MAXIMIZATION OF TOTAL PV OUTPUTS FOR DISTRIBUTION NETWORK TOPOLOGY WITH A NOVEL SEARCH METHOD

Tsunaki Sato and Hiroumi Saitoh
Tohoku University
Miyagi, Japan
epsys15@ec.ecei.tohoku.ac.jp, saitoh@ecei.tohoku.ac.jp

Abstract –This paper proposes a novel method to search the optimal topology of radial distribution network which maximizes total output power of photovoltaic generators (PV) on the network and keeps the network voltage within a regulated range. The method is based on a correlation between total PV output and the largest difference in the voltage profile and the principle of superposition for the radial network. The optimal topology is found out so as to form a tree by extending distribution feeders from a distribution substation. In order to verify the effectiveness of the method, a comparison study with tabu search approach was carried out by using a multi loop distribution network model. The result shows that the method can find a suboptimal topology faster than tabu search.

Keywords: photovoltaic, distribution network, topology, combinatorial optimization

1 INTRODUCTION

Electricity generation from renewable energy such as solar and wind is a desirable approach for the reduction of CO_2 emission. Therefore, photovoltaic generation (PV) is expected to be increased in the future [1]. When a large number of PV generators are connected to distribution networks, the generated power from PV will cause reverse power flow on the network. In consequence, the distribution voltage will rise and it will be difficult to keep the voltage within a regulated range [2]. One of the countermeasures is to adjust power factor of PV output by controlling the power conditioning system (PCS) installed in individual PV. Although this approach is practical, it is undesirable to reduce active power outputs of PVs, from the viewpoint of effective use of renewable energy.

One of the possible approaches for the radial distribution network to avoid reducing the active power output of PV is to change the network topology. Since the voltage profile depends on the topology, there exists the optimal topology which can maximize total outputs of PVs on the network without voltage violation. The simple method to search such optimal topology is to compare total PV outputs with respect to all combinations of state of section switch. However, this is time consuming method.

This paper proposes a novel method to search the optimal topology of radial distribution network which maximizes total output power of PV on the network and keeps the network voltage within a regulated range. The method is based on a correlation between total PV output and the largest difference in the voltage profile and the principle of superposition for the radial distribution network. The optimal topology is found out so as to form a tree by extending distribution feeders from a distribution substation.

Determining distribution network topology is a combinatorial optimization problem. To solve this problem, tabu search and genetic algorithm have been applied [3, 4 & 5]. In order to verify the effectiveness of our proposed method, a comparison study with tabu search approach was carried out by using a multi loop distribution network model. The result shows that the proposed method can find a suboptimal topology faster than tabu search.

This paper is organized as follows. In section 2, we define the problem to search optimum topology which maximizes total outputs of PV without voltage violation. In section 3, a novel search method is described. In section 4, we discuss the effectiveness of the proposed method. Section 5 concludes the paper.

2 PROBLEM FORMULATION

When a large number of PVs are connected to a feeder of distribution network, the generated power from PVs will flow reversely and the voltage at the end of the feeder will rise. As the result, the PCSs of individual PVs reduce the active power outputs of PVs by controlling the power factor to keep the voltage within the regulated range of 101 ± 6 V on the low-voltage distribution network.

In order to avoid the reduction of PV output and maximize the total outputs of all PVs connected with the feeder, it is desirable that PVs and loads are connected with each feeder evenly. Such allocation of PVs and loads can be achieved by changing the states of section switches on distribution network, that is, changing the distribution network topology [6, 7]. The optimal topology which maximizes total outputs of PVs can be found out by solving the following combinatorial optimization problem.

Objective

$$\max_{S_j \in S} \left\{ \sum_{i \in Vertices} P_{oi}\left(S_j\right) \right\} \quad (1)$$

Subject to

PV output constraint

$$0 \le P_{oi}\left(S_j\right) \le P_{gi} \quad (2)$$

Voltage constraint

$$\mathrm{V}_{\mathrm{lower}} \le V_i\left(\mathrm{S}_j\right) \le \mathrm{V}_{\mathrm{upper}} \quad (3)$$

Constraint for nodes connection

$$\sum_{b \in Vertices} t^j{}_{ab} \ge 1, (\forall a \in Vertices) \quad (4)$$

Constraint for radial configuration

$$\sum_{a \in Vertices} \sum_{b \in Vertices} t^j{}_{ab} = 2(n-1) \quad (5)$$

Where

S_j	:	Topology #*j* (Combination of section switch states)
$P_{oi}(\mathrm{S}_j)$	:	Active power of PV output on node #*i* for topology S_j
P_{gi}	:	Maximum active power which PV on node #*i* can generate
$V_i(\mathrm{S}_j)$	:	Distribution voltage at node #*i* for topology S_j
V_{upper}	:	Upper bound of voltage $V_i(\mathrm{S}_j)$
V_{lower}	:	Lower bound of voltage $V_i(\mathrm{S}_j)$
$\mathrm{T}(\mathrm{S}_j)$	:	Matrix about a formation of topology S_j. The element of $\mathrm{T}(\mathrm{S}_j)$ in the *a*th row and *b*th column is expressed as $t^j{}_{\mathrm{ab}}$, it is 1 when node #*a* and node #*b* is connected, and it has 0 when these are not connected. On-diagonal element is 0.
n	:	The total number of nodes
S	:	A set of topology
Vertices	:	A set of all nodes

In the above formulation, node means the point where a group of PVs or loads are connected. P_{gi} is the maximum active power which the PV at node #*i* can generate without the voltage constraints of Eq.(3). Eq.(4) shows that there is no isolated node. Eq.(5) shows that the distribution network has radial topology. In this paper, the abovementioned optimization problem is called "maximization problem of total PV outputs".

3 A SEARCH METHOD OF THE TOPOLOGY

3.1 Exhaustive search approach

The simplest method to solve the maximization problem of total PV outputs is an exhaustive search approach. By this approach, the optimum topology can be searched by comparing total PV outputs of all combinations of states of section switches as shown in Fig. 1. First, to determine the total PV output of topology Sj without voltage violation, the processes on the loop A are done. Next, the topology is changed along the loop B, then the processes on the loop A are carried out again. The optimum topology is found out through the repeat of those processes on the loop A and loop B.

The defect of the exhaustive search approach is to take enormous computation time by the following two factors. The first one is the loop A. In this loop, AC power flow calculation must be done many times. The second factor is the loop B. Since the computation time also depends on the number of combinations of states of section switches, in case of studying a large scale distribution network, the computation time will increase enormously.

3.2 Problem modification

In order to develop the optimum topology search method which can reduce the enormous computation time caused by the two factors in the previous section, the maximization problem of total PV outputs is modified to the other new optimization problem. The modification is based on the correlation between total PV outputs and voltage profile of distribution network. The topology which minimizes the largest difference among node voltages tends to maximize total PV outputs, so that the optimum topology will be found out by solving the problem of minimizing the largest difference among node voltages. For the illustration of the correlation, a simulation study has been done using a simple distribution network model as shown in Fig. 2 before we describe the modified new optimization problem and our proposal of solving the modified problem.

When the correlation between total PV outputs $P_{sum}(\mathrm{S}_j)$ and node voltage $V_i(\mathrm{S}_j)$ is investigated, $P_{sum}(\mathrm{S}_j)$ is calculated with the voltage constraints of Eq.(3) but

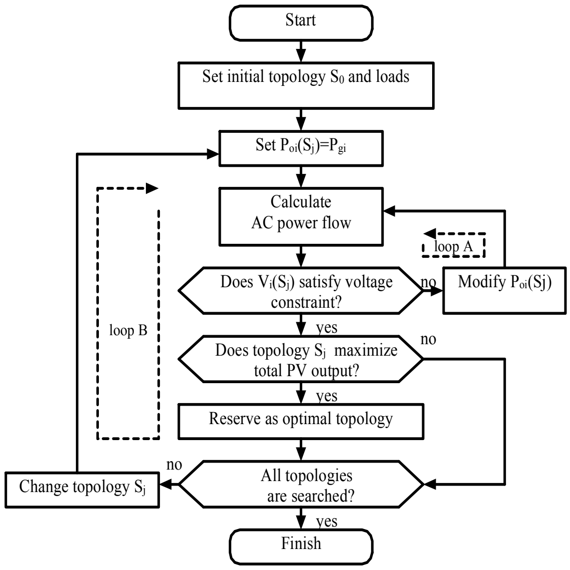


Figure 1: A flowchart of exhaustive search

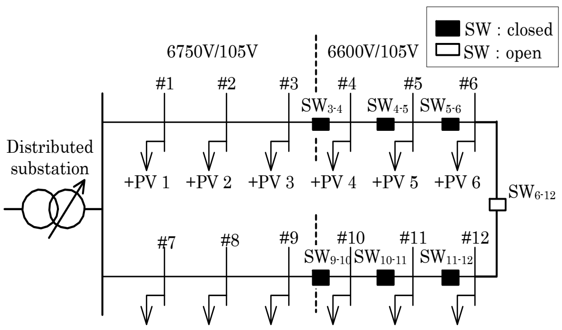


Figure 2: A distribution network model for explanation of the proposed method

$V_i(S_j)$ is calculated without the constraints. The parameters of the distribution network are described in section 4. In the case of the distribution model in Fig.2, the number of all combinations of section switch states is seven. Fig. 3 shows the voltage profiles of the following different states of section switches.

Open $SW_{6\text{-}12}$: topology that same amount of load is connected on each feeder
Open $SW_{3\text{-}4}$: topology which maximizes $P_{sum}(S_j)$

In the case of Open $SW_{6\text{-}12}$, the largest voltage difference $V_{dif}(S_j)$ between the maximum node voltage and the minimum one is 11.4 V. In the case of Open $SW_{3\text{-}4}$, $V_{dif}(S_j)$ is 5.5 V. By computing the largest voltage differences and total PV outputs for the all combinations of section switch states, the correlation between them was obtained as the regression line plotted in Fig. 4.

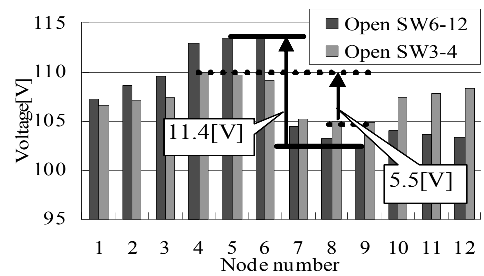


Figure 3: Voltage profiles of the different states of section switch

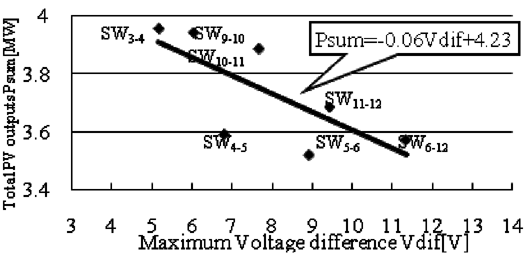


Figure 4: Corelation between the largest voltage difference and total PV output

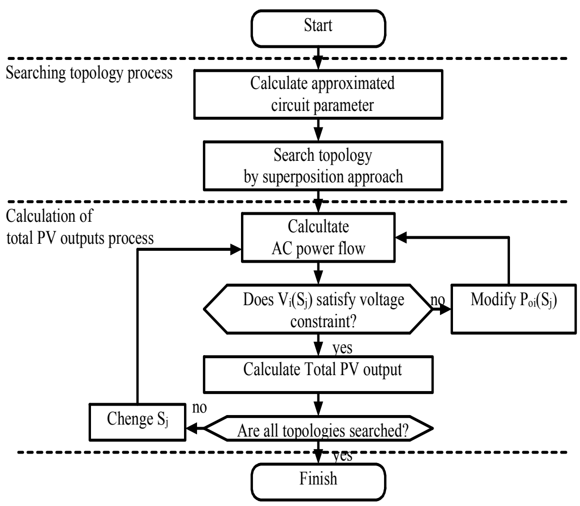


Figure 5: A flowchart of proposed method

The regression line shows that a negative correlation exists between $P_{sum}(S_j)$ and $V_{dif}(S_j)$. In the figure, $SW_{i\text{-}j}$ indicates that the section switch equipped between node #*i* and #*j* is open. By utilizing the negative correlation, the maximization problem of total PV outputs $P_{sum}(S_j)$ formulated in section 2 can be modified to the minimization problem of the largest voltage difference $V_{dif}(S_j)$. The modified problem is formulated as follows:

Objective

$$\min_{S_j \in S}\left[V_{dif}\left(S_j\right)\right] \quad (6)$$

$$V_{dif}\left(S_j\right) = \max_{x \in Vertices}\left(V_x\left(S_j\right)\right) - \min_{y \in Verticies}\left(V_y\left(S_j\right)\right) \quad (7)$$

Subject to

PV output constraint

$$P_{oi}\left(S_j\right) = P_{gi} \quad (8)$$

Constraint for nodes connection

$$\sum_{b \in Vertices} t^j{}_{ab} \geq 1, \left(\forall a \in Vertices\right) \quad (9)$$

Constraint for radial configuration

$$\sum_{a \in Vertices} \sum_{b \in Vertices} t^j{}_{ab} = 2(n-1) \quad (10)$$

where $V_x(S_j)$ and $V_y(S_j)$ are the voltage magnitudes of node #x and #y on topology S_j.

Fig. 5 shows the proposed method for solving the above modified problem. In the first part, "Searching topology process", the candidates of topology which should be evaluated in the next part of the method, are selected. The selection process is based on the superposition of radial electric circuit. In the second part, "Calculation of total PV output process", the total PV outputs are computed for all the selected candidates of topology, then the topology which minimizes the total PV output is determined as the optimum one.

3.3 Approximated electric power circuit model

In the first part "Searching topology process" of the proposed method, all power sources in distribution network are expressed approximately by current sources because of the following reason.

An injected current at node #*i* is given by

$$I_i\left(S_j\right) = \frac{\left(P_{gi} - P_{li} - jQ_{li}\right)^*}{V_i\left(S_j\right)} \quad (11)$$

where, $I_i(S_j)$ is the injected current and P_{li} and Q_{li} are the injected active power and reactive power at node #*i* . The injected current $I_i(S_j)$ depends on node voltage $V_i(S_j)$ of topology S_j . However, in the proposed method, the node voltage is assumed to be constant, that is, independent on topology because node voltage is much larger than the difference between $V_{dif}(S_j)$ and $V_{dif}(S_k)$ of different topology. Therefore, the node voltage of specified topology is used as the constant node voltage. As the specified one, the topology that all section switches are closed is adopted because it has loop configuration, so that total PV output will be maximized.

By this approximation, voltage profile of radial distribution network can be calculated easily and successively with the principle of superposition whenever a new node is added to the network.

The procedure for calculating voltage profile is explained by using a simple model of radial distribution network as shown in Fig.6. According to the above approximation, the distribution networks in Fig. 6(a) and (b) are represented by the equivalent circuits in Fig. 6(c) and (d) respectively. Therefore, the voltage V_{a1} in Fig. 6(a) is given by the following equation.

$$\dot{V}_{a1} = \dot{V}_0 + z_{01}\dot{I}_1 \quad (12)$$

where V_0 is the voltage on the distribution substation (node #0). z_{01} is impedance between node #0 and node #1. I_1 is the injected current source at node #1 which is approximated with Eq.(11).

Next, a new node #2 with a load is added to the network in Fig. 6(a). Each node voltage in the network including the new node #2 in Fig.6(b) can be obtained by utilizing the node voltage of the network in Fig. 6(a) as shown in the following equations.

$$\begin{aligned}\dot{V}_{b1} &= \dot{V}_0 + z_{01}\left(\dot{I}_1 + \dot{I}_2\right)\\ &= \dot{V}_0 + z_{01}\dot{I}_1 + z_{01}\dot{I}_2\\ &= \dot{V}_{a1} + z_{01}\dot{I}_2 \quad (13)\end{aligned}$$

$$\dot{V}_{b2} = \dot{V}_{b1} + z_{12}\dot{I}_2 \quad (14)$$

where V_{bi} is the voltage at node #i (i =1, 2) as shown in Fig.6(b). z_{12} is the impedance between node #1 and node #2. I_2 is the injected current source at node #2 which is approximated with Eq.(11).

Eq.(13) and Eq.(14) show that the voltage profile of m node radial network can be successively computed by using the voltage profile of (m-1) node radial network and the newly added current source. In the "Searching topology process", this computation procedure is used.

3.4 *Search algorithm*

The algorithm of "Searching topology process" in Fig. 5 is explained. For the simple explanation, the algorithm for searching one network configuration that minimizes the largest voltage difference $V_{dif}(S_j)$ is explained in detail using a simple network in Fig. 7.

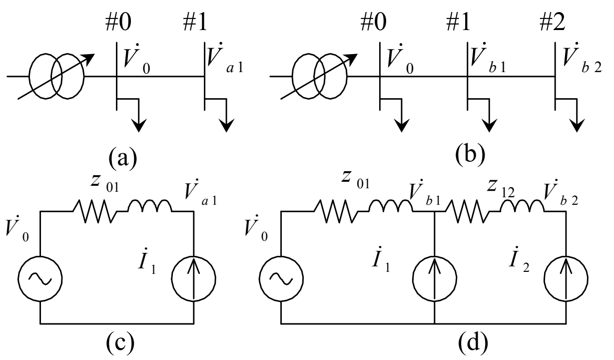


Figure 6: Voltage profile of the radial network that a new node is added

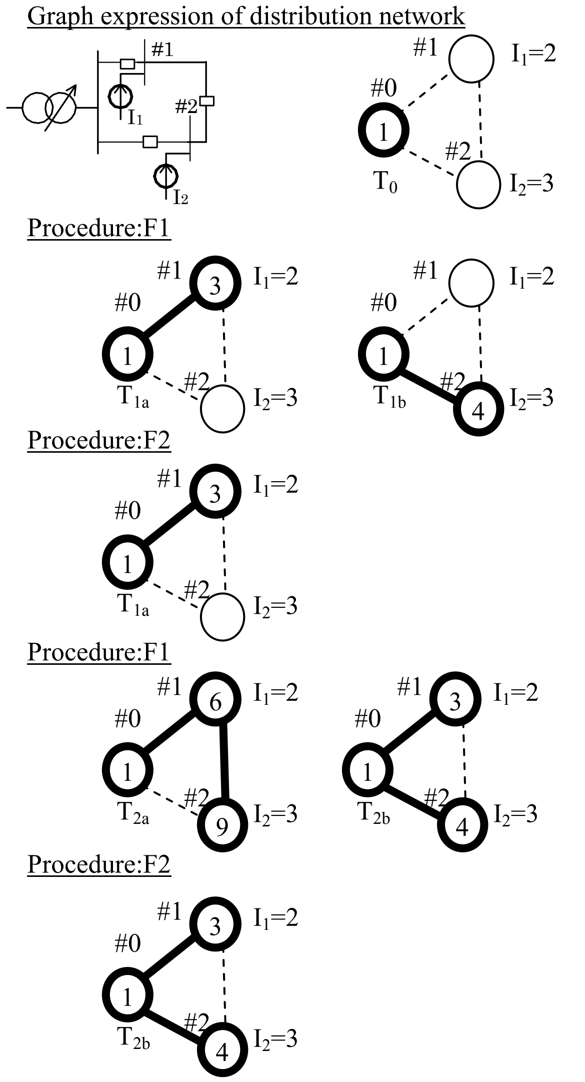
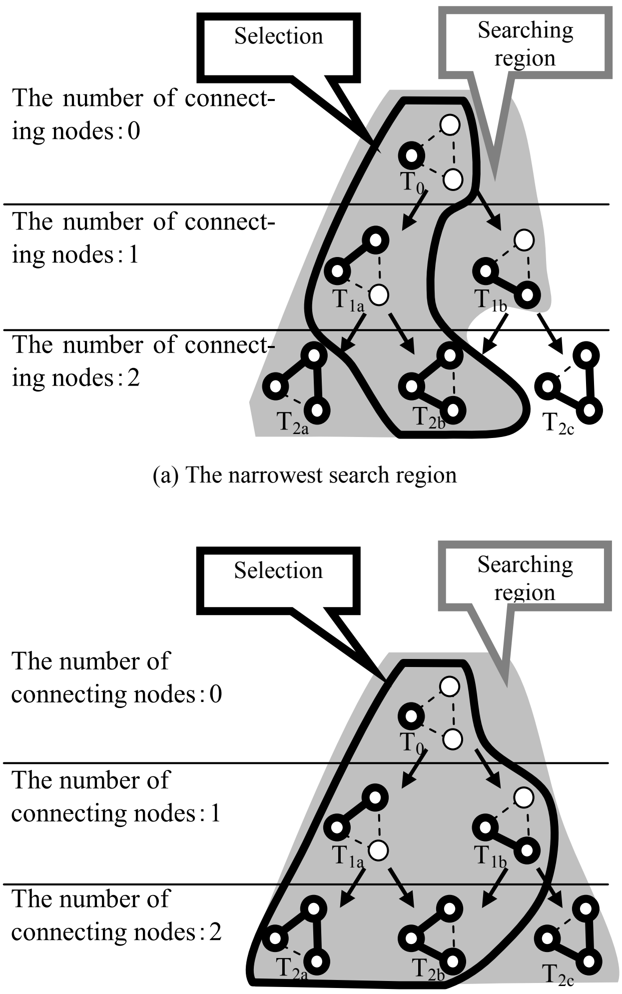


Figure 7: Schematic representation of the proposed topology search algorithm

In Fig. 7, node #0 represents a distribution substation. The other nodes show the connection points of PV and load. For the simple explanation, the impedance between two nodes is assumed to be z= 1 Ω and the voltage at node #0 is set V_0=1 V. In addition, node #0 is called subtree T_0 in the following explanation.

Procedure (F1): New subtrees are made by connecting a new node with the subtree T_0. As shown in Fig.7, a new subtree T_{1a} is made by connecting node #1 with the subtree T_0. In the same way, T_{1b} is made by connecting node #2 with the subtree T_0. Next, voltage profile $\mathbf{V}_{1a}$ of T_{1a} and $\mathbf{V}_{1b}$ of T_{1b} are calculated. Voltage V_{1a1} at node #1 in T_{1a} is obtained as $V_{1a1}=V_0+z_{01}I_1=3$ by substituting I_1=2 for Eq.(12). In the same way, the voltage V_{1b2} at node#2 in T_{1b} is calculated to be $V_{1b2}=V_0+z_{02}I_2=4$ by using I_2=3.

Procedure (F2): For the individual subtrees made in Procedure (F1), the largest voltage difference V_{dif} is calculated and the subtree which has minimum V_{dif} is



(a) The narrowest search region

(b) An extended search region

Figure 8: An image of variable search region

selected. V_{dif} of T_{1a} is computed to be 2 with Eq.(7) because the maximum node voltage is V_{1a1}=3 and minimum voltage is V_0=1. In the case of T_{1b}, V_{dif} is computed to be 3. Therefore T_{1a} is selected as the subtree with minimum V_{dif}.

The above two procedures are applied successively to the selected subtree until all nodes are connected. As the result, the topology that the section switch between node #1 and node #2 is opened is selected as the topology which should be evaluated in "Calculation of total PV output process" of the proposed method.

In the proposed method, AC power flow calculation is done only a few times when the power sources are replaced with current sources and the total PV output of the selected topology is calculated. Therefore, the computation time of the proposed method will be shorter than that of the exhaustive search approach. However, there is some possibility that a local optimum is obtained because the proposed method can not necessarily search broad solution space of the original problem, "maximization problem of total PV outputs".

In order to avoid selecting the local optimum, the proposed method can be improved so as to search the broader solution space. Fig. 8(a) represents schematically the abovementioned algorithm for searching one network configuration as shown in Fig. 7. On the other hand, Fig. 8(b) illustrates the improved algorithm that two subtrees with smaller V_{dif} are selected. Although the improved method can search broader solution space by increasing the number of selected subtrees, the computation time will increase. In the next section, the result of the comparison study with tabu search is described in order to verify the effectiveness of the improved method.

4 CASE STUDIES

In order to verify the effectiveness of proposed method, the comparison study has been done by using distribution network model as shown in Fig.9. The base capacity and voltage of this model is 1 MVA and 6.6 kV. The line impedance between the two adjacent nodes in the model is 0.31+j0.36 Ω. The network has multi-loop configuration, including 4 substations, 61 nodes, and 67 section switches. Among the section switches, 10 switches are in open state. As shown in Fig. A.1, the load at node #*i* that is equivalent to the 300 houses, P_{li} , and the power generation that is equivalent to 200 PVs are used in this case study [8]. According to the reference [8], the maximum active power generation of PV at node #*i*, P_{gi} , is set 0.69 MW. Then, the active power output of PV, P_{oi}, and reactive power output of PV, Q_{oi}, can be expressed by the following equations.

$$P_{oi} = C_i P_{gi} \cos\theta_i \quad (15)$$

$$Q_{oi} = C_i P_{gi} \sin\theta_i \quad (16)$$

where $\cos\theta_i$ is the power factor of PV output on node #*i*, C_i is the index of PV output on node #*i*. In the case studies, C_i is specified as 1 when the voltage on node #*i* is within the regulated range. If the voltage violation occurs, the P_{oi} and Q_{oi} are modulated as follows. When node voltage is lower than 105 V, the power factor $\cos\theta_i$ is reduced to 0.85. When node voltage exceeds 107 V, P_{oi} and Q_{oi} are modulated by decreasing the C_i to 0.

We also consider the influence of the Line voltage Drop Compensator (LDC) at substation. LDC adjusts the voltage at the sending end of feeder when one of the average voltage levels at monitoring nodes does not exist between 102 [V] and 104 [V]. These monitoring nodes are marked by M in Fig.9.

As the first case study, the computation time to find optimum topology by tabu search method [9] and the proposed method and the total PV outputs of the optimum topology are compared. The tabu search is applied so as to improve the process of loop B in the exhaustive search in Fig.1. The outline of the tabu search application is explained in the appendix.

Table 1 summarizes the total PV outputs and computation time for the two methods. Fig. 9 shows the initial topology. Fig. 10 illustrates the final optimal topology found by tabu search. The gray arrows in the figure indicate the changes in section switch states from

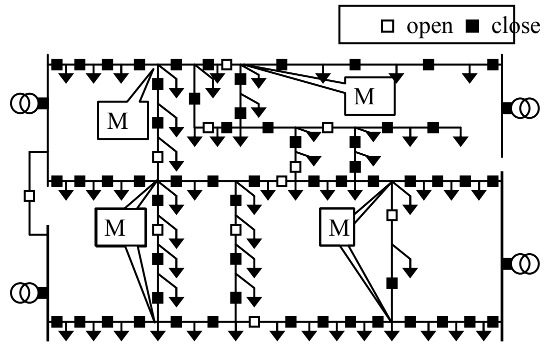


Figure 9: A distribution network of multi loop configuration

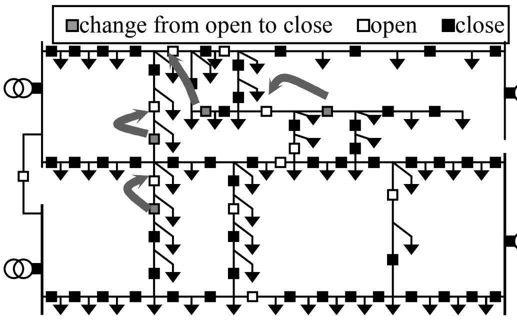


Figure 10: Topology found by tabu search method

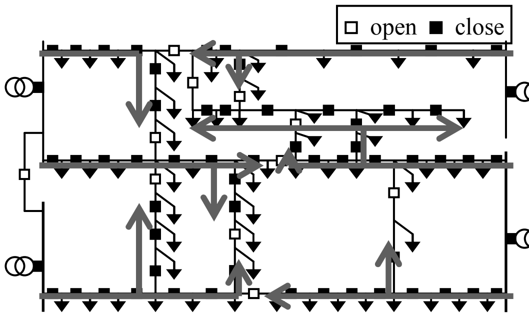


Figure 11: Topology found by the proposed method

Adopted method	Total PV Output [MW]	Computation Time[sec]
Tabu search	21.63	13.36
Proposed method	**21.69**	**2.71**

Table 1: Comparison of total PV output and computation time

the initial topology to the final one. The computation time is 13.36 sec and the total PV output is 21.63 MW.

On the other hand, the final optimal topology found by the proposed method is illustrated in Fig.11. The gray arrows show the directions of searching process from the initial switch states to the final ones. The proposed method can find the optimum topology with the computation time of 2.71 sec and the total PV output of 21.69 MW can be achieved with this topology.

Those results show that the proposed method can search the optimum topology about 5 times faster than that of tabu search approach. This is because the tabu search needs many times of AC power flow calculation in every searching step.

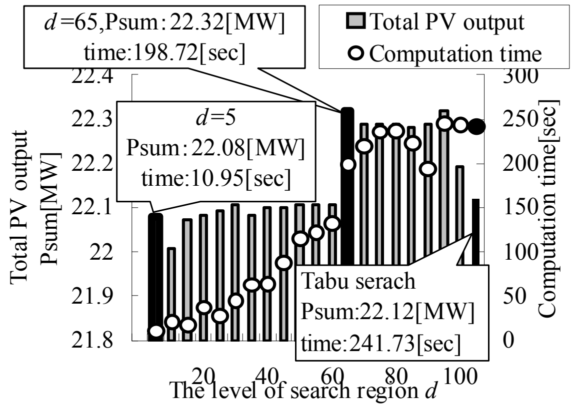


Figure 12: Topology found by the proposed method

However, the proposed method only needs the AC power flow calculation for the approximation of the electric power circuit and the computation of total PV output of the final topology.

Next, we show the effectiveness of the improved algorithm that is explained in Fig. 8(b). When the number d of subtrees in the topology searching process is set 5 to 100, total PV outputs and computation time of the proposed method change as shown in Fig.12. In the figure, the results of tabu search are also indicated. In the case of d=5, the total PV outputs is 22.08 MW and its computation time is 10.95 sec. In the case of d =65, the total PV output increases up to 22.32 MW and the computation time is 198.72 sec. It can be said that the total PV output tends to increase by expanding the solution space that is searched even if the computation time becomes long. Compared with tabu search, the proposed method can find the optimal topology faster than that of tabu search when the number d of subtrees is set optimally.

5 CONCLUSION

This paper proposes a novel method for radial distribution network to search the optimal topology which maximizes total PV outputs. The features of the proposed method are as follows. (1) The maximization problem of total PV output is replaced with a simplified problem of minimizing the largest voltage difference. (2) By extending distribution feeders by adding a new node to the feeders one by one, suboptimum network configurations can be found out without AC power flow computation.

Although the case studies in section 4 are based on the assumption that PV output is predictable completely, the proposed method can give distribution system operators how to determine the distribution network configuration that maximizes total PV output without voltage violation. In addition, the fast computation time of proposed method allows searching the sub-optimal topology with respect to the varying PV outputs. Therefore, this method is very useful for the application of scheduling the distribution network operation by comparing several network configurations with different PV outputs.

It is necessary to take into account the uncertainty of PV outputs due to dependency on weather and operational limits of section switches in actual distribution network. Therefore, consideration of these constraints is left as a future work.

APPENDIX

A.1 Load and PV Output data [8]

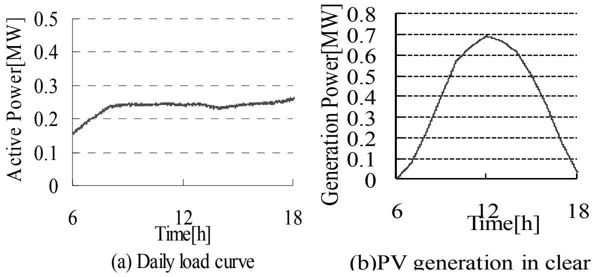


Figure. A1. Load and PV generation data

A.2. Outline of Tabu search approach

Tabu search approach is the method which improves the search process (loop B) in Fig.1. The search process is as follows.

At first, total PV output of initial topology (network configuration) is calculated and the topology is added to tabu list. Next, all the network configurations close to the initial one are created, and their total PV outputs are calculated. The configuration that maximizes total PV output is selected. Then the tabu list is updated and the suboptimal configurations are searched repeatedly until stopping condition is satisfied.

The process of searching the configuration S_{near} close to the initial one can be explained as follows. At first, an opened section switch between adjacent two nodes #a and #b is changed to be closed. Next, another closed section switch between adjacent two nodes #a and #c is changed to be opened. If the topology S obtained by the above steps satisfies the network constraints of Eq.(4) and Eq.(5), then the topology S is regarded as local optimum topology S_{near}. Although any nodes in the network can be selected as node #b and #c, as node #a, the substation node cannot be selected.

ACKNOWLEDGEMENT

The authors wish to express the appreciation to lab member, Mr. Aung Ko Thet, Tohoku University, Japan for helping in proof reading of this paper.

REFERENCES

[1] Y. Hayashi, ”Trend and Future View of Voltage Control for Distribution Systems with Distributed Generators”, IEEJ Trans. PE, Vol. 129, No. 4, pp.491-494, 2009 (in Japanese).

[2] M. Kouzuki, R. Suyama, K. Moriya and H. Saitoh, “Agent based Cooperative Control of Photovoltaic Systems for Voltage Adjustment in Distribution Networks”, The 2008 ANNUAL MEETING RECORD IEE Japan, pp240-241, 2008 (in Japanese).

[3] K. Nara, A. Shinose, M. Kitagawa and T. Ishihara, “Implementation of Genetic Algorithm for Distribution Systems Loss Minimum Re-configuration for Operation Cost Reduction”, IEEE Trans. on PWRS, Vol.7, No.3, pp.1044-1051, Aug. 1992.

[4] Y. Hayashi and J. Matsuki, “Loss Minimum Configuration of Distributed System Considering N-1 Security of Dispersed Generators”, IEEE Trans on PS, Vol. 19, No. 1, pp. 636-642, Feb. 2004.

[5] Y. Misima, K. Nara, T. Sato, T. Ito and H. Kaneda, “Method for Loss Minimum Re-configuration Problem of Distribution System by Tabu Search”, IEEJ Trans. PE, Vol.123, No10, pp.1149-1154, 2003 (in Japanese).

[6] T. Sato and H. Saitoh, “A Novel Searching Method for Distribution Network Topology to Maximize Total Outputs of PV Clusters”, The Papers of Joint Technical Meeting on Power Engineering and Power Systems Engineering, IEE Japan, PE-06-95, 2009 (in Japanese).

[7] T. Sato and H. Saitoh, “A Searching Method for Distribution Topology to Accurate Maximum Outputs of PV clusters”, The 2010 ANNUAL Conf. of Power & Energy Society IEE Japan, 203, 2010 (in Japanese).

[8] The Institute of Applied Energy : “A Report of Study on Power System” (2003), pp.24 (in Japanese), http://www.iae.or.jp/PROJECT/nws/pdf/a1/IAE-C0243.pdf, download in 2009.

[9] Kwang Y. Lee, Mohamed A. El-Sharkawi “Modern Heuristic Optimization Techniques”, John Wiley & Sons, pp.101-119(2008)