A CASE STUDY ON REACTIVE POWER IMPACT ON DETERMINATION OF CRITICAL LOAD LEVELS

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***Abstract* —LMP versus System Load curve, as well as system status trajectory curve, may exhibit significant change in curve pattern at some load levels, called Critical Load Levels. Majority of the critical load levels can be identified through an established criterion which is defined on marginal unit set and congested branch set. However, exceptions have been observed in a revised IEEE 118-bus system where the existing criterion fails to identify a CLL. A detailed case study has been performed around the missing CLL. Study shows that this CLL is caused by status change in generator reactive power constraint and voltage magnitude constraint. Results suggest to expand the existing criterion to include the generator reactive power constraint in order to capture potential missing CLLs.**

Key Words: Locational Marginal Price (LMP), Optimal Power Flow (OPF), AC-OPF, DC-OPF, binding constraint, Critical Load Level (CLL)

I. INTRODUCTION

Locational marginal price (LMP) has been a widely used pricing mechanism in US electricity markets. LMPs change over time in an ever-changing market, and price spikes can be observed from time to time. One of the major contributors to this LMP behavior is load variation combined with piece-wise constant bidding rule adopted in most power markets [1-4]. In some markets, piece-wise linear bidding curve is also accepted. Nevertheless, in normal practices it is often approximated internally by piece-wise constant curve for the benefit of utilizing linear programming algorithms. In general, LMP changes steadily when load varies in a certain range, and it may change radically, and sometimes jump, at some load levels. These load levels have been studied and named Critical Load Levels (CLLs) around which significant change of LMP variation pattern may occur [5, 6].

Studies have shown that over the course of load change, LMP jump may happen when there is a shift of marginal units or change in branch congestion statuses [7]. For linear programming model such as a lossless DCOPF, the optimal solution is the vertex intersected by all binding constraints. When model parameter such as system load changes slightly, the optimal solution will remain at the same vertex although the polyhedron denoting the feasibility region has changed its shape. When system load changes beyond a certain level (i.e., a CLL), the optimal solution may jump to one of the adjacent vertices. This may result in a significant change of solution pattern with respect to the system load variation. This translates to a good criterion for determining the location of CLLs, which is defined on the status of marginal unit set and congested branch set. The criterion has been successfully employed in identifying CLLs in both lossless DC-OPF framework [6] and with-loss DC-OPF framework [17]. In AC-OPF framework, the above criterion generally works well. However, exceptions have been observed which tend to suggest the reactive power and voltage may also play a role in affecting LMP pattern and locations of CLLs.

Figure 1 shows LMP versus Load curve for eight selected buses of a revised IEEE 118-bus system [8, 9]. In load range from 5000MW to 5800MW, 10 CLLs are identified using the above criterion. Significant LMP pattern changes can be perceived at some of the CLLs while LMPs change steadily at other CLLs. For instance, LMP at bus 43 exhibits a bump starting at a CLL of 5233.29MW and starts to drop sharply at another CLL of 5264.66MW. Likewise, LMPs at bus 22 and 72 start to increase quickly at CLL of 5264.66MW. No radical LMP pattern changes are present at other identified CLLs. While it demonstrates the usefulness of the existing criterion, load level 5336MW, however, is not identified as a CLL even though a noticeable LMP pattern change occurs at that load level. For instance, LMPs at bus 22 and 72 quit rapid growth at 5336MW and start increasing much slowly, while the sharp drop of LMP at bus 43 stops at the same load level.

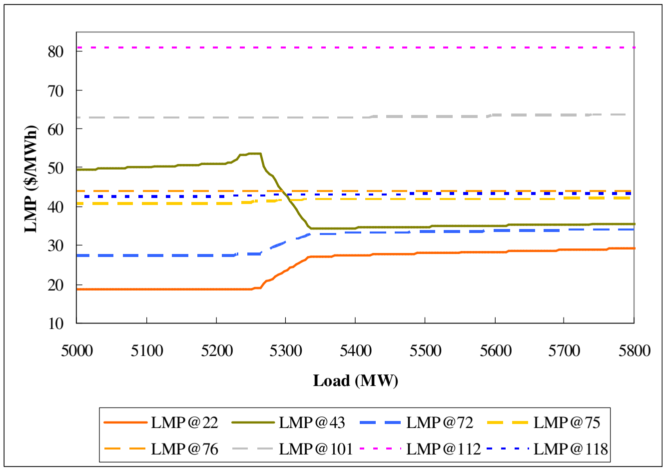

Fig. 1. LMP-Load curves for eight selected buses of a revised IEEE 118-bus system.

It is interesting to study why the existing criterion fails to identify the potential CLL. In fact, around the potential CLL, neither marginal unit set nor congested branch set has changed while LMPs at some buses, as well as some of the generator outputs and line flows, change their pattern

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considerably. Therefore, this paper presents a detailed study on this particular case and attempts to find out additional factors that may impact the determination of CLLs.

This paper is organized as follows. In section II, CLL concept is reviewed and the criterion for identifying CLLs is introduced. Using this criterion, identified CLLs on a revised IEEE 118-bus test system, as previously shown in Figure 1, are listed in section III. A detailed case study focusing on the missing CLL is subsequently presented. Section IV concludes the paper.

II. REVIEW OF CRITICAL LOAD LEVEL

Critical Load Level is initially proposed to represent load levels where significant LMP pattern change, such as a step change, is present. When LMP changes its pattern with respect to system load variation, what happens internally is the change of binding constraint or unbinding constraint. When system load varies sufficiently small around a given operating point, the set of binding constraints (such as congested branch set) and set of unbinding constraints (such as marginal unit set) remain unchanged. However, when system load varies significantly, the binding constraint set and unbinding constraint set may change (e.g., when a marginal unit becomes non-marginal). The load level when such change occurs is called Critical Load Level (CLL).

The load level is ***critical*** because across this point system statuses, as well as LMP, may change significantly from their past trajectory. For instance, in a congestion-free lossless DCOPF dispatch model, there is only one marginal unit serving any variation in system load. When system load grows to a CLL where the cheap unit reaches its generation upper bound, the unit will become a non-marginal unit and another more expensive generator will become marginal. Thus the generation dispatch pattern will change. In addition, the entire system will observe a price jump due to the higher bid of the new marginal unit.

Based on the definition of CLL, a criterion for identifying CLL can be derived as follows:

Criterion A*: A system load level is a Critical Load Level where the binding constraint set or non-binding constraint set changes when system load varies right across this load level.*

As LMP pattern change is of our primary interest and it is mainly related to transmission congestions and active power component of generation dispatch, Criterion A can be simplified by focusing on the active power related constraints. For example, non-marginal unit set and congested branch set are the major binding constraint sets while marginal unit set and uncongested branch set are the major unbinding constraint sets. Among these major constraint sets, a change in binding constraint set will always lead to a change in unbinding constraint set. For instance, when a non-marginal unit becomes marginal, the unit will be removed from the binding constraint set and added to the unbinding constraint set. For a branch becoming congested, it will be moved from unbinding constraint set to binding constraint set. Therefore, it is not necessary to check both binding constraint set and unbinding constraint set for changes. Criterion A is consequently simplified as follows:

Criterion B*: A system load level is a Critical Load Level where the marginal unit set or congested branch set changes when system load varies right across this load level.*

Criterion B is in a more explicit form and therefore easier for implementation. In addition, Criterion B is very effective in determining CLL because it involves much less constraint status checking as typically there are only a handful of marginal units and congested branches.

It should be noted that aforementioned marginal unit is defined with respect to its active power generation output. Numerically speaking, a unit is marginal when its active power output is within the open interval formed by the lower bound and upper bound of its active power generation capability.

Based on Criterion B, algorithms for locating CLLs have been developed. For lossless DCOPF model, [6] utilizes the nice characteristics of the linear programming model and proposes a simple-like method to efficiently and accurately solve for the two immediate adjacent CLLs around any given operating point. For ACOPF framework, [10] proposes a quadratic interpolation method to get the trajectory of system statuses such as generation dispatch and line flows, and extrapolates the trajectory to estimate CLLs. Results show that quadratic polynomial is a good approximation to system status trajectories. The same methods can be applied to with-loss DCOPF model where marginal loss factor is often used for modeling power losses [11, 13-14]. Although marginal loss factor introduces nonlinearity into the with-loss DCOPF model, the model in nature is much closer to lossless DCOPF model than to ACOPF model and therefore it is possible to develop more efficient method utilizing the features of with-loss DCOPF models. For with-loss DCOPF model, a variable substitution method is proposed in [17], which approximates the generation dispatch using quadratic polynomials.

It should be noted that, although [12] offers a good method for calculating the LMP sensitivity at any operating point, sensitivity is essentially a localized and linearized information and therefore it is not advisable to apply it to a broader range for the purpose of CLL estimation.

A binary search algorithm [18] can be employed to locate CLLs, when efficiency is not of concern. This method can obtain CLL to any given precision requirement and therefore it is utilized in this paper for identifying CLLs.

III. CASE STUDY

A case study is performed on a revised IEEE 118-bus system. The system consists of 118 buses, 54 generators and 186 branches/transformers. System total load is 4242MW with 9966.2MW total generation capacity. The detailed system data are available in [8]. OPF related data are obtained from [9]. Specifically, five line limits are added for the transmission system: 345MW for line 69-77, 630MW for line 68-81, 106MW for line 83-85 and 94-100, 230MW for line 80-98. For illustration purpose, let the generators be sorted in the order of the bus numbers they are connected to. The first 20 generators are assumed to be the cheapest generators with bids from \$10 to \$19.5 with \$0.5 increment; the next most expensive 20 generators have bids from \$30 to \$49 with a \$1 increment; and the remaining most expensive 14 generators have bids from \$70 to \$83 with \$1 increment. It should be noted that, in this case study, bus shunt admittance, branch charging capacitance, transformer tap ratios are all kept as they are in the original case.

A one-line diagram of the IEEE 118-bus system is obtained from [15], and shown in Figure 2. ACOPF is performed using MATPOWER simulation package [16].

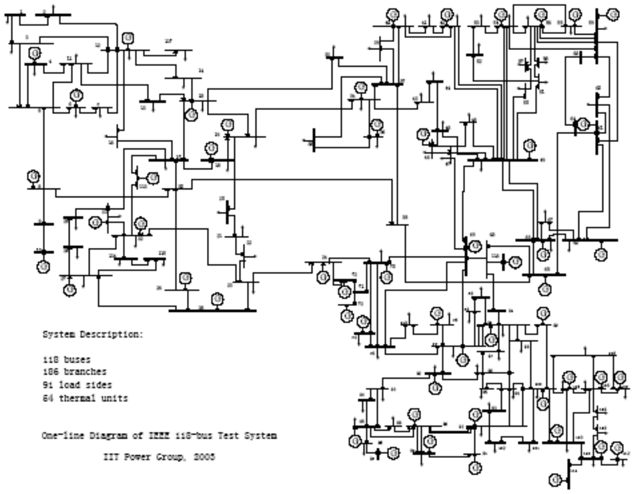

Fig. 2. A one-line diagram of the IEEE 118-bus system [15].

For load range from 5000MW to 5800MW as shown in Figure 1, there are totally 10 CLLs identified using binary search algorithm with precision of 0.001 p.u. of base load (i.e., 4.242MW). Criterion B is used in the algorithm.

CLL(MW)	CLL(MW)	CLL(MW)
5002.73	5229.05	5404.31
5055.41	5233.29	5794.88
5133.57	5264.66	
5219.85	5273.77	

Table 1: Identified CLLs in load range from 5000MW to 5800MW using Criterion B.

By intention and definition of Critical Load Level, 5336MW should be a CLL, as seen in Figure 1. However, it is not identified through Criterion B. To focus on what happens across this load level, we will narrow the studied load range to a close neighborhood around 5336MW, from 5302.50MW to 5408.55MW (i.e., 1.25 p.u. to 1.275 p.u. of base case load).

The LMP-Load curves for the same eight buses are shown in Figure 3. It should be noted that the pattern change for LMP at bus 22 does no appear as significant as in Figure 1 simply because Figure 3 plots the same data for a narrower window of system load.

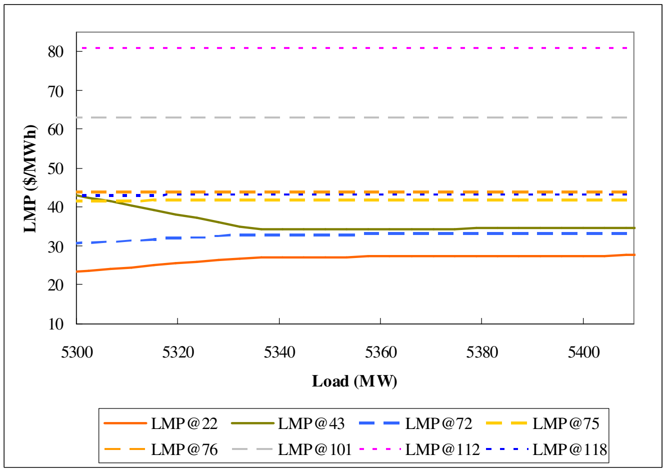

Fig. 3. LMP-Load curves for eight selected buses of a revised IEEE 118-bus system for system load from 5300MW to 5410MW.

When system load varies across 5336MW, there is no change in marginal unit set and congested branch set. There are 10 marginal units and 2 congested branches in the system. The marginal units are units at bus 69, 76, 77, 87, 89, 104,105, 107, 110, and 112. The two congested branches are line 68-81 and line 94-100. Since neither marginal unit set nor congested branch set changes when system varies across 5336MW, algorithms using Criterion B will not be able to identify this load level as a CLL.

Normally LMP pattern change is a resultant of change in marginal unit generation pattern. Therefore, we carefully examine the generation pattern of the 10 marginal units. All marginal units except for the marginal units at bus 76 and bus 77 suggest a near linear pattern with respect to system load. The dispatched generations of marginal units at bus 76 and bus 77 are shown in Figure 4 and Figure 5 respectively.

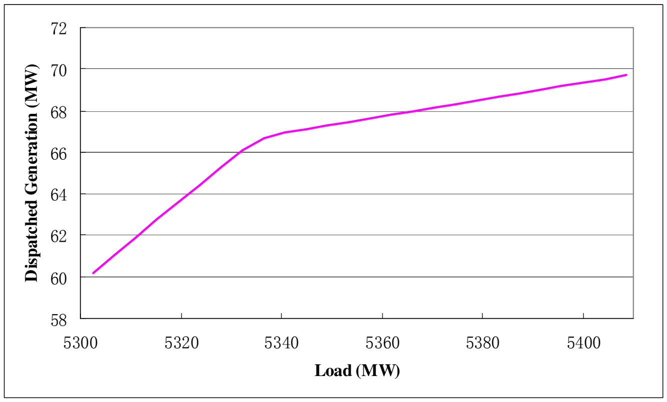

Fig. 4. Dispatched generation of marginal unit at bus 76 for load from 5300MW to 5410MW.

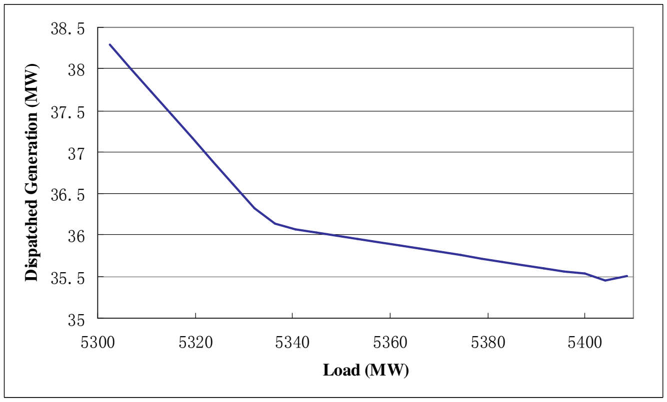

Fig. 5. Dispatched generation of marginal unit at bus 77 for load from 5300MW to 5410MW.

Generation outputs of both the marginal units appear to have pattern change around 5336MW. It should also be noted that in Figure 5 there is a second pattern change around 5404MW for the marginal unit generation at bus 77. This is caused by change of marginal unit set and has been identified as a CLL of 5404.31MW as shown in Table 1. More specifically, generator at bus 66 becomes marginal when system load grows to that load level.

We also examine the MVA flow of each uncongested branch. Flows on quite a few branches suggest pattern change around 5336MW. Figure 6 shows the flows on two of such branches, line 37-38 and line 69-75. For comparison purpose, Figure 6 also draws flows on two other lines, line 19-34 and line 25-27, whose flows do not demonstrate significant change of pattern.

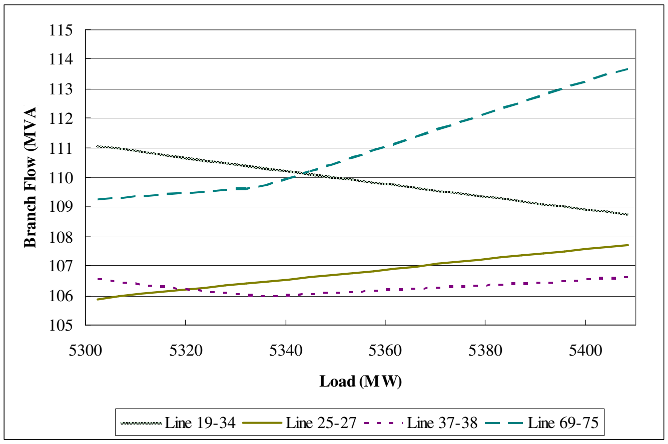

Fig. 6. Flows at selected branches for load from 5300MW to 5410MW.

Pattern changes in both marginal unit generation and branch flows around the missing CLL of 5336MW have been observed, and they are not caused by change of constraint sets that are considered in Criterion B, namely, marginal unit set and congested branch set. There must be other factors that have led to the pattern changes. In that regard, we examine the rest of the constraint sets such as generator reactive power constraint and voltage magnitude constraint.

Figure 7 shows the shadow price of reactive power lower bound constraint of generator at bus 62. It reflects the reduced total generation cost with respect to incremental relaxation of the generator reactive power generation lower limit. Shadow prices can be calculated from solving the dual problem of the original OPF dispatch model.

Figure 7 shows that the shadow price continuously drops and reaches zero at around 5333MW. It implies the reactive power of generator at bus 62 is no longer binding at lower bound when system load grows beyond 5333MW. This subsequently results in that voltage magnitude constraint at bus 43 changes from binding to unbinding at around 5337MW, as can be seen from Figure 8 which shows the shadow price of the lower bound constraint of voltage magnitude at bus 43.

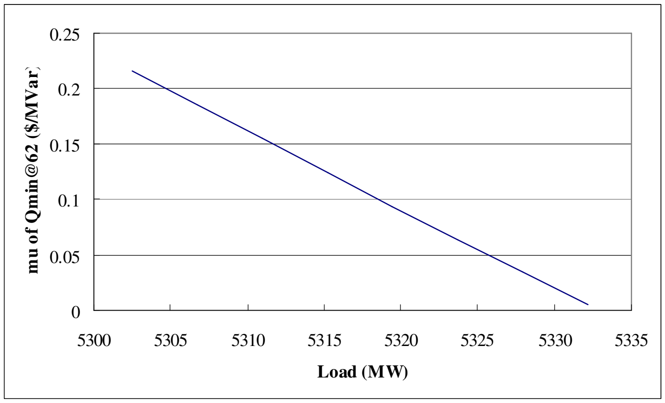

Fig. 7. Shadow price of reactive power lower bound constraint of generator at bus 62 for load from 5300MW to 5335MW.

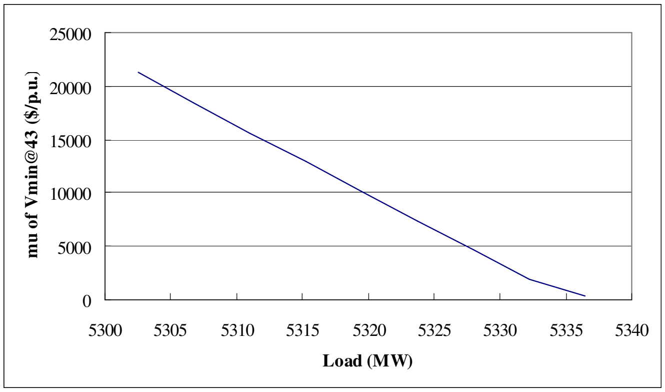

Fig. 8. Shadow price of lower bound constraint of voltage magnitude at bus 43 for load from 5300MW to 5340MW.

The load level where we see a change in generator reactive power constraint set and voltage magnitude constraint set coincides with the missing CLL at 5336MW. It should be noted that the simulation step for locating CLLs in this study is 0.001 p.u., or, 4.242MW. Therefore, the actual value of the missing CLL may be in the 4MW range around 5336MW.

Similar situation is observed at other load levels. For example, there is a little bump for LMP at some buses when load level is around 3520MW. Therefore 3520MW is a CLL by definition. However, it cannot be identified through algorithms using Criterion B because there is no change in marginal unit set and congested branch set across this load level. Before load grows to the unidentified CLL, voltage at bus 99 keeps staying at the maximum limit and the generation at the bus has reactive power close to the maximum reactive power generation limit. When load grows to the unidentified CLL, the generator reaches its reactive power upper-bound which subsequently causes the voltage magnitude constraint to be unbinding. This change of binding and unbinding statuses in generator reactive power constraint and voltage magnitude constraint leads to the change of generation pattern of certain marginal units and therefore results in the bump of LMP at some buses.

Although LMP is primarily related to system congestions and dispatch of active power component of generators, the case study has shown that reactive power constraint and voltage constraint may affect the active power dispatch pattern and subsequently LMP pattern, and therefore play a role in determining CLLs. In fact, reactive power component of the dispatch problem can affect real power problem through power losses and congestions.

In order to identify missing CLLs due to using Criterion B, one can define a new marginal unit set based on reactive power output and limit, and name it Reactive Marginal Unit Set. Then, expand Criterion B to include the change of Reactive Marginal Unit Set in determining CLLs. The effectiveness should be further studied and verified.

IV. CONCLUSIONS

This paper presents a case where existing criterion for determining CLLs fails to identify a CLL where significant LMP pattern change occurs. The existing criterion is then elaborated and discussed.

A detailed study is performed around the missing CLL. First, significant change in generation pattern of some marginal unit and flow pattern of a few branches are identified. Then, reactive power constraint and voltage magnitude constraint are examined around the missing CLL. Results show that the missing CLL coincides with the load level where a reactive power constraint, as well as a voltage magnitude constraint, becomes unbinding. The study suggests including reactive power constraint in the conditions of the existing criterion in order to capture missing CLLs.

It is also interesting to see in the case study how reactive component of the dispatch problem can affect the active power dispatch, line flows and LMPs.

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VI. BIOGRAPHIES

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