So that the main interest of analysis is along the tolerance limit in the classic illustration version. In respect to this fact, the second illustration version demonstrates the behaviour of the required reserve capacity in dependency of two varied input parameters and a fixed tolerance limit of 0.1%.

The remaining input parameter is fixed to its standard value thus the regarded matrix is cut down on three dimensions. For each combination of the two varied parameters the first power value is searched, that has a deficit probability of less than 0.1%. Figure 4 shows the resulting diagram for the required reserve capacity in variation of the  $\sigma_{LPE}$  and  $\sigma_{WPE}$ .

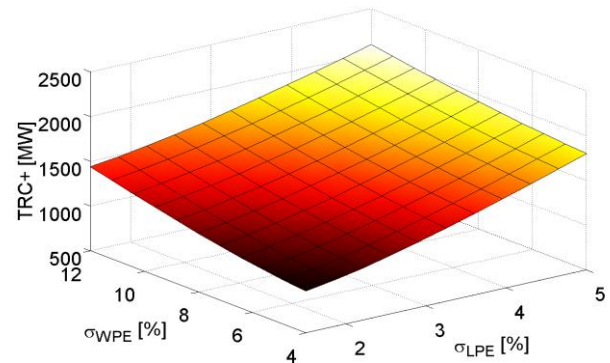


Figure 4: Required TRC+ with variation of  $\sigma_{LPE}$  and  $\sigma_{WPE}$

The second illustration version is suitable for the determination of the curves characteristics between the lower and upper limits of the both varied input parameters. With this given knowledge, the estimation of possible savings due to improved prediction methods is permitted. Additionally, the inclination of the area along the two input parameter axes demonstrates, whose prediction method improvement is more efficient regarding the required reserve capacity.

The reading of the exact values for a certain parameter configuration is complicated in this illustration version. Therefore, the both versions complement one another.

#### 4 DEVELOPMENT OF CONFIDENCE INTERVALS

The extended convolution-based method demonstrates, that even little variations of the prediction errors change the amount of required reserve capacity significantly. The provision of reserve capacity with a resolution of 1 MW seems to be questionable.

These uncertainties result from the dispersion of the values of the prediction errors. The determination of a certain value of the reserve capacity, that fulfils the tolerance limit, is impossible. Considering this fact, probability density functions are implemented on the input parameter intervals for the identification of confidence intervals. They qualify for the determination of the required reserve capacity rather than calculating on single value, having in mind the input parameter uncertainties.

##### 4.1 Assumptions

The distributions along the intervals of the standard deviation of the input parameters need to be defined on a closed interval, for what three distributions exist in stochastic:

- Rectangular distribution
- Triangular distribution
- Beta distribution

Assuming a little probability of the standard deviation of the prediction errors at the edges of the intervals, rectangular distributions are inappropriate. In comparison to the triangular distribution, the beta distribution provides more possibilities of modelling the distribution.

##### 4.2 Parameterization of the beta distribution

The probability density function of a beta distribution  $\beta(x)$  in an interval  $[c, d]$  is described in equation (19) with  $B(a, b)$  being the beta function (20).

$$\beta(x) = \begin{cases} \frac{(x-c)^{a-1}(d-x)^{b-1}}{B(a,b)(d-c)^{a+b-1}}; x \in [c, d] \\ 0; \text{else} \end{cases} \quad (19)$$

$$B(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx; a, b > 0 \quad (20)$$

The equations for the variance  $Var(x)$  (21) and the expectation  $E(x)$  (22) are converted for the calculation of the two missing parameters  $a$  and  $b$ .

$$Var(x) = \frac{(d-c)^2 ab}{(a+b)^2(a+b+1)} \quad (21)$$

$$E(x) = c + (d-c) \frac{a}{a+b} \quad (22)$$

For the determination of  $a$  and  $b$  assumptions for the variance and the expectation need to be done. They are presented in Table 6. The interval  $[c, d]$  is defined with the lower and upper limits  $\sigma_{LL}$  and  $\sigma_{UL}$  for the standard deviation of the prediction errors.

|     | $c$<br>[%] | $d$<br>[%] | $E(x)$<br>[%] | $Var(x)$<br>[%] |
|-----|------------|------------|---------------|-----------------|
| LN  | 0.5        | 1.5        | 1.2           | 0.01            |
| LPE | 1.5        | 5.0        | 2.5           | 0.1             |
| WPE | 4.0        | 12.0       | 6.0           | 0.1             |

Table 6: Parameters of the beta distributions

Figure 5 shows for example the resulting beta distribution for the interval of the standard distribution of the load prediction error. The vertical lines represent the confidence intervals of 50%, 95% and 99%.

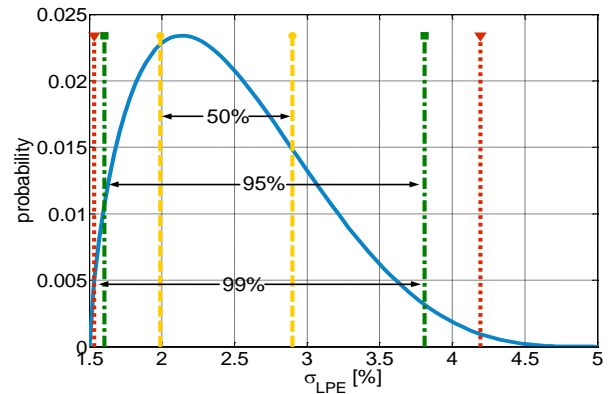


Figure 5: Beta distribution of  $\sigma_{LPE}$

##### 4.3 Resulting distribution

The n-dimensional matrix of the deficit probability is the starting point for the determination of a resulting distribution. For every power value  $P_x$  in the matrix every combination of the standard deviations  $\sigma_{LN}$ ,  $\sigma_{LPE}$ ,  $\sigma_{WPE}$  is detected, where the deficit probability is fallen below the tolerance limit  $D_{TL}$  of 0.1%. For this combination, the probabilities in the specific beta distributions are taken and multiplied (23). The integral of every possible, the tolerance limit respecting, combination is the resulting probability  $W$  for  $P_x$  (24).

$$\beta_{res} = \prod_n \beta_i(\sigma_{i,x}) \quad (23)$$

$$W(P = P_x, D = D_{TL}) = \int_{-\infty}^{\infty} \beta_{res} dx \quad (24)$$

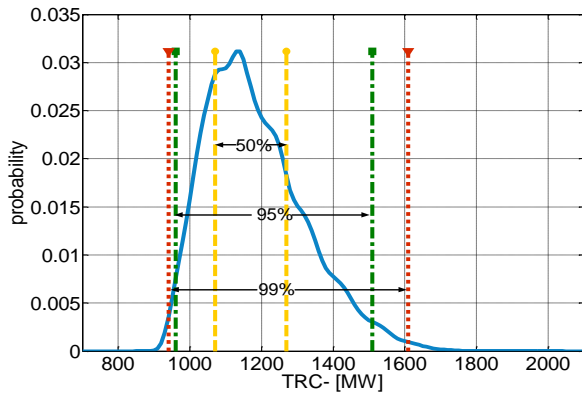


Figure 6: Probability density function for TRC-

The resulting probability density  $W$  is printed in Figure 6. The vertical lines are once again the confidence intervals of 50%, 95% and 99%. This diagram allows the determination of the most probable required amount of reserve capacity having in mind a given tolerance limit.

Even the 50%-confidence interval has a width of about 100 MW in a 10-GW-system. This diagram clarifies the need of better prediction tools with less variability in the prediction errors to reduce the width of the presented probability curve.

## 5 CONCLUSION

The paper presents an amelioration of the convolution-based method for the determination of the required control reserve capacities in a control area. Due to the great variations of the input parameters, the comparability among different studies is difficult. Instead of using variational calculus for the examination of influences, the presented method allows the utilization of intervals for the uncertainties of the input parameters. So that the effect of can directly be studied after one fast calculation. The utilization of the convolution allows the usage of almost every probability density function – even if new research on the prediction error evokes new distributions, the tool is still usable.

The chosen diagrams permit the extensive analysis of the needed reserve capacity. Especially the comparison of the influences of two parameters on the amount of reserve capacity identifies the benefit of improved prediction tools.

Due to the implementation of a distribution on the prediction error intervals, this tool is able to give a confidence interval of the calculated needed reserve capacity. The further lack of knowledge, whether the determined value is reliable is solved with the presented approach. It demonstrates additionally that a resolution of 10-20 MW is suitable in the acquisition of reserve capacity of the transport system operators, because of the relatively wide confidence intervals.

The paper demonstrates, that the prediction quality is really influential on the amount of reserve capacity. Only with a high quality improvement, especially in

regard on the renewable energies, the required amount of control reserve capacity is reducible.

A little limitation is the static power plant fleet behaviour. Improvements on the modelling of the power plant fleet are already initialized. Additionally research on the insufficiently described influencing parameters, like steps in net schedules, is needed to improve the prediction. However, the implementation in the presented calculation tool is without any difficulty.

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