ARES: AN ENERGY PRICE SIMULATION MODEL INCLUDING STRATEGIC BEHAVIOR

Frédérique Verrier — Panagiota Tsamasfyrou — Samuel Scolari
RTE
Versailles, France
frederique.verrier@rte-france.com panagiota.tsamasphyrou@rte-france.com samuel.scolari@rte-france.com

Peter Borre Eriksen — Berit Bitsch Kristoffersen — Bjarne Donslund
Energinet.dk
Fredericia, Denmark
peter.borre.eriksen@eltra.dk berit.bitsch.kristoffersen@eltra.dk bjarne.donslund@eltra.dk

Abstract - RTE and Energinet.dk developed a common market simulation tool called ARES. It is a supply function equilibrium model that simulates an integrated market of several zones with different prices interconnected by lines whose flows are restricted. Many companies can be implemented, that may have assets in different zones, in order to represent the economical reality in Europe. Different production mixes are considered and a simple representation of the network is used. We apply this tool to a quite simple case and show the results one can obtain.

Keywords - market power, market maker, simulator, oligopoly, Nash Equilibrium

1 INTRODUCTION

THE power markets in Europe are more and more opening to competition. Moreover, there is a growing interest among the continental european countries about market coupling. However, in most european countries there exist big incumbent generators that hold power on a large part of the country's generation capacity. Thus, competition may be weak and markets may fail to force prices down to marginal prices. Many questions arise then as to what may happen to prices subject to the actual rules and actual situation of markets. Another question concerns the impact of changes on the potential prices: what may be the impact of changes of rules or concentration of the market on the potential behavior of the marketers ?

Oligopoly simulation models [1, 2, 3, 4, 5] are perhaps one of the most powerful tools in exploring market power and its impact on markets by explicitly incorporating into one model many of the structural, behavioural and market design factors that are related to market power, including concentration, demand elasticity, supply curve bidding, forward contracting, and in some cases transmission constraints. Although no modelling approach can be used to predict prices in oligopolistic markets, there appears to be agreement that equilibrium models are valuable for gaining insights on modes of behaviour and relative differences in efficiency, prices, and other outcomes of different market structures and designs. As a result, these models can then be used to explore the above questions.

Equilibrium market models differ in many ways, including the market mechanisms modelled, the type of network constraints taken into account, and computational methods. Analysis of the theoretical and practical differences between different approaches are given in [6, 7].

To study the questions about the impact of changes of rules or concentration of the market, Energinet.dk[1] and RTE developed a common simulation tool, ARES. This tool is based on an existing tool, MARS [8], developed by ELTRA. ARES is a supply function equilibrium model that simulates an integrated market of several zones with different prices interconnected by lines whose flows are restricted. Many companies are implemented, that may have assets in different zones, in order to represent the economical reality in Europe. Different production mixes are considered, such as hydropower, nuclear power and conventional means. An elastic representation of the demand by a Cobb-Douglas function has been chosen. The bid of each generator is composed of the marginal cost plus a markup, that each company fixes so as to optimize its profit. A Nash equilibrium is reached when no company has any incentive to unilaterally change its bids. In the current formulation of the problem, we assume that the market players are price takers with respect to the price of transmission charged by the TSOs and therefore the price of transmission is not a decision variable for them.

In this paper, we first describe the two-level formulation of the problem: one optimisation problem to determine the marginal price of the market taking into account all the strategic decisions of the companies and one where each company is maximising its profits by considering the decisions of the other companies as fixed. We then propose an original formulation of the problem that transforms the initial bi-level problems into one problem for each company: maximize the profit of one company while the mark-ups of the other companies are held constant. We eventually show the results obtained on a very simple test. We illustrate, on this example, the modifications introduced by the exercise of market power.

2 FORMULATION

The model includes a number of zones, interconnected by pipes representing tie-lines and a number of market players (companies). Each company may have assets in many zones and thus must have a common policy. We

[1]Energinet.dk is the result of a merger between Eltra, Elkraft and Gastra.

assume that the demand is elastic and represented by a Cobb-Douglas function. There isn't any representation of the underlying network, except for the tie-lines.

Each company bids a quantity of production in the market at a price based on the marginal cost plus a markup. It seeks the optimal markup value so as to optimize its profit. Then, the central authority that clears the market decides how much power to buy from which units, how much power to deliver to customers and what prices to charge, based on the optimisation of the socio-economic-surplus. To optimise the socio-economic-surplus the central authority uses the bids of the different players rather than the true cost functions. A Nash equilibrium is reached when no company has incentive to unilaterally change its bids.

2.1 Notations

We present hereafter the different notations used to formulate the problems:

- i is a zone of the network;
- g is a generator;
- c is a company;
- $i \rightarrow j$ is the interconnection from zone i to zone j;
- x^* is the optimal value of variable x.
- $C_g(.)$ is the marginal cost function of generator g. In general, we assume that the marginal cost function is constant, that is $C_g(p_g) = MC_g$(in €/MW) . For some generators, the marginal cost is production dependant in a linear way, that is $C_g(p_g) = MC_g + \lambda p_g$. For the sake of simplicity, we will consider that $C_g(p_g) = MC_g$ throughout this paper. Cost functions for entire companies are step functions. Since it is easier to have only constants, each generator is virtually divided in many generators, with as many generators as steps;
- $\mu_{c,i}$ is the markup of company c for zone i;
- p_g is the production of generator g;
- $\bar{p}_g$ is the maximum production of generator g;
- $bid_g(.)$ is the bid function of generator g owned by company i. The bid function depends on the production of this generator, but also on the production of other generators owned by company i that cost less than generator g;
- $Q_{i\rightarrow j}$ is the directional flow on the interconnection line flowing from zone i to zone j;
- $\bar{Q}_{i\rightarrow j}$ is the maximum exchange permitted in the interconnection from zone i to zone j;
- $\tau_{i\rightarrow j}$ is the exchange tariff for exchange from zone i to zone j;
- π_i the price of zone i;
- $f_i(x) = kx^{-\frac{1}{\beta}}$ is the constant elasticity demand curve in zone i, where x is the total production in zone i plus the imports minus the exports. We have $\pi_i = f_i(x^*)$; The parameters k and β are calibrated to fit the data we want to simulate and to ensure that the derivative of the function is always negative. Calibration parameters are the average load and the corresponding average price, as well as the elasticity of demand.

For hydro power producers, we chose to use water values rather than production costs, in order to obtain a more realistic bidding curve. Indeed, actual variable production costs for hydro producers are in the short term either equal or very close to zero, and using these costs would not enable us to mimic the behaviour that can be seen from most hydro power producers. These water values are calculated ex ante and given to the model.

The companies that adopt a strategic behaviour submit a linear non-decreasing bid function that is the sum of the standard marginal cost function and the total production of the (strategic) generators owned by the company in the zone times the chosen markup μ_c, see Fig. 1. Therefore, the bid for the production of generator g situated at zone i and owned by company c is the following:

$$bid_g(p_g) = C_g + \mu_{c,i}\left(\sum_{\substack{k\in c,i\\ C_k<C_g}} p_k + p_g\right)$$

The graphs below explain how the bid curve for generator g is derivated from the agregated bid curve for company c (that owns generator g) :

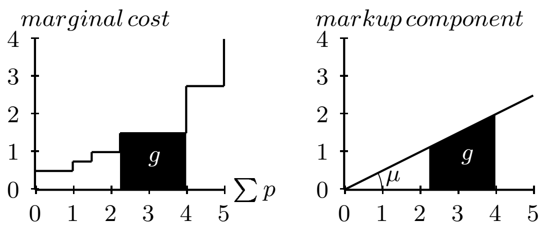

Figure 1: The agregated bid curve for company c is the sum of the marginal cost curve (left) and of the markup component (right)

The expression of the bid curve for generator g is then straightforward: it is the sum of the marginal cost MC_g (dotted line on the graph below) plus the corresponding markup (shaded areas on the two graphs above).

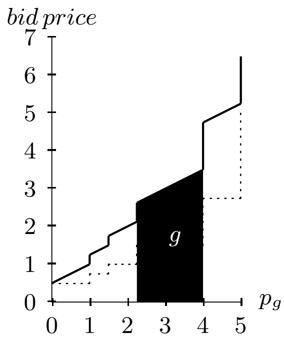

Figure 2: Total bid curve for company c

We will hereafter use these notations to write the two levels of the initial problem: one level of maximisation of the socio-economic-surplus to determine the marginal price of the market taking into account the bids of the companies and one level of maximisation of the profits for each company by choosing adequately its markup.

2.2 Maximisation of socio-economic-surplus

The as bid socio-economic-surplus is defined as follows:

$$z = s - pc - ic \qquad (1)$$

where

- s is the consummers' surplus:

$$s = \sum_i \left(\int_0^{\sum_{g\in i} p_g + imp_i - exp_i} f_i(p)dp \right) \qquad (2)$$

  Cobb Douglas function's integral is infinite near 0, we thus choose to compute this integral from an arbitrary production level and not from 0. For each area, this level is arbitrarily chosen as the energy demand at the average price. This creates a shift in the socio-economic-surplus value that has no influence on the results.

- pc is the production cost (as bid):

$$\begin{aligned} pc &= \sum_{i,c} \sum_{g\in i,c} \int_0^{p_g} bid_g dp_g \\ &= \sum_i \sum_{g\in i} p_g MC_g \\ &+ \frac{1}{2} \sum_i \sum_c \mu_{c,i} \left(\sum_{g\in c,i} p_g \right)^2 \end{aligned} \qquad (3)$$

  where c is the owner of the generator g.

- ic is the interconnection costs :

$$ic = \sum_{i,j i\neq j} Q_{i\to j} \tau_{i\to j} \qquad (4)$$

To determine the marginal price of the market we maximise the socio-economic-surplus by taking into account all the strategic decisions of the companies. In this step, mark-up factors are fixed to the value given by the different companies. The maximisation of the socio-economic-surplus is formulated as follows:

$$(SW) \begin{cases} \max z \\ s.t. \\ 0 \le p_g \le \quad \bar{p}_g \qquad \mu_g^1, \quad \mu_g^2 \\ 0 \le Q_{i\to j} \le \quad \bar{Q}_{i\to j} \qquad \nu_{i\to j}^1, \quad \nu_{i\to j}^2 \end{cases} \qquad (5)$$

where μ_g^1 (respectively μ_g^2) are the Lagrange variables associated with the constraint $p_g \le \bar{p}_g$ (respectively $0 \le p_g$) and $\nu_{i\to j}^1$ (resp. $\nu_{i\to j}^2$) the Lagrange variables associated with the constraint $Q_{i\to j} \le \bar{Q}_{i\to j}$ (resp. $0 \le Q_{i\to j}$).

The resolution of problem (SW) provides the vector π^* of equilibrium prices for each zone, such that $\pi_i^* = f_i(x^*)$.

2.3 Profit maximisation

Each company fixes its markups (one for each zone in which it has some assets) so as to optimise its profits. The profit Π_c of company c is:

$$\Pi_c = \sum_i \sum_{g\in c} p_g (\pi_i - MC_g) \qquad (6)$$

However, for a company c with market power the price π_i in the above definition of the profit is an endogenous variable, since the market makers may alter the price of some zones by adjusting their markups. Indeed, this variable is the result of the optimisation of the (SW) problem defined above, which uses the markup and the production of company c. Then, the problem each strategic company faces is the maximisation of Π_c under the constraint "z is maximal". The structure of this problem is a bilevel optimisation, i.e. it is a mathematical program with equilibrium constraints for each company, and the resulting market equilibrium problem is an Equilibrium Problem under Equilibrium Constraints (EPEC).

Each company must therefore find the right balance between:

- adding too high a mark-up and loosing market shares;

- adding a small mark-up and not taking enough advantage of its position.

2.4 Compact formulation of the problem

The problem described above is a two-level one: the first level is the profit maximisation and the second level is the maximisation of the socio-economic surplus. The constraint "z is maximal" in the profit maximisation problem can be converted into equations. Indeed, the initial problem (SW) is a convex one (see demonstration in §5). Therefore, forming the Karush-Kuhn-Tucker optimality conditions for the (SW) problem and using the Lagrange multipliers corresponding to its constraints, we obtain the following mixed linear complementarity formulation of the second-level problem:

$\forall g$

$$\left.\begin{array}{ll} -\pi_i + MC_g + \mu_c \sum_{k\in c} p_k + \mu_g^1 - \mu_g^2 & = 0 \\ \mu_g^1 (p_g - \bar{p}_g) & = 0 \\ \mu_g^1 & \ge 0 \\ \mu_g^2 p_g & = 0 \\ \mu_g^2 & \ge 0 \end{array}\right\} \qquad (7)$$

and $\forall i \to j$

$$\left.\begin{array}{ll} -\pi_j + \pi_i + \tau_{i\to j} + \nu_{i\to j}^1 - \nu_{i\to j}^2 & = 0 \\ \nu_{i\to j}^1 (Q_{i\to j} - \bar{Q}_{i\to j}) & = 0 \\ \nu_{i\to j}^1 & \ge 0 \\ \nu_{i\to j}^2 Q_{i\to j} & = 0 \\ \nu_{i\to j}^2 & \ge 0 \end{array}\right\} \qquad (8)$$

By adequately eliminating the Lagrange multipliers (based on the above constraints), we obtain the following set of conditions:

$$\left.\begin{array}{lr} p_g(\pi_i - MC_g - \mu_{c,i}\sum p_k) & \geq 0 \\ (p_g - \bar{p}_g)(\pi - MC_g - \mu_{c,i}\sum p_k) & \geq 0 \\ (\pi_j - \pi_i - \tau_{i\to j})(Q_{i\to j} - \bar{Q}_{i\to j}) & \geq 0 \\ (\pi_j - \pi_i - \tau_{i\to j})Q_{i\to j} & \geq 0 \\ 0 \leq p_g \leq \bar{p}_g & \\ 0 \leq Q_{i\to j} \leq \bar{Q}_{i\to j} & \end{array}\right\} \qquad (9)$$

Those equations are in fact intuitive:

- If the bid of a generator (marginal cost + mark-up times total production) is strictly greater than the price in the area, then $p_g = 0$. Else, if it is strictly lower, then $p_g = \bar{p}_g$;
- If the price difference between two areas is greater than exchange tariff, then $Q_{i\to j} = \bar{Q}_{i\to j}$ and else $Q_{i\to j} = 0$.

Therefore, each company solves the following problem:

$$\left.\begin{array}{l} \max \sum_i \sum_{g\in c} p_g(\pi_i - MC_g) \\ \text{under the constraints (9)} \end{array}\right\} (PM) \qquad (10)$$

The above problem is highly non-linear, since both the objective function and the constraints (equilibrium constraints) are non-linear, as well as nonconvex. This formulation has the advantage to have less variables and constraints than the standard one: maximising the profits under the constraints (7,8).

3 RESOLUTION

3.1 Algorithm

The problems (10) are highly non-linear, thus the solution (local optimum) depends greatly on the starting point. To generate good starting points, we use the solution arguments of the (SW) problem (5) for the case of perfect competition (by setting the mark-up factors of all companies equal to 0) and other points, generated to cover as large a space as possible. We then choose the best result from the different optimisations. For the algorithm implemented above we used 6 starting points. We use the platform GAMS [10] to write this problem, and we use the solver CONOPT [11, 12] , version Conopt3, provided by GAMS to solve it. Conopt is found to be an efficient algorithm to solve such highly non-linear problems.

Let c now denote the index of companies with market power. The maximum number of companies with market power in the problem is $\bar{c}$. Respectively, l is the index of iterations for the research for a Nash equilibrium and $\bar{l}$ is the maximum number of such iterations. In our model, we set $\bar{l} = 7$.

In order to find the Nash supply function equilibrium, the following algorithm, using a Gauss-Seidel iterative method to solve the multi-company problem, has been implemented:

1. Initialisation: fix all markups to 0: $\forall c, i$ set $\mu_{c,i} = 0$; set c to 0 and l to 1;

2. Iteration l:

   (a) Increment c: $c = c + 1$. Is $c > \bar{c}$?

   - YES. We set c back to 0. GO TO STEP 2c;
   - NO. At the end of this first step, company c has market power;

   (b) Is $c == 1$?

   - YES. Test for a Nash equilibrium: solve for all starting points the problem (10) of maximisation of profit for the company $c = 1$. The markups of the other companies are fixed at the values found from the last optimisation of their own profit, $\mu_{c',i} = \mu^{l-1}_{c',i}, \forall c' > c$. We keep the value of markup corresponding to the greatest profit found for the different starting points, $\mu^{*,l}_{c,i}$.
     i. If the profit and the markup are identical to the ones found at iteration $l - 1$, a Nash equilibrium is found. STOP;
     ii. If the profit and the markup are identical to the ones found at iteration $l - k$, with $k > 1$,then it is a cyclic Nash equilibrium. STOP;
     iii. If the profits are not identical to the ones found at iteration $l - k$, then GO TO 2a;
   - NO. Solve the problem (10) for company c for all starting points. The markups of the other companies are fixed to the values found after the last optimization of their own profit (or to 0 if it has not been optimized yet): $\mu_{c',i} = \mu^{l}_{c',i}, \forall c' < c$ and $\mu_{c',i} = \mu^{l-1}_{c',i}, \forall c' > c$. We keep the value of markup corresponding to the greatest profit found for company c among the different starting points, $\mu^{*\,l}_{c,i}$. GO TO STEP 2a.

   (c) Increment l: $l = l + 1$. Is $l + 1 \leq \bar{l}$?;

   - YES: go to step 2;
   - NO: the maximum number of iteration has been reached without finding a Nash equilibrium. STOP.

As one can see, there are three ways to exit the program: finding a Nash equilibrium at step 2(b)i, a cyclic Nash equilibrium at step 2(b)ii, or exceeding the maximum number of iterations.

In general, a Nash supply function equilibrium in pure strategies does not necessarily exist, nor is it necessarily unique [13]. The Nash equilibrium we are trying to find using this algorithm is thus of course a semi-heuristic Nash equilibrium, as we cannot prove that the solution found is indeed a Nash equilibrium. The only thing we are sure of when we reach what we call a Nash equilibrium is that, given the starting points we chose, the solver does not see any incentive for any of the company with market power to adopt a different strategy than the one found in the Nash equilibrium solution. However, even if we cannot guarantee that a Nash equilibrium will be found, the results we have tend to show that in most cases we succeed in finding one.

3.2 Test on a Simple Network

The model has been tested on a data set of 4 interconnected areas, A, B, C and D. Care was taken to include in this case, although quite simple, wind generation, CHP and a realistic representation of hydro producers using water values.

We used Cobb-Douglas functions with the same calibration parameters for all four zones, except for the average demand.

Transmission lines exist between the zones A and D, B and C, B and D and C and D. All these pipelines are modelled as exchange capabilities; the four limits have been fixed at the same value.

We model conventional means (thermal and nuclear) and hydro producers, as well as price independant generation (such as wind producers and hydroplants without reservoir). The supply curve of these later means is a vertical line. The supply function of the hydro production (with reservoirs) is computed as weekly values based on water values given to the model. For the simulations, the hydro producers are considered competitive. Out of the 6 power producers of the data set, 3 are assumed to have the possibility to exercise market power. The three companies with market power are located in zones A, B and D.

The different starting points chosen, as mentioned in 3.1 are the following :

1. $p^{(k)^0} = p^{(k-1)^*} - \epsilon$
   $Q^{(k)^0} = Q^{(k-1)^*} - \epsilon$
   $\mu^{(k)^0} = \mu^{(k-1)^*} - \epsilon$;
2. $p^{(k)^0} = p^{(k-1)^*} - \epsilon$
   $Q^{(k)^0} = Q^{(k-1)^*} - \epsilon$
   $\mu^{(k)^0} = 0.1$;
3. $p^{(k)^0} = p^{(k-1)^*} - \epsilon$
   $Q^{(k)^0} = Q^{(k-1)^*} - \epsilon$
   $\mu^{(k)^0} = \max(0.1, \min(\bar{\mu}, 2 \cdot \mu^{(k-1)^*}))$;
4. $p^{(k)^0} = 0.7 \cdot p^{(k-1)^*} + 0.1 \cdot \bar{p}$
   $Q^{(k)^0} = 0.8 \cdot Q^{(k-1)^*} + 0.1 \cdot \bar{Q}$
   $\mu^{(k)^0} = \min(1.01 \cdot \mu^{(k-1)^*}, \bar{\mu})$;
5. $p^{(k)^0} = 0.3 \cdot \bar{p}$
   $Q^{(k)^0} = 0.5 \cdot \bar{Q}$
   $\mu^{(k)^0} = 0.03$;
6. $p^{(k)^0} = \min(0.8 \cdot p^{(k-1)^*} + 0.3 \cdot \bar{p}, \bar{p})$
   $Q^{(k)^0} = 0.8 \cdot Q^{(k-1)^*} + 0.1 \cdot \bar{Q}$
   $\mu^{(k)^0} = 0.0$.

where $x^{(k-1)^*}$ denotes the previous solution and $x^{(k)^0}$ the initial value taken for step k.

Using different starting points enables us to cover a wide range in the solution space. Different values as well as a different number of starting point have been tried before these values were chosen; the table below shows the results obtained at the end of the first iteration of the algorithm, when only the markup of the first company, that has market power in zone A, is being optimised, the markup of the others being fixed to 0, with a different number of starting points :

	1	3	6	7
Π_1	53217	53134	71710	71710
π_A	21.79	21.80	29.03	29.03
π_B	21.99	22.00	29.23	29.03
π_C	31.83	31.83	31.83	31.83
π_D	21.89	21.90	29.13	29.13

Table 1: Values of the profit of company 1, the one with market power in zone A, as well as zonal prices, for different numbers of starting points, for the first hour

Using 6 starting points seemed to achieve an acceptable balance between computation time and accuracy of results. Using 6 points, we have done many runs for different starting points: in the table below are some examples of the results that can be found - at the end of the first iteration as well as for the first hour - by modifying starting point #5:

$p^{(k)^0}$	$0.3 \cdot \bar{p}$	$0.8 \cdot \bar{p}$	$0.8 \cdot \bar{p}$
$Q^{(k)^0}$	$0.5 \cdot \bar{Q}$	$0.5 \cdot \bar{Q}$	$0.5 \cdot \bar{Q}$
$\mu^{(k)^0}$	0.03	0.03	0.93
Π_1 first iteration	71710	26735	53134
Π_1 equilibrium	71710	-	75869
Π_2 equilibrium	54316	-	48217
Π_5 equilibrium	71050	-	71050

Table 2: Profit, for the first hour, for company 1 at the end of the first iteration and for the Nash equilibrium using different values for starting point #5

At the end of the resolution and for the solution that is found to be the Nash solution, we ran two resolutions of the socio-economic-surplus for a value of μ equal to $\mu + \epsilon$ and $\mu - \epsilon$ and found that the profits thus obtained are not better than the previous profits. In the above table, no Nash equilibrium can be found using the starting point proposed in the middle. For the other two values of starting point #5, a Nash equilibrium has been found, but it is not the same : company 2 is better off in the first case, whereas it is the opposite for company 1. In this example, two Nash equilibria have been found ; it is very likely that there are even more than just 2 Nash equilibrias. However, we have no right to say that the first equilibrium is better than the second. We could use mixed strategies in our formulation in order to account for the case where multiple Nash equilibria are found ; this has not been done yet.

Using the standard starting points, the average number of global iterations required to reach the Nash equilibrium is 4.

The graph below (Fig. 3) shows the difference between the two prices for zone A.

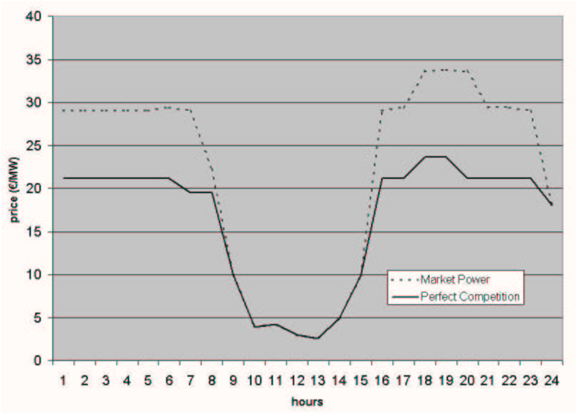

Figure 3: Comparison in zone A between the competitive price and the case where market power is exercised

Exercising market power results, as one could have expected, in higher prices than the ones obtained with perfect competition. The price distortion reaches an average of 8€, except between hours 18 and 21, where it goes up to 12€, and between hours 9 and 16, where it goes down to 0€.

The prices in zone A are very low around peak hours; the explanation is that a lot a wind power is available at that point of time, and that, as a result, very little power needs to be produced using other production means. It then becomes very difficult for power producers to exercise market power. This fact leads to a tiny difference between the competitive price and the price achieved under the exercise of market power.

We also show in the graph below (Fig. 4) the shift in the profit of the company with market power in zone A.

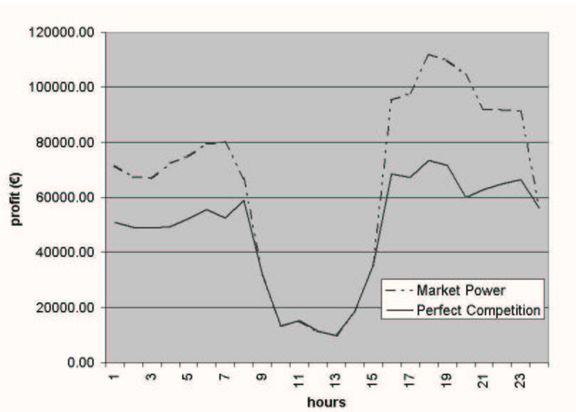

Figure 4: Profit shift for company with market power in zone A

By exercising its market power, this company has managed to increase its markup by 34% on this single day. Although simple, this example shows that some companies could have huge incentives to exercise their market power, when they have some.

4 CONCLUSION

We have shown an alternative formulation of the initial two-stage problem of maximisation of profit for strategic producers that operate in a market with central clearing. Based on this formulation we developed, using GAMS, a tool for the resolution of such problems.

We then tested this tool on a simple case to assess its efficiency and robustness; we also performed a first analysis of the exercise of market power on this example.

The next steps of our work are to test this tool on more detailed data sets, and to enhance its robustness. This tool can then be used to analyze the market as well as any modification in the market's rules and design.

Further work will implement the transmission rights and study their impact on the profit and prices, as well as the description of the network using physical limits and implementing loop flows rather than exchange limits.

5 APPENDIX: CONVEXITY PROOF

The constraints of the problem (5) are linear. Therefore, to prove its convexity, we must show that the Hessian of the objective function is semi-positive definite. In the beginning, we assume that the markup is 0 for all companies. Then,

$$\left.\begin{array}{lll}\dfrac{\partial(-z)}{\partial p_g} = -\pi_i + MC_g & \forall g \in i \\ \dfrac{\partial(-z)}{\partial Q_{i\to j}} = \pi_i - \pi_j + \tau_{i\to j} & \forall i, \forall j,\, i \neq j\end{array}\right\}$$

Because of the Cobb-Douglas function, we have:

$$-\frac{\partial \pi_i}{\partial p_g} = -\frac{\partial \pi_i}{\partial Q_{j\to i}} = \frac{\partial \pi_i}{\partial Q_{i\to j}} =: y_i \geq 0$$

Then, the coefficients of the Hessian are:

$$\left.\begin{array}{ll}\dfrac{\partial^2(-z)}{\partial p_g^2} = y_i & \forall g \in i \\ \dfrac{\partial^2(-z)}{\partial p_g \partial p_k} = 0 & k, g \text{ not in the same area} \\ \dfrac{\partial^2(-z)}{\partial Q_{i\to j}^2} = y_i + y_j & \forall i, \forall j,\, i \neq j \\ \dfrac{\partial^2(-z)}{\partial p_g \partial Q_{i\to j}} = -y_i & \forall g \in i \\ \dfrac{\partial^2(-z)}{\partial p_g \partial Q_{j\to i}} = y_i & \forall g \in i \\ \dfrac{\partial^2(-z)}{\partial p_k \partial Q_{i\to j}} = 0 & \forall g \notin i, j\end{array}\right\}$$

For example, the Hessian of $-z$ in the case of three areas takes the form $\mathfrak{H}^0 =$:

$$\begin{array}{c} p_{g1} \\ p_{g2} \\ p_{g3} \\ Q_{1\to2} \\ Q_{2\to3} \\ Q_{1\to3} \end{array} \begin{pmatrix} y_1 & 0 & 0 & -y_1 & 0 & -y_1 \\ 0 & y_2 & 0 & y_2 & -y_2 & 0 \\ 0 & 0 & y_3 & 0 & y_3 & -y_3 \\ -y_1 & y_2 & 0 & y_1+y_2 & -y_2 & y_1 \\ 0 & -y_2 & y_3 & -y_2 & y_2+y_3 & y_3 \\ -y_1 & 0 & y_3 & y_1 & y_3 & y_1+y_3 \end{pmatrix}$$

In the above matrice, we omitted the lines and corresponding columns associated with the lines in the opposite direction (p.e. $Q_{j\to i}$), as they have the same coefficients as the lignes for $Q_{i\to j}$ with opposite sign. Moreover, we only represent one generator per area, since the lines corresponding to the generators in the area have the same coefficients. We can see that the line corresponding to $Q_{i\to j}$ is the sum of line p_{gj} minus line p_{gi}. More generally, if we compute the Hessian for a problem with n zones, the lines ranging from $n+1$ to $n+\frac{n(n-1)}{2}$ are linear combinations of the first n lines. Thus, the principal minors of this Hessian are positive or null and its rankis the number of zones of the problem I. We can see that all the leading principal minors of a principal minor of dimension I are positive (since a principal minor is a diagonal matrix with y_i at the diagonal position). Therefore, the function is semi-positive. If we were to compute the Hessian of the objective function of the problem (5) in the case where the markup μ is different from zero, we would obtain $\mathfrak{H}^\mu$:

$$\mathfrak{H}^\mu = \mathfrak{H}^0 + \begin{pmatrix} \mu_{c_{g1},i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_{c_{g2},j} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{c_{g3},k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where μ is positive or null[2]. We then have

$$\mathfrak{H}^\mu = \mathfrak{H}^0 + \text{diag}(\mu_1, \mu_2, \dots, \mu_n, 0, \dots, 0)$$

thus the Hessian is still semi-definite positive, even in the case where the markup is non-null.

As a result, $-z$ is convex.

Acknowledgements

We would like to thank our reviewers for their remarks that helped us improve the present paper.

References

[1] B.F. Hobbs, C.B. Metzler, J.-S. Pang, "Strategic Gaming Analysis for Electric Power Systems: An MPEC Approach," IEEE Transactions on Power Systems, Vol. 15, No 2, May 2000.

[2] B.F. Hobbs, F.A.M. Rijkers, "Strategic Generation with Conjectured Transmission Price Responses in a Mixed Transmission Pricing System – Part I: Formulation," IEEE Transactions on Power Systems, Vol. 19, No 2, May 2004.

[3] B.F. Hobbs, F.A.M. Rijkers, A.F. Wals, "Strategic Generation with Conjectured Transmission Price Responses in a Mixed Transmission Pricing System – Part II: Application," IEEE Transactions on Power Systems, Vol. 19, No 2, May 2004.

[4] B.F. Hobbs, "Linear Complementarity Models of Nash-Cournot Competition in Bilateral and POOLCO Power Markets," IEEE Transactions on Power Systems, Vol. 16, No 2, May 2001.

[5] W. Xian, L. Yuzeng and W. Shaohua, "Oligopolistic Equilibrium Analysis for Electricity Markets: a Nonlinear Complementarity Approach," IEEE Transactions on Power Systems, Vol. 19, No 3, August 2004.

[6] K. Neuhoff, J. Barquin, M.G. Boots, A. Ehrenmann, B.F. Hobbs, F.A.M. Rijkers, "Network-constrained Cournot models of liberalized electricity markets: The devil is in the details,", Energy Economics, 2005.

[7] O. Daxhelet, Y. Smeers, "Variational inequality models of restructured electricity systems," in M.C. Ferries, O.L. Mangasarian and J.S. Pang, Applications and Algorithms of Complementarity, Kluwer, Dordrecht.

[8] ELTRA, "A Brief Description of ELTRA's Market Model MARS," http://www.eltra.dk/media(15185,1033)/Beskrivelse-_af_Eltra%27s_Markedsmodel_MARS-GB-okt03.pdf

[9] Metzler, C., B.F. Hobbs, and J.-S. Pang "Nash-Cournot Equilibria in Power Markets on a Linearized DC Network with Arbitrage: Formulations and Properties," Networks & Spatial Economics. vol.3. 2 (2003). pp. 123 - 150.

[10] GAMS, "General Algebraic Modeling System, Modeling for the real world!," http://www.gams.com

[11] A. S. Drud, "CONOPT – A Large Scale GRG Code," ORSA Journal on Computing 6, pp. 207-216, 1994

[12] K. Holmström, A. O. Göran, M. M. Edvall, "User's Guide for Tomlab/Conopt," http://tomlab.biz

[13] P.D. Klemperer, A.M. Meyer, "Supply Function Equilibria," Econometrica, vol. 57, pp. 1247-1277, 1989.

[2]As before, only one line and column corresponding to one generator in one zone and belonging to the one company are given, as the other lines and columns corresponding to the other generators have the same coefficients