Measurement Gross Error Composition, Considering the Residual Vector Correlation for Power System State Estimation

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***Abstract* – This paper focus on a topological and geometrical based approach to recover the measurement masked errors in power system state estimation. An approach to compose the measurement error *(CME)* is then proposed. It is shown that, using the measurement residual in gross error (GE) detection test, instead of a hyper sphere to guarantee if the measurement set has error or not a hyper ellipsoid is the one to be used. Indeed it is shown also that instead of GE detection it is detected if there are measurements with residuals equal or superior to a chosen value. Then the ellipsoid radii in each measurement direction are calculated. Those radii are, for each measurement, the threshold value to declare a measurement as suspicious of having error. With those suspicious measurements in hands, the filtering index *(FI)* is used in order to identify which of the measurements in fact have residual superior to the chosen threshold value. Then that measurement is corrected using the Composed Normalized Error (*CNE)*, and another state estimation is performed.**

Also the gross error detection test is made using the *J(x)* index, with the *CMEs.* Then, the normalized form of the CME is obtained, in the measurement error space. They are not correlated to each other, and in this case, a hyper sphere is used to identify the measurements containing errors.

To test the gross error detection, identification, and correction efficiency, the classical three bus system [4,5] available in many papers of this field is used.

***Index Terms:* State Estimation, Orthogonal Projections, Gross Errors Analysis, Recovering Errors, Innovation Information, Filtering Index.**

I. INTRODUCTION

Based on a geometric interpretation of the residual estimation, a method for the detection and identification of gross errors has been developed in [9]. The authors discuss that their method is able to determine if the residual vector lies in a subspace determined by the suspect measurements (suspect in terms of having gross errors) and if any part of that subspace is orthogonal to the residual vector. They also discuss that their proposition is able to find the suspect measurements of the measurement set.

Recently, Bretas at al [11-15] have developed, a family of papers related to Power Systems State Estimation using of geometrical properties. In those papers it was shown that the proposals which use of the measurements residual as a measure of the measurement gross errors will fail quite frequently. It is shown that part of the measurements errors are in the null space related to the projection matrix of the measurements residual. That leads the state estimation process masking part of the measurement error.

In this paper, using of topological and geometric approaches as in [11-15], methodologies to tackle gross errors in power system state estimation are provided. This is achieved by decomposing the measurement error of every measurement of the measurement set into two components: the first component orthogonal to the Jacobean range space whose amplitude is equal to standard measurement residual and the other component contained in that space, which does not contribute to the measurement residual. The ratio between the norms of those quantities, the Innovation Index (*II*), which provides the new information that measurement contains related to the others measurements of the measurement set. Moreover, based on the *II* index, a technique to compute the measurement error not reflected in the measurement residual, the masked error, is proposed. Then each measurement error is composed (CME). It is shown that using the *J(x)* index, with the measurement residuals, the current gross error detection test guarantee, with a certain uncertainty, that at least one of the measurements of the measurement set have residual equal or superior to a threshold value. Also it is shown that instead of a hyper sphere to detect if the measurement set has “error” or not, with a certain uncertainty, in the measurement error space it should be a hyper ellipsoid. And, finally, using the measurement residuals the radii of that hyper ellipsoid, in each measurement direction, are calculated. In normalized form, they will be the threshold values in order to declare the measurements containing residuals equal or superior to a threshold value with a predefined degree of confidence.

The classical three bus system avalable in many papers of this research field [4,5] will be used to test the proposed solution accuracy.

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II. Demonstration to Show that the Measurements Residuals are not Appropriate, in a Direct Way, as Measure of the Measurements Errors

The ith-measurement CME_i will be given by:

$$CME_i^{\,2} = (1+1/II_i^{\,2})*\left\|e_{miR(H)^{\perp}}\right\|^2 = (1+1/II_i^{\,2})*r_i^{\,2} \qquad (1)$$

with the ith-measurement Innovation Index [14] being given by:

$$II_i = \left\|e_{iR(H)^{\perp}}\right\|_W / \left\|e_{iR(H)}\right\|_W \qquad (2)$$

As can be seen, the measurement CME_i magnitude, when compared to the r_i magnitude, may be significantly different from each other, according to the measurement II_i.
Below it is presented the definition and interpretation of the Innovation Index as presented in [14]

Definition- Given an observable measurement set for state estimation purposes, the innovation of a measurement [14], may be defined as the new information it contains, related to the other measurements of the measurement set.
The previous definition suggests that the measurement innovation is the part of the measurement that is independent of the other measurements of the measurement set. As a consequence, the new information contained in a measurement, for state estimation purposes, is the part of the measurement which is orthogonal to the Jacobian range space

III. Demonstration to Show that the a Hyper Sphere in the Residual Space is not Appropriate to Identify Measurements Containing Gross Errors

Assume two variables X_1 and X_2 and that they are normally bivariate distributed with mean vector components μ_1 and μ_2, a correlation ρ, and variance-covariance matrix taking the form as shown below.

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^{\,2} & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^{\,2} \end{pmatrix}\right] \qquad (3)$$

Taking the joint probability density function of (X_1, X_2) for the multivariate normal distribution one will get:

$$X \sim N(\mu, \Sigma)$$

$$f(X) = \frac{1}{\sqrt{(2\pi)^p|\Sigma|}} e^{\left\{-\frac{1}{2}(X-\mu)^T\Sigma^{-1}(X-\mu)\right\}} \qquad (4)$$

With

$$p = 2 \quad \Rightarrow \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

In graphic form this distribution will be, in terms of the X_1 and X_2 variables shown in *Fig. 1*.
As can be seen in that *Fig. 2,* the length of the line segment starting on a fixed ellipsoid center until the interception of the X_1 and X_2 variables will change according to the correlation that may exist between those variables. Otherwise, the length of that line segment is the threshold value to declare a measurement as being suspicious of having gross error. However when the variables X_1 and X_2 have the same σ, but with $\rho = 0$, that graphic shape will be, as in *Fig.3*. In this case the measurements gross errors identification may be performed through a hyper sphere in the $X_1 x X_2$ plane.

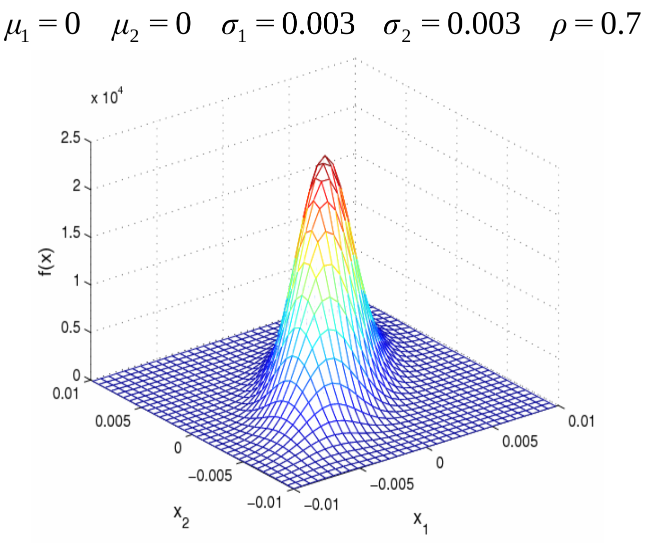

Fig. 1: f(x) with $\mu_1 = \mu_2 = 0, \sigma_1 = \sigma_2 = 0.003, \rho = 0.7$

If a cut in the X_1 X_2 plane is made it will give:

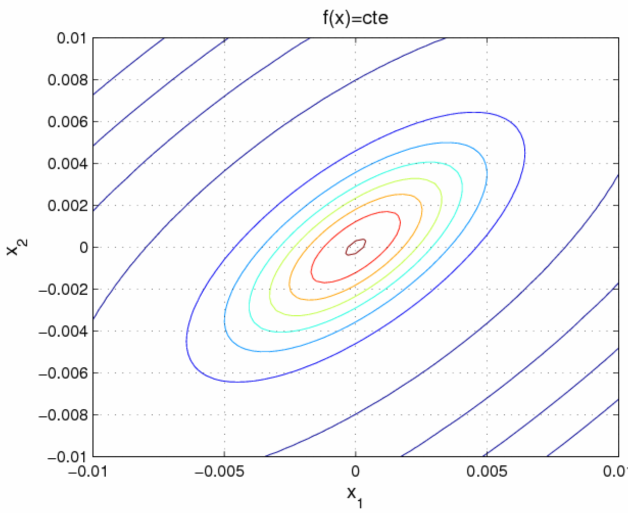

Fig. 2: Cut of f(x) in the $X_1 x X_2$ plane of Fig. 1.

Based on the previous statements, two solutions for the measurements gross errors identification may be proposed:

(i)To calculate the hyper ellipsoid ray in each measurement direction, *Fig. 2*, for a predefined value for the measurement grosses errors;

(ii) To make a mapping from the measurement residual space, to the measurement error space where it does not exist any correlation between the measurements errors and then applying the measurements gross errors identification using the measurements CME^N. These approaches will be presented in this paper. Before that, however, it is required to develop the statistical properties of the variables that appear in this process.

$\mu_1 = 0 \quad \mu_2 = 0 \quad \sigma_1 = 0.003 \quad \sigma_2 = 0.003 \quad \rho = 0.0$

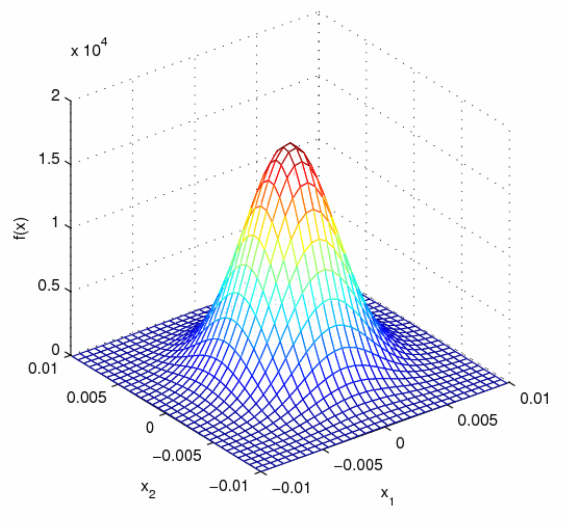

Fig. 3: f(x) with $\mu_1 = \mu_2 = 0, \sigma_1 = \sigma_2 = 0.003, \rho = 0.0$

If a cut in the X_1xX_2 plane, in Fig. 3, is made it will give:

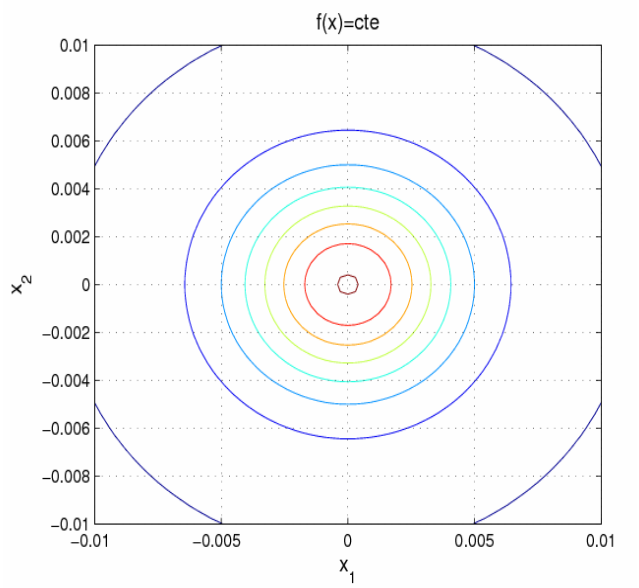

Fig. 4: Cut of f(x) in the X_1x X_2 planes for Fig. 3.

IV. Statistical Properties of the Composed Measurements Errors

Recently, it was developed, a family of papers related to Power Systems State Estimation using geometrical background [11-15], where it was shown that the proposals which use of the measurements residual as a metric for the measurement gross error will fail. It was demonstrated that the ith-measurement CME_i is described by:

$$CME_i^2 = (1 + 1/II_i^2) * \left\| e_{miR(H)^\perp} \right\|^2 = (1 + 1/II_i^2) * r_i^2 \qquad (5)$$

with the ith-measurement Innovation Index given by:

$$II_i = \left\| e_{iR(H)^\perp} \right\|_W / \left\| e_{iR(H)} \right\|_W \qquad (6)$$

Then:

$CME_i = \sqrt{(1 + 1/II_i^2)} * r_i$ and $CME_i^N = CME_i / \sigma_i$

with σ_i the ith measurement standard deviation. The normalization of the CME it is easily obtained using the measurement residual correlation and from the equation for CME and the property of finding the covariance of a function that is equal to a constant times another function of known covariance.

Also it can be calculated the measurement CNE, that is:

$CNE_i = \sqrt{(1 + 1/II_i^2)} * r_i^N$.

V. Proposed Solutions for the Measurement Gross Errors Detection and Identification

For the measurement gross errors detection, in this paper it is proposed the same classical gross error detection methodology but using the measurements CMEs with *m* degrees of freedom being *m* the measurements number. For more details see Appendix below.

For the measurements gross errors identification, two methodologies are proposed:

(i) Gross Errors identification through the computation of the ellipsoid ray in each measurement direction, as in Fig. 2. This proposition can be placed as:

Which would be, for each measurement of the measurement set, the minimum residual it should contain so that one can accept, within a predefined uncertainty, that the measurement normalized residual will be at least a predefined value k?

Proposition: That measurement residual Threshold Value to declare a measurement as having, normalized residual, with predefined value of *k, with* a chosen uncertainty, is given by:

$$TV_i = k * \sqrt{(1 + 1/II_i^2)}. \qquad (6)$$

The correction being made in k is due to the existing measurement residual correlation.

Proof:

When the measurement gross error detection and identification, using the measurements residual, is applied, although not stated, the assumption is that no correlation between them exist. Then the factor $\sqrt{(1 + 1/II_i^2)}$ is nothing more than the correction due to that effect. For example, in case a measurement contains infinite innovation $(II_i \to \infty)$ or, equivalently, being orthogonal to the Jacobian range space, the threshold value would be the proper *k*. In the case of gross error analysis, the correction, in the residual it is equivalent to compute the ellipsoid ray in all the measurements directions as shown in Fig. 2. One should be aware that in this case the problem that is being solved is that one in which the normalized residual is equal or superior to k, but not the measurement error.

- q.e.d.

(ii) Gross Error identification through the Composed Measurement Error, CME.

PROPOSITION: In this case the proposed solution is quite obvious, that is, it is required to normalize the measurements CME and directly retain the measurements which have CME_i^N equal to or superior to a chosen predefined value. Those measurements will be the ones suspicious of having gross errors. This solution corresponds to the Fig. 4 situation.

PROOF:

Compute:

$$CME_i^{\ N} = CME_i / \sigma_i \qquad (7)$$

The suspicious measurements will be the ones with the Measurement Composed Normalized Error superior to the chosen constant *k,* previously defined in the detection test. That is the case of Fig. 4 above, where no correlation among the measurement errors does exist. Or in other words, since the measurements errors are orthogonal to each other, they attend all the requirements of the classical Hypothesis Test Identification (HTI).

- q.e.d.

In what follow a small example will be presented in order to better explain a paper propositions in gross errors detection and identification but using previous proposed solution (i).

EXAMPLE: The measurement values used in this example were obtained from a load flow solution (z^{lf}) to which normally distributed noises will be added. The measurements noise was assumed to have zero mean and a standard deviation σ given by: $\sigma = \dfrac{pr * |z^{lf}|}{3}$; with *pr,* the meter precision, equal to 3%.

Using the measurement set as indicated in Fig.5, with values as in Table I, for each of those measurements, once at a time, error is increased at steps of 0.1 until the gross error is flagged using the confidence degree of 95% that at least one measurement has its r^N equal or superior to three has occurred. For each of those measurements it is also computed: the measurement CNE_i, and the TV_i respectively. Also the added error at that time is available at that table.

In what follows the corresponding sequence of computation for the active measurement I(A):1 will be described (The other cases will be just repetition).

After performing the state estimation analysis, the r_1^N (3.04) for that measurement is obtained; the II_1=1.03 as well as the CNE_1=4.34 is also calculated then the TV_1=4.18 is obtained.

In all the cases the measurement with gross error was correctly identified. It should be observed that in Table I the added gross error is a little bit different from the corresponding CNE_i, the reason being that random errors has been previously added to the measurements After correcting that measurement identified as the one containing error, and performing another state estimation, no gross error has been flagged anymore.

One should see that as much innovation a measurement has, closer it will be the measurements quantities r_i^N, CNE_i, TV, as well as the added error.

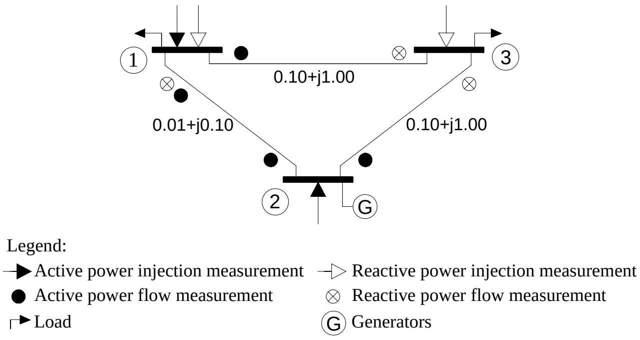

Fig. 5: The Three-bus-system

TABLE I
RESULTS CORRESPONDING TO THE THREE -BUS SYSTEM: FIG. 5

Meas.	II	r^N Det. level	Added Error	CNE	TV (σ)
I(A):1 -1.6234	1.03	3.04	3.80	4.24	4.18
I(A):2 2.5962	1.88	3.04	3.90	3.44	3.40
F(A):2-1 2.0332	1.58	3.02	4.20	3.58	3.39
F(A):1-3 0.3661	0.76	3.06	2.80	5.07	4.97
F(A):2-3 0.5629	1.71	3.06	3.10	3.54	3.47
I(R):1 0.3726	0.70	3.01	4.40	5.22	5.20
I(R):3 0.4047	1.49	3. 02	3.20	3.64	3.61
F(R):1-2 0.3743	0.72	3.04	5.20	5.21	5.14
F(R):3-1 0.1446	0.97	3.03	3.90	4.35	4.31
F(R):3-2 0.2583	1.47	3.07	3.60	3.71	3.62

From the Table I one should observe: (i) In the added error column it is not included the measurement random error; (ii).Although the residual detection test worked well, column of the $r^N s$, the measurement error at that time is quite different from the r^N as shown at the column of the CNE; (iii) Also, the measurements threshold values are different from the corresponding $r^N s$, but very close to the *CNEs*; (iv) As can be seen at column of the $r^{N,}$ in all the cases at least one measurement had its residual superior to three sigma. That confirms that the test is for the measurements residual and not for the errors.

With the increase in the information degree, all those quantities will be closer and closer to each other.

REMARK: observe the great advantage of our proposition when compared to the available classical state estimation, in terms of correcting the measurement with gross error instead of eliminating that measurement.

With that correction no risk of turning the measurement set unobservable will exist any time.

VI. CONCLUSIONS

In this paper, using topological and geometrical approaches, as previously proposed in [11-15], methodologies to deal with measurements gross errors detection, identification and composition using of the fact that the measurement residuals are correlated have been proposed. A three-bus system was used as an example in order to show in steps the computation procedure.

The paper results have shown that the same threshold value for the measurement residuals to declare them as containing error is not correct, as it is made in the standard literature of state estimation. On the contrary that threshold value is a specific value for each measurement and function of the measurement topology and the quantity of new information that measurement contains.

The paper results also indicate that in order to have an adequate detection, identification, and composition of measurements gross errors, the proposed *FI* is required as well as the composed measurement error must be used in the tests.

As result of this research one comes out with the conclusion that a minimum redundancy level for the measurement set is required in order to have a reliable state estimation process. Even more one can design such measurement set so that a minimum threshold values for the measurements errors can be detected and identified.

Appendix: Detection and Identification of Gross Errors

Given a measurement vector *z*, the estimated state $\hat{x}$ will depend on the projection operator *P*, which depends only on the inner product choice *(W)* and matrix *H*.

Matrix "*W*" is given by the inverse of the measurement covariance matrix. After finding $\hat{x}$, it is interesting to verify the existence of gross errors in the measurements. Therefore a routine for error detection is required. At this point only statistic concepts are needed.

Assuming that measurement errors have a normal distribution, it is easy to show that index $J(\hat{x})$, i.e., the function to be minimized in (7), has a Chi-square distribution (χ^2) with *(m-n)* degrees of freedom. Choosing a probability "1-α" of false alarm and being "α" the significance level of the test, a number "*C*" is obtained (via Chi-square distribution table for $\chi^2_{m-n,1-\alpha}$) such that, in the presence of gross errors $J(\hat{x}) > C$.

Another way to detect the presence of gross errors is by the largest normalized residuals test. Based on the same assumption about measurement errors, the vector of residuals *r* is normalized and subjected to a validation test: $r(k)^N = |r(k)| / \sigma_r(k) \leq \lambda$ (threshold value), where $r(k)^N$ is the largest among all $r(i)^N, i = 1,\ldots,m$; $\sigma_r(k) = \sqrt{\Omega(k,k)}$ is the Standard Deviation vector of the k^{th} component of the residuals, and Ω is the residual covariance matrix given by $\Omega = (I - H(H^T H)^{-1} H^T.R$, that is $\Omega = W^{-1} - H(H^T W H)^{-1} H^T$. If $r(k)^N > \lambda$, the measurement with gross error is detected and the k^{th} measurement will be the one with gross error (usually $\lambda = 3$ [14]).

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