A MODEL TO ACCOUNTING FOR LOSSES IN THE UNIT COMMITMENT PROBLEM AT BRAZILIAN HYDRO PLANTS

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Abstract — We describe a Hydro Unit Commitment (HUC) model that has been developed to be used as a support tool in the Brazilian system. The objective is to determine the unit commitment and generation schedules for cascaded plants with multiple units and head-dependent function production. We solve the HUC problem in a two-phase approach based on dual decomposition. The computational tool allows the model to effectively schedule hydro units for the problem in the Brazilian regulatory framework. Application of the approach is demonstrated by determining the 24-time-steps HUC for four cascaded plants with 4,170 MW of installed capacity.

Keywords: Hydro Unit Commitment, Hydro Production Function, Lagrangian Relaxation, Inexact Augmented Lagrangian.

I. INTRODUCTION

Brazil has a modern electricity industry which depends on hydropower and its hydrothermal system has the largest capacity for water storage in the world. In this context, an Independent System Operator (ISO) is responsible for the system scheduling managing optimization studies which extent from the long/medium-term to short-term operation scheduling models. In the short-term model, the ISO perform a week-ahead planning to yield a scheduling for all plants in order to meet the load and satisfy other constraints. However, the huge number of reservoirs precludes the ISO from precisely take into account the complex modeling associated with the hydro units. In other words, in the short-term the hydro plants are modeled by a continuous piecewise linear function; therefore, all nonlinearities and unit commitment constraints are not considered. As a result, the generation distribution among the machines in a hydro plant regarding specific modeling and constraints is a local decision. In this sense, modeling and optimization algorithms improvements is an important issue, since studies have shown that significant gains can be reached with an efficient allocation of generation among hydro units [1]. Given that hydro plants possess distinctive characteristics[1] one important challenge is to represent a precise modeling which takes into account these characteristics.

The HUC problem has been extensively studied and various modeling were proposed [2] – [11]. Some of then represents the hydro production function in a simplified manner. On the other hand, other works [12] – [14] take into account nonlinear head variations, hydraulic losses and hydraulic efficiency in their formulation. This paper present a most detailed representation of the energy conversion process, which takes into account mechanical losses in the turbine as well as mechanical and electrical losses in the generator. An accurate modeling of these issues plays an important role in the problem, in particular for the Brazilian hydro-dominated system where every tenth of a percentage increased in the energy conversion is welcome.

In this paper we propose a new mathematical model for the hydro production function which allows modeling all relevant factors that affect the unit output. We represent hydraulic losses in the conduits and in the turbine suction tube, nonlinear forebay and tailrace functions, hydraulic efficiency and mechanical losses in the turbine, and mechanical and electrical losses in the generator. Additionally, the problem is formulated to maximize the overall energy conversion efficiency while meeting an hourly generation targets defined by the ISO. This is a mixed-integer nonlinear programming problem and to solve it efficiently, we propose a two-phase decomposition approach similar to [14]. Initially, we use the Lagrangian Relaxation (LR) to obtain an infeasible solution, and in the second phase we use an inexact Augmented Lagrangian (AL) to find a feasible solution.

This paper is organized as following. In Section II we present a new modeling for the hydro function production. The optimal HUC problem formulation is shown in Section III. In Section IV, we focus on the solution strategy and, in Section V the test results are provided. In the last section, we present the main conclusions.

II. THE HYDRO PRODUCTION FUNCTION

Initially, consider Figure 1 below:

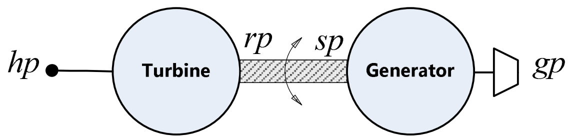

Figure 1: Schematic diagram of a typical hydro generating unit.

The power variables (MW) shown above are:

hp hydraulic power available for producing electrical energy:

$$hp = G \cdot gh(v, Q, q, s) \cdot q, \qquad (1)$$

where G is a constant and $gh(.)$ is the gross head (m) which is a function of the unit turbined discharge (m^3/s), q, the reservoir volume (hm^3), v, the plant water discharge (m^3/s), Q, and, in some plants, the spillage (m^3/s), s.

rp mechanical power transmitted through the cou-

[1] Especially due to the presence of diverse kinds of conduits, turbines, generators, etc.

pling of the runner and the turbine shaft:

$$rp = G \cdot \rho(nh, q) \cdot nh(v, Q, q, s) \cdot q, \quad (2)$$

where $\rho(\cdot)$ is the turbine hydraulic efficiency and nh is the net head (m), i.e., the part of the gross head which is available for the turbine.

- *sp* mechanical power delivered by the turbine shaft, assigning the mechanical losses of the relevant bearings and shaft seals to the hydraulic machine [15];
- *gp* power output at the generator terminals.

As it will be shown, the hydro function production depends on many factors. We detail the influence of different parameters on unit efficiency bellow.

2.1 Forebay and Tailrace Levels

As it is known $gh(\cdot)$ is given by the difference between the forebay, *fbl*, and tailrace, *trl*, levels:

$$gh = fbl(v) - trl(Q, s), \quad (3)$$

In the Brazilian case, the $fbl(\cdot)$ and $trl(\cdot)$ functions are modeled by a forth order polynomial.

2.2 Head Loss

The head loss (m), *hl*, is composed by:

$$hl = hlp(q) + hls(q), \quad (4)$$

The $hlp(\cdot)$ is the head loss associated with the penstock water friction and is modeled as a quadratic function of q, $k_p \cdot q^2$. The k_p is a constant which depends on the penstock characteristics (s^2/m^5). The term $hls(\cdot)$ is the head loss associated with the hydraulic energy dissipated between the tailrace level and the low pressure reference section of the turbine. This term is also a quadratic function $k_s \cdot q^2$, where k_s depends on the section area of the low pressure reference section and the gravitational acceleration. At this point, it is important to notice that the net head, *nh*(.), is given by $fbl(v) - trl(Q,s) - hlp(q) - hls(q)$.

2.3 Turbine Hydraulic Efficiency

The turbine hydraulic efficiency, ρ, represents the transfer effectiveness to the runner of the available power in water that flows through it [15]. It depends on $nh(\cdot)$ and *rp* and is furnished by the turbine manufacturer as a set of triples (ρ, *rp*, *nh*). We use the following model [15] to represent the hydraulic efficiency:

$$\rho(q, nh) = c_0 + c_1 \cdot q + c_2 \cdot nh + c_3 \cdot nh \cdot q + c_4 \cdot q^2 + c_5 \cdot nh^2, \quad (5)$$

where $c_0,...,c_5$ are constants. In order to estimate these values, we take the set (ρ, *rp*, *nh*), and use (2) to compute q and, in the sequence, with the new triple set (ρ, *q*, *nh*) the constants in (5) are computed with a regression technique.

2.4 Turbine Mechanical Losses

The turbine mechanical efficiency is the ratio of the power available at the turbine shaft to the power transmitted from the water to the runner [15], that is:

$$\frac{sp}{rp} = \frac{sp}{sp + tml(gp)}. \quad (6)$$

Above *tml*(*gp*) is the turbine mechanical losses (MW) associated with the power consumed by the mechanical friction in the guide bearings, thrust bearing and shaft seals. By means of unit field tests, a set of points (*tml*, *gp*) can be obtained and we can adjust, for example, a polynomial function:

$$tml(gp) = b_0 + b_1 \cdot gp + b_2 \cdot gp^2, \quad (7)$$

where b_0, b_1, b_2 are constants.

2.5 Generator Global Losses

The generator efficiency is the ratio of the power available at the generator terminals to the mechanical power delivered by the turbine shaft, that is:

$$\frac{gp}{sp} = \frac{gp}{gp + ggl(gp)}. \quad (8)$$

Above, *ggl*(*gp*) is the generator global losses. As $tml(\cdot)$ a set of points (*ggl*, *gp*) is obtained by means of field tests and we adjust an exponential function:

$$ggl(gp) = a_0 \cdot e^{a_1 \cdot gp}, \quad (9)$$

where a_0 and a_1 are constants.

2.6 Hydro Production Function

According (1)-(9), the new hydro production function proposed in this paper is given by:

$$gp - G \cdot \rho(v, Q, q, s) \cdot nh(v, Q, q, s) \cdot q + tml(gp) + ggl(gp) = 0. \quad (10)$$

III. THE PROBLEM FORMULATION

The model seeks to maximize the overall generation efficiency while attempts to meet the hydro plant load requirement and others constraints. The main result is a set with the commitment state of each generating unit, and the respective generation level, over a planning horizon. So, the HUC optimization problem related to this paper is given by:

$$\min \Theta = -100 \cdot \left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G \cdot gh_{rt}(v_{rt}, Q_{rt}, s_{rt}) \cdot Q_{rt}} \right] \quad (11)$$

$$\text{s.t.:} \quad \sum_{i=1}^{n_{rt}} gp_{irt} = L_{rt}, \quad (12)$$

$$gp_{irt} - G \cdot \rho_{irt}(v_{rt}, Q_{rt}, q_{irt}, s_{rt}) \cdot nh_{irt}(v_{rt}, Q_{rt}, q_{irt}, s_{rt}) \cdot q_{irt} + tml_{irt}(gp_{irt}) + ggl_{irt}(gp_{irt}) = 0, \quad (13)$$

$$v_{rt} - v_{r,t-1} + c_1 \left[Q_{rt} + s_{rt} - \sum_{m \in \Re_r^{up}} (Q_{m,t-\tau_{mr}} + s_{m,t-\tau_{mr}}) - y_{rt} \right] = 0 \quad (14)$$

$$v_r^{\min} \le v_{rt} \le v_r^{\max}, \quad (15)$$

$$\sum_{i=1}^{n_r} q_{irt} - Q_{rt} = 0, \quad (16)$$

$$q_{ir}^{\min}(v_{rt}, Q_{rt}, s_{rt}, q_{irt}) \le q_{irt} \le q_{ir}^{\max}(v_{rt}, Q_{rt}, s_{rt}, q_{irt}), \quad (17)$$

$$u_{irt}\cdot gp^{\min}_{irtk} \le gp_{irt} \le u_{irt}\cdot gp^{\max}_{irtk}, \quad (18)$$

$$u_{irt} = \sum_{k=1}^{\Phi_{ir}} z_{irtk},\ \sum_{k=1}^{\Phi_{ir}} z_{irtk} \le 1,\ \ z_{irtk}\in\{0,1\}. \quad (19)$$

Where:

- T number of stages (h), so that $t = 1,T$;
- R number of hydro plants, so that $r = 1,R$;
- c_1 conversion factor of water discharge units (m^3/s) in volume units (hm^3);
- L_{rt} plant load requirement of the plant r and stage t (MW);
- n_{rt} number of units available for operation in the plant r and stage t;
- i is the hydro unit index, so that $i = 1,n_{rt}$ for each plant r and stage t;
- $v_r^{\min(\max)}$ minimum (maximum) volume of the reservoir r [hm^3];
- m index of reservoir upstream of the reservoir r;
- τ_{mr} number of time stages that the total outflow in the upstream hydro plant m takes to reach the downstream hydro plant r;
- $\Re_r^{up}$ set of reservoirs immediately upstream to the reservoir r;
- y_{rt} incremental inflow of reservoir r during stage t [m^3/s];
- Φ_{ir} number of non-forbidden zones of the unit i and plant r, so that $k = 1, \Phi_{ir}$;
- $gp_{ikrt}^{\min(\max)}$ minimum (maximum) power of the unit i, zone k, plant r and stage t (MW);
- v_{rt} volume of the reservoir r and stage t (hm^3);
- Q_{rt} turbined outflow in the reservoir r and stage t (m^3/s);
- s_{rt} spillage of the reservoir r and stage t (m^3/s);
- q_{irt} turbined outflow of the unit i, reservoir r and stage t (m^3/s);
- gp_{irt} generator power output of the unit i, reservoir r and stage t (MW);
- u_{it} binary variable which indicates if unit i is operating ($u_{it} = 1$) or not ($u_{it} = 0$) during the stage t;
- z_{ikrt} binary variable which indicates if the unit i of the reservoir r is operating ($z_{jkrt} = 1$) or not ($z_{jkrt} = 0$) in the non-forbidden zone k during the stage t;
- $q_{ir}^{\min(\max)}$ (.) minimum (maximum) discharge of the unit i and plant r as a function of the net head[2].

In (11) the objective is to maximize the efficiency energy conversion over the planning horizon. The constraint set (12) is the plants load requirement. Each constraint (13) is the hydro production function. Constraints (14) represent the stream-flow balance equations. Constraints (15) and (16) describe the volume variables limits and the penstock water balance in each reservoir, respectively. Constraints (17) express the units discharge limits as a function of the net head. Constraints (18) are related to power limits for each non-forbidden operating zone [16]. Finally, Equations (19) represent the integrality constraints.

[2] These expressions can be obtained by means of the set of points (ρ, *nh*, *rp*) supplied by the turbine manufacturer.

IV. SOLUTION ALGORITHM

We use a two-phase decomposition approach. In the first phase, a LR algorithm is used to obtain a solution (infeasible) and, in the second phase, we use this information as a starting point for the AL to find a feasible solution. To formulate the LR and AL dual functions, we introduce a new set of "copy" constraints where the volume, plant turbined outflow and spillage variables are duplicated. The overall solution strategy is detailed in the next two sections.

2.7 Phase I: Decomposition by Lagrangian Relaxation

In the LR phase we include in (11), (14) and (15) artificial variables va_{rt}, Qa_{rt}, and sa_{rt}, in the following way:

$$\min \Theta_1 = -100\cdot\left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G\cdot gh_{rt}(va_{rt},Qa_{rt},sa_{rt})\cdot Qa_{rt}}\right] \quad (20)$$

$$\text{s.t.: } va_{rt} - va_{r,t-1} + c_1\left[\begin{matrix} Qa_{rt} + sa_{rt} - \\ \sum_{m\in\Re_+^{(r)}} (Qa_{m,t-\tau_{mr}} + sa_{m,t-\tau_{mr}}) - y_{rt}\end{matrix}\right] = 0 \quad (21)$$

$$v_r^{\min} \le va_{rt} \le v_r^{\max}, \quad (22)$$

$$va_{rt} = v_{rt},\ Qa_{rt} = Q_{rt}, sa_{rt} = s_{rt}, \quad (23)$$

(12), (13), (16)-(19). (24)

Relaxing the constraints (23) leads to:

$$\begin{aligned}\Theta_{RL} = \min\ & -100\cdot\left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G\cdot gh_{rt}(va_{rt},Qa_{rt},sa_{rt})\cdot Qa_{rt}}\right] \\ & -\sum_{t=1}^{T}\sum_{r=1}^{R}[\lambda v_{rt}(va_{rt}-v_{rt}) + \lambda Q_{rt}(Qa_{rt}-Q_{rt}) \\ & + \lambda s_{rt}(sa_{rt}-s_{rt})],\end{aligned} \quad (25)$$

s.t.: (21), (22) and (24). (26)

The function above can be evaluated by means of R+1 subproblems:

$$\begin{aligned}\Theta^{RL\text{-}HS} = \min\ & -100\cdot\left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G\cdot gh_{rt}(va_{rt},Qa_{rt},sa_{rt})\cdot Qa_{rt}}\right] \\ & -\sum_{t=1}^{T}\sum_{r=1}^{R}[\lambda v_{rt}\cdot va_{rt} + \lambda Q_{rt}\cdot Qa_{rt} + \lambda s_{rt}\cdot sa_{rt}]\end{aligned} \quad (27)$$

s.t.: (21)-(22), (28)

and,

$$\Theta_r^{RL\text{-}HC} = \min \sum_{t=1}^{T}(\lambda v_{rt}\cdot v_{rt} + \lambda Q_{rt}\cdot Q_{rt} + \lambda s_{rt}\cdot s_{rt}) \quad (29)$$

s.t.: (12), (13), (16)-(19). (30)

The Subproblem (27)-(28) is a nonlinear programming (NP) and its size depends on number of hydro plants in a same cascade. The Subproblem (29)-(30) is an integer-mixed NP problem with variables and constraints related with the hydro plant *r*. We use a Dynamic Programming algorithm which exploits the feasible set characteristics to decrease the state space associated with the binary variables. To sum up our LR description, we implemented a Bundle algorithm [17] to find a dual optimal efficiently.

2.8 *Phase II: Primal Recovery by an Inexact Augmented Lagrangian*

In this phase we apply an inexact AL which uses the Auxiliary Problem Principle (APP) [18]. The APP is necessary to keep the same separability accomplished in LR. The AL function has the same formulation as (25)-(26), except that is necessary to include quadratic terms in (25) in the following way:

$$\Theta_{\text{AL}} = \min -100 \cdot \left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G \cdot gh_{rt}(va_{rt}, Qa_{rt}, sa_{rt}) \cdot Qa_{rt}} \right] - \sum_{t=1}^{T}\sum_{r=1}^{R}\left[\lambda v_{rt}(va_{rt} - v_{rt}) - \frac{1}{2\mu}(va_{rt} - v_{rt})^2\right] - \sum_{t=1}^{T}\sum_{r=1}^{R}\left[\lambda Q_{rt}(Qa_{rt} - Q_{rt}) - \frac{1}{2\mu}(Qa_{rt} - Q_{rt})^2\right] - \sum_{t=1}^{T}\sum_{r=1}^{R}\left[\lambda s_{rt}(sa_{rt} - s_{rt}) - \frac{1}{2\mu}(sa_{rt} - s_{rt})^2\right] \quad (31)$$

s.t.: (12), (13), (16)-(19), (21) and (22). (32)

Above, $\mu > 0$ is the penalty parameter. To achieve the separability obtained by the LR, the APP approximates each quadratic term of (31) as follows. Suppose at iteration *it* we have calculated va_{rt}^{it}, v_{rt}^{it}, Qa_{rt}^{it}, Q_{rt}^{it}, sa_{rt}^{it} and s_{rt}^{it}. Then, using the APP at iteration $it + 1$, we approximate:

$$(va_{rt} - v_{rt})^2 = \left(va_{rt} - \frac{v_{rt}^{it} + va_{rt}^{it}}{2}\right)^2 + \left(v_{rt} - \frac{v_{rt}^{it} + va_{rt}^{it}}{2}\right)^2, \quad (33)$$

and similarly for the other quadratic terms. Using this approximation, the function (31)-(32) can be evaluated by the sum of R+1 subproblems:

$$\Theta^{\text{AL-HS}} = \min -100 \cdot \left[\frac{\sum_{t=1}^{T}\sum_{r=1}^{R} L_{rt}}{\sum_{t=1}^{T}\sum_{r=1}^{R} G \cdot gh_{rt}(va_{rt}, Qa_{rt}, sa_{rt}) \cdot Qa_{rt}} \right] - \sum_{t=1}^{T}\sum_{r=1}^{R}[\lambda v_{rt} \cdot va_{rt} + \lambda Q_{rt} \cdot Qa_{rt} + \lambda s_{rt} \cdot sa_{rt}] + \frac{1}{2\mu}\sum_{t=1}^{T}\sum_{r=1}^{R}\left[\left(va_{rt} - \frac{v_{rt}^{it} + va_{rt}^{it}}{2}\right)^2 + \left(Qa_{rt} - \frac{Q_{rt}^{it} + Qa_{rt}^{it}}{2}\right)^2 + \left(sa_{rt} - \frac{s_{rt}^{it} + sa_{rt}^{it}}{2}\right)^2\right] \quad (34)$$

s.t.: (21)-(22), (35)

and,

$$\Theta_r^{\text{AL-HC}} = \min \sum_{t=1}^{T}(\lambda v_{rt} \cdot v_{rt} + \lambda Q_{rt} \cdot Q_{rt} + \lambda s_{rt} \cdot s_{rt}) + \frac{1}{2\mu}\sum_{t=1}^{T}\left[\left(v_{rt} - \frac{v_{rt}^{it} + va_{rt}^{it}}{2}\right)^2 + \left(Q_{rt} - \frac{Q_{rt}^{it} + Qa_{rt}^{it}}{2}\right)^2 + \left(s_{rt} - \frac{s_{rt}^{it} + sa_{rt}^{it}}{2}\right)^2\right] \quad (36)$$

s.t.: (12), (13), (16)-(19). (37)

The iterative process in the AL can be summarized as follows. To solve the dual problem we must decrease μ over the iterations in order to force the primal feasibility. At the same time, it is necessary to update the Lagrange multipliers and we use an inexact gradient-like method; for more details, see [18] and references therein.

V. NUMERICAL RESULTS

The proposed solution strategy has been applied to obtain a daily unit commitment and operation schedule of a cascade with four hydro plants[3], whose diagram is presented in Figure 2. In this figure it is shown the installed capacity in each plant, the water traveling time (in brackets), and the number of units in each plant (in parentheses).

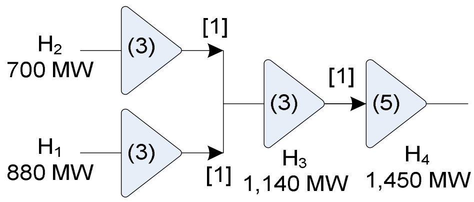

Figure 2: Schematic diagram of the cascaded hydro system.

Except for H_4, all hydro plants possess identical units with a single non-forbidden zone. The H_4 plant possesses two groups of different units. The first one is composed by units 1 and 2 and the remaining units belong to the

[3] Detailed data are too lengthy to list in this paper.

second group two. The only difference among the units of these groups is the turbine hydraulic efficiency function. Figure 3 illustratively shows the *tml* and *ggl* functions of a unit of H_4 hydro plant.

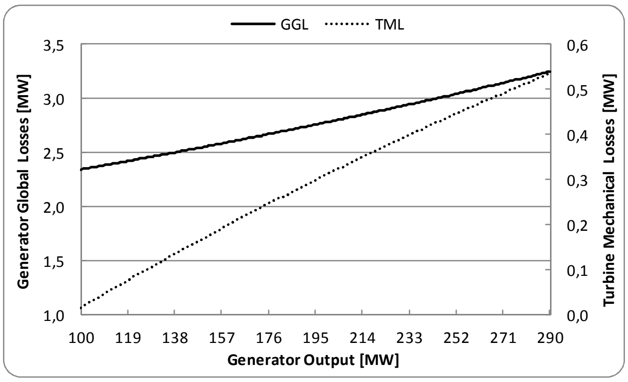

Figure 3: Mechanical and electrical losses – H_4 unit.

Figure 4 shows the load requirement of each plant over the planning horizon.

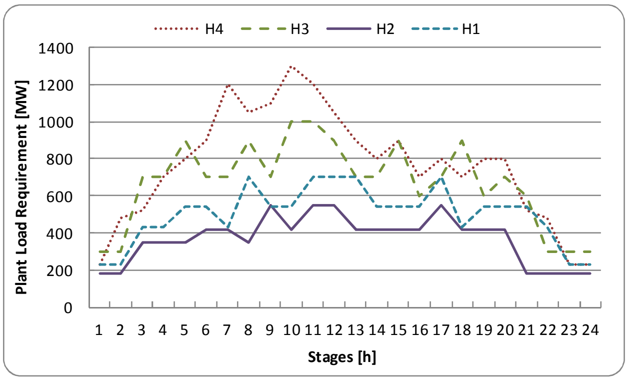

Figure 4: Hydro plants load requirement.

The numerical experiments are organized in two parts. Firstly, we focus on a particular test problem and analyze the performance of the two-phase decomposition approach. We are interested in the solution behavior and the algorithm performance in relation to mechanical and electrical losses of the units. Afterwards, we analyze the numerical performance accomplishing a sensitivity analysis when different initial volumes are supplied to the problem. The model was implemented in LabVIEW 9.0, and the tests were executed in an Intel Quadcore *i*7 2.80 GHz machine. All the NP problems were solved by the LabVIEW constrained NP subroutine, which uses a Sequential Quadratic Programming algorithm.

5.1 Mechanical and Electrical Losses Analysis

The LR performance of this first case, not considering the units mechanical and electrical losses, is shown in Figure 5. The algorithm was started with λv_{rt}, λQ_{rt} and λs_{rt} equal to 0.1 and it performed 40 iterations to find the optimal LR function equal to –91.1473 %. This value assures that the optimal overall efficiency is smaller than 91.1473 %, although the primal solution associated with this efficiency is not feasible, as it can be noticed in Figure 5. In the last LR iteration the subgradient vector norm is 5,295.43.

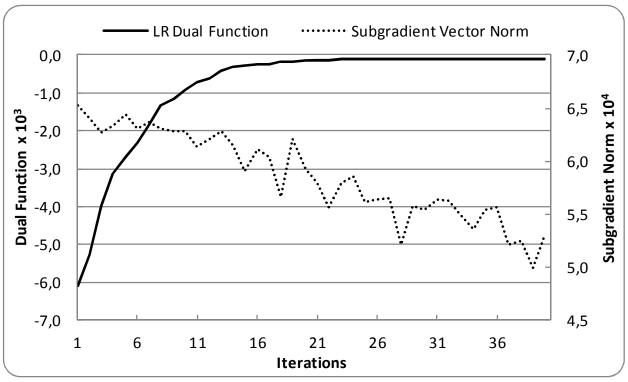

Figure 5: LR algorithm performance.

To find a feasible solution we use the inexact AL where the initial penalty parameter is $5\cdot 10^2$ and its update is performed by $\mu^{it+1} = 0.4\cdot\mu^{it}$ for $\mu > 0.5$ and $\mu^{it+1} = 0.98\cdot\mu^{it}$ for $\mu \leq 0.5$. The algorithm was stopped when $\|AC\|_2 \leq 0.1$ where $\|\cdot\|_2$ represents the Euclidean vector norm of the artificial constraints (23).

The AL performance is shown in Figure 6. This algorithm performed 37 iterations and the optimal value found was –90.9644 %. The associated efficiency was 90.9774 %.

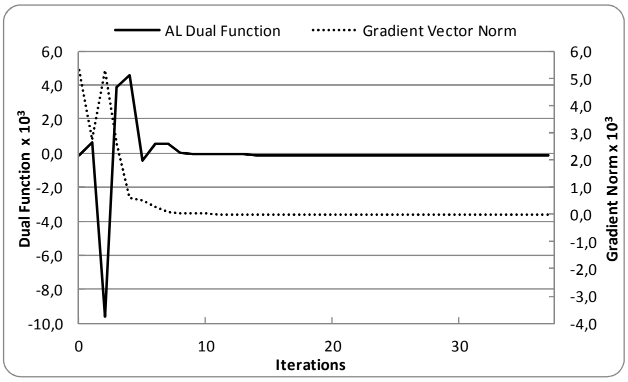

Figure 6: AL algorithm performance.

In Figure 7 we present the vector norm of the artificial constraints (23) in each stage in the last iteration. As it can be seen, a high precision feasible solution is supplied by the AL.

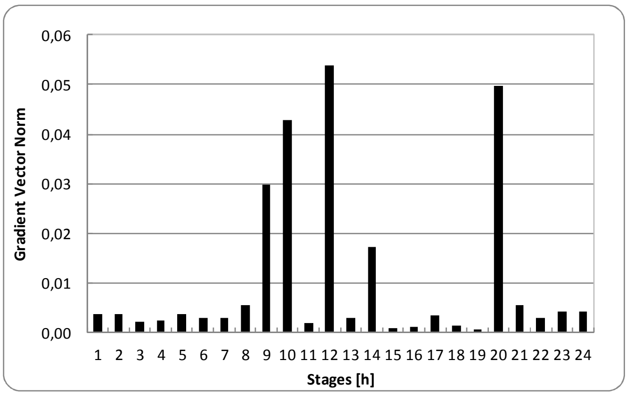

Figure 7: Vector norm of the artificial constraints in the last iteration.

In the previous analysis, the mechanical and electrical losses of the units were not taken into account. Including this modeling issue, the algorithm performed 42 iterations and the associated optimal value of the LR function found was –90.3753 %. Afterwards, the inexact AL found the optimal value equal to –89.3646 % in 43 iterations. In this case the overall efficiency was 89.3815 %. Compared to the previous results the efficiency was decreased in 1.75 % due to the inclusion of *tml*(·) and *ggl*(·) functions. In terms of primal solution, the main result is related to the difference between plants turbined outflow, as presented in Figure 8, where:

$$Qdif_t = Q_{1t} - Q_{0t}. \quad (38)$$

As it can be seen above, Q_{0t} (Q_{1t}) is the sum of all plants turbined outflow obtained without (with) losses in stage *t*.

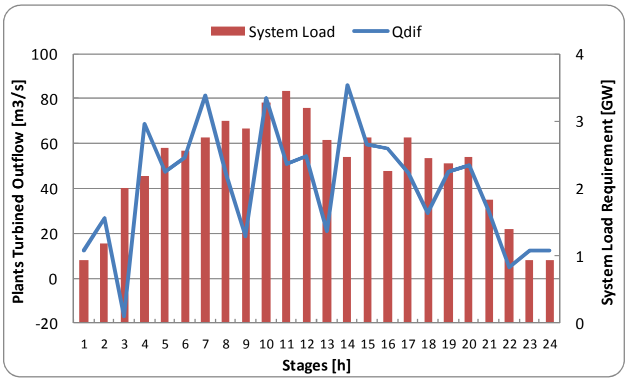

Figure 8: Difference between plants turbined outflow over the planning horizon.

From this point on, we pay attention to the effect of the losses on the final volumes. In Table 1 we compare some results:

Plant	Initial Volume (m^3/s)	Final Volume (m^3/s)	
		No Losses	*With Losses*
H_1	1,398.50	1,419.87	1,419.90
H_2	3,808.12	3,824.50	3,823.96
H_3	2,815.50	2,843.46	2,842.90
H_4	4,700.00	4,819.73	4,820.83

Table 1: Losses effect on the final volumes.

5.2 *Initial Volumes Analysis*

In this section we present an analysis where three initial volumes are considered. In this analysis we take into account the *tml*(·) and *ggl*(·) in the problem modeling. Table 2 shows the initial volume condition considered. All the remaining data is kept identical.

Plant	Initial Useful Volume (%)		
	Scenario 1	Scenario 2	Scenario 3
$H_{1,2,3,4}$	40	50	60

Table 2: Different Initial Volumes.

The AL performance of these cases is shown in figures 9 and 10.

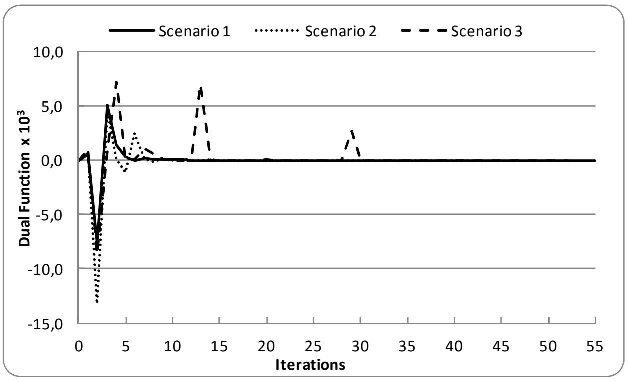

Figure 9: AL Dual Function performance.

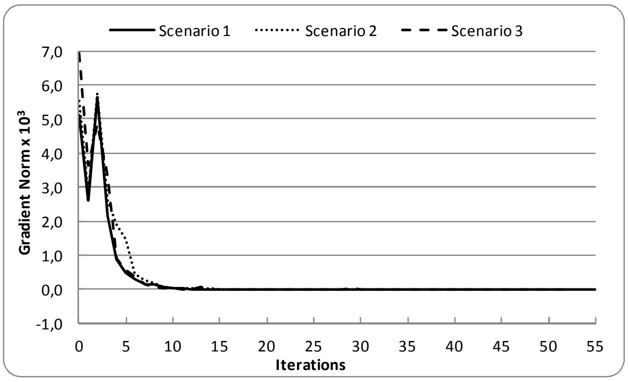

Figure 10: AL Gradient Vector Norm.

As it can be seen in the last two figures, the inexact AL algorithm can supply high precision feasible solutions using the same update criteria for the penalty parameter. These results indicate that the algorithm is robust in relation to the variation in the parameters of the problem.

In order to summarize the results Table 3 shows the optimal values (LR and AL Dual Functions) as well as the overall efficiency found by the solution strategy in all initial volumes condition.

Scenario	Results (%)		
	LR	AL	Overall Efficiency
1	-89.8846	-88.9003	89.0921
2	-90.3753	-89.3646	89.3815
3	-90.6261	-89.5005	89.5103

Table 3: Initial Volumes Analysis.

Some important conclusions can be assured by the table above and confirm the good algorithm performance. Firstly, the optimization by means of a Bundle method provides a good lower bound to the objective function since the AL results are very close to the LR. Additionally, the overall efficiency is also very close to the absolute AL dual optimal, which indicates that a high precision feasible solution is available for all scenarios.

VI. CONCLUSIONS

In this paper we have presented an efficient computational model for the HUC problem that has been developed to be used as a support tool of operation in the Brazilian system. The aim is to assist the operators to determine the optimal unit commitment and generation schedules for cascaded plants with multiple units and head-dependent hydro function production. The main contribution of this paper is the presentation of a new mathematical model for the hydro production function, where all relevant factors that affect the unit output are included, especially the hydraulic efficiency and mechanical losses in the turbine as well as the mechanical and electrical losses in the generator. The computational tool acts in accordance with the sustainable hydroelectric initiatives of the Brazilian regulatory framework to minimize waste of renewable energy.

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