OPTIMIZATION OF PENALTIES ASSOCIATED WITH UNAVAILABILITIES OF TRANSMISSION LINES

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Abstract – Currently, the Brazilian Agency for Electric Energy applies penalties to the transmission utilities due to scheduled and non-scheduled outages. The use of penalties is aimed to assure the reliability of the transmission service in a competitive environment. A method to minimize the penalties associated with transmission equipment outages through the scheduled maintenance interval optimization is proposed. The proposed optimization model is based on a new analytical method that estimates the expected penalties accurately.

Keywords: Reliability, Maintenance, Markovian Models, Monte Carlo Simulation, Optimization

1 INTRODUCTION

The economic model of the Brazilian electric sector is based on the unbundling of the activities performed by the agents responsible for energy generation, transmission, distribution and commercialization. The separation of the activities of energy generation and transportation has led to auctions for transmission line concessions and the participation of transmission companies in these auctions. The concessions of transmission lines are granted to transmission companies that offer the lowest transmission tariffs (lowest annual revenues), provided the charges informed by the other participants are higher than a given percentage established for the auction. If the bid is lower than the percentage established, the auction will continue by means of successive bids with discount rates. Due to their nature, the investments on transmission lines are characterized by relatively low implementation costs and by a pre-established operational revenue, according to the winning bid, that meet the maximum limits stipulated by the Brazilian Agency for Electric Energy (ANEEL). The winning transmission utility signs a Contract for the Provision of Transmission Services (CPTS) with the Brazilian ISO (ONS). The ONS is then authorized to manage and coordinate the use of the electricity supply network, and, to the winning utility the costs and investments will be fully refunded. The investments can be considered of low-risk, once the associated revenue is ensured by the CPTS signed. Consequently, the economic model that guides the electric power supply services is based on a regulatory structure. Unlike the Generation Companies (GENCOs) and Power Marketers, which have freedom to set their prices within a competitive environment, the income from power transmission agent is defined by ANEEL, as Allowed Annual Revenue (AAR) paid for the “rental” of its assets.

The system users (GENCOs, distribution companies and large consumers) incur the necessary investments for connection to the bulk transmission network (substations and transmission lines with voltages higher than 230 kV) and pay one tariff to the ONS for the use of the transmission network. This tariff must cover the concession contracts between the ONS and Transmission Companies (TRANSCOs) plus the system operation costs.

One of duties of the ANEEL is to estimate the penalty (discount) on the AAR due to transmission equipment unavailability. This discount is called *Variable Penalty Regard to Availability of Bulk Transmission Facilities*, or merely *Variable Penalty* (VP). The VP definition has been under discussion among ANEEL, TRANCOs, ONS NAEE, transmission utilities, ONS and financial institutions which have invested in the expansion of electric sector. The unavailability of transmission assets can occur in two ways: voluntary or scheduled (maintenances, operational adequacy, maneuvering and others) and by forced or nonscheduled outages (failures). Due to that, ANEEL [1] has established the following formula to evaluate the VP in the bidding processes:

$$VP = \frac{EP}{1440 \cdot D}\left(\sum_{i=1}^{N_s} g(DSO_i)\right) + \frac{EP}{1440 \cdot D}\left(\sum_{i=1}^{N_f} h(DFO_i)\right) \quad (1)$$

where:

EP is the payment equivalent to the twelfth of the AAR, associated with the full availability of assets that integrate the facilities of a TRANSCO;

$$h(DFO_i) = \begin{cases} K_f DFO_i, \text{ para } DFO_i \le 300 \\ 300K_f + K_s(DFO_i - 300), \\ \text{para } DFO_i > 300 \end{cases}$$

$$g(DSO_i) = K_s DSO_i$$

K_s factor for scheduled outages (K_s = 10);
K_f factor for forced outages (K_f = 150);
N_s number of scheduled outages in a given month;
N_f number of forced outages in a given month;
D number of days of a month;

DSO_i duration (in minutes) of the scheduled outage i for a given month;
DFO_i duration (in minutes) of the forced outage i for a given month;

From (1), it can be noticed that the VP is a random variable, that is, it depends on the scheduled (preventive maintenance) and non-scheduled outages. The latter, in turn, depend on the durations of the failures of transmission assets, that is: permanent failures (tower fall, equipment breakdown, human errors, etc.), where the intervention of the maintenance crew is needed, and momentary failures (atmospheric discharge, fires), when the asset returns to operation by manual or automatic reclosing operations. In this context, it is evident that the transmission utility TRANSCOs must reduce the cost incurred by a VP in order to maximize their annual revenues. One of the main variables associated to the minimization of the VP is the maintenance interval. However, this variable has conflicting effects on the VP. The maintenance activities improve the equipment conditions. Therefore, it is possible to minimize the penalties due to non-scheduled outages by reducing the maintenance intervals. On the other hand, the increase in maintenance frequency leads to more penalties for scheduled outages. Due to this, the TRANSCOs must define the maintenance intervals in order to provide a balance between the penalties caused by scheduled and non-scheduled outages. In this way, the main purpose of this article is to present a method that minimizes the VP by adjusting the maintenance interval of the protection devices of the transmission lines. The main characteristic of the optimization model proposed is the establishment of an acceptable compromise between the VP values associated to scheduled and non-scheduled outages. Additionally, the VP optimization model uses an analytic method that allows precise estimation of the expected VP value. The results demonstrated that the solutions generated by the algorithm of VP minimization are better than those obtained via availability maximization [2]. Moreover, the VP estimations shown in the present article are more accurate than those calculated using analytical methods for estimating the Customer Interruption Costs (CIC) [3]. The rest of the paper is organized as follows:
i) Section 2 shows the Markovian model used to model the unavailability of transmission lines;
ii) The analytical method for estimating the VP is demonstrated in Section 3;
iii) the model of VP optimization is described in Section 4; and
iv) the results and conclusions are presented in Sections 5 and 6.

2 MARKOVIAN MODEL FOR PROTECTION DEVICES OF TRANSMISSION LINES

Failure modes for power system protection devices were defined in [4]. From the analysis of the failure modes it is possible to identify the set of possible states of a protection system. Initially, four states are identified, when the protection device is available for operation:

i) Normal operation: when the protection device is ready for executing its mission, performing as expected. In other words, it does not have any internal or intrinsic defect and can carry out the following functions: tripping of the protected equipment in case of a fault and lockout the tripping in the absence of a fault or for external faults.

ii) Failure caused by inadvertent tripping of the protection device: when there is unplanned operation of the protection device without a fault in the protected equipment.

iii) Failure caused by protection device stuck: when the protection device has a failure that prevents it from operating when there is a fault in the protected equipment. The protection device will resume its normal operation state after corrective maintenance. In this case, the protected equipment will be tripped by backup protection of other equipments or by the safeguard protection for failures in the circuit breaker. Corrective maintenance of the protected equipment will also be required.

iv) Test: when the protection device is under inspection or periodical test, preventive maintenance or self-test.

The Markov diagram used to represent the transitions between the four defined operational states is shown in Figure 1 [5], where:
λ_{12} transition rate between the normal operation and test state;
μ_{21} transition rate between the test and normal operation state;
λ_{13} failure rate of normal operation state to inadvertent tripping state;
μ_{31} repair rate for restoring the inadvertent tripping state to normal operation state;
λ_{14} failure rate of normal operation state to stuck state;
μ_{41} repair rate for restoring the failure caused by stuck to normal operation state.

Here, the system states associated with failures due to stuck and inadvertent tripping are merged in a single non-conform state, hereafter referred to as failure state. This merging is feasible only if the new model is still Markovian with constant transition rates. To make it possible, it is sufficient that the transition rates of the merged state to any other state remain equal to those of the original chain [6], [7]. The values of the probabilities calculated after merging the states are only accurate long-term (steady state) solutions, that is, the grouping of states cannot be used when short-term (transient) probabilities values are calculated [6]. In Figure 2 [5] is illustrated the transition between states of the resulting Markov processes, where the merged state is represented by number five, the other states maintain the original numbers.

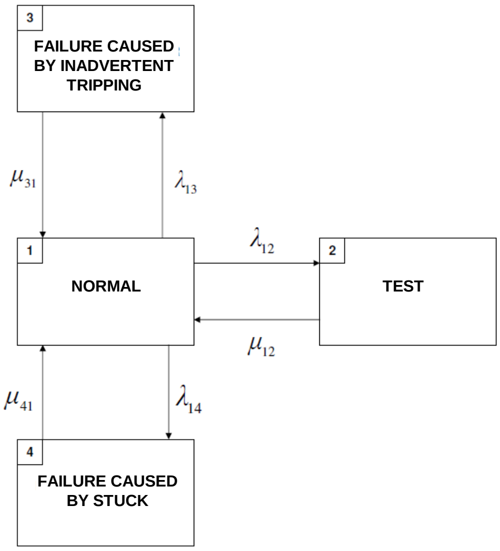

Figure 1: Markov model for the protection system

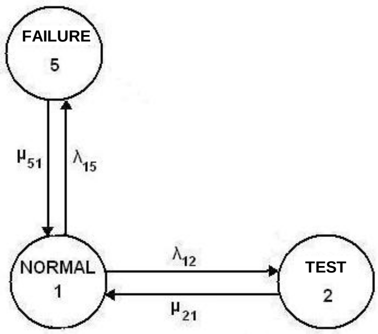

Figure 2: Simplified Markov model

In general, the transition rate from the normal operation state to the failure state (λ_{15}), cannot be directly observed, once an internal defect of the protection device remains invisible to the operator [4]. Consequently, the failure rate of transmission lines protection ($\lambda_{15} = \lambda_f$), the main variable of the reliability of the protection system, shall be calculated through other observable variables, using the model equations. The observable variables are occurrences and durations of visible events such as: frequency of stuck and inadvertent tripping and the average duration of test and repair. These parameters were calculated in a study carried out by I. P. Siqueira [4] for an observation period of eleven years. Consequently, the failure rate of the protection system is determined by:

$$\lambda_f = \frac{(F_{stu} + F_{ina})}{1 - F_{test} \cdot m - r \cdot (F_{stu} + F_{ina})} \qquad (2)$$

where:
F_{stu} is the frequency of stucks;
F_{ina} is the frequency of inadvertent tripping;
$m = 1/\mu_{21}$ average duration of the test;
$r = 1/\mu_{51}$ average duration of the repair;
F_{test} is the maintenance periodicity.

In (2), all the elements of the right side can be assessed by statistics of historical data of forced outages. F_{stu} and F_{ina} represent, respectively, the frequencies observed for stuck and inadvertent tripping in the period, both observable after a long period of implementation of the frequency of test F_{test}. Except for these frequencies, the other data are typical of the protection systems and independent from the frequency of the F_{test}. The steady state probabilities of the normal state and failure are determined through the transition matrix of the state diagram [8] illustrated in Figure 2:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_5 \end{bmatrix}^T \begin{bmatrix} 1-\lambda_{12}-\lambda_f & \lambda_{12} & \lambda_f \\ \mu_{21} & 1-\mu_{21} & 0 \\ \mu_{51} & 0 & 1-\mu_{51} \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_5 \end{bmatrix}^T \qquad (3)$$

$$P_1 + P_2 + P_5 = 1 \qquad (4)$$

From (3) e (4):

$$P_1 = \frac{1 - F_{test} \cdot m}{1 + \lambda_f \cdot r} \qquad (5)$$

$$P_2 = F_{test} \cdot m \qquad (6)$$

$$P_5 = \frac{\lambda_f \cdot (1 - F_{test} \cdot m) \cdot r}{1 + \lambda_f \cdot r} \qquad (7)$$

Finally, it must be said that this paper considered that frequencies of inadvertent tripping (F_{ina}) and stuck (F_{stu}) are correlated with maintenance intervals (F_{test}^{-1}) through:

$$F_{ina} = F_{ina}^0 e^{\delta_{ina}/F_{test}} \qquad (8)$$

$$F_{stu} = F_{stu}^0 e^{\delta_{stu}/F_{test}} \qquad (9)$$

where:

F_{ina}^0 (F_{stu}^0) and δ_{ina} (δ_{stu}) are the initial value and the growth factor, respectively, associated with the frequency of inadvertent tripping and (stuck). Some papers model the impact of the maintenance directly in the failure rate [9], [10]. In this paper, the impact of the maintenance is considered in the frequency of failure rates. This approach is used due to Brazilian transmission utilities store only the frequency of failure states in their outage management systems.

Here the unavailability of transmission lines is only caused by failures and maintenance activities in the protection devices. However, the presented Markovian model can be expanded to include other types of failures.

3 VP ESTIMATION USING THE PROPOSED MARKOVIAN MODEL

The analytical methods used to estimate the expected value of the VP is described in this section. The subsection 3.1 describes a technique called Approximate Analytical Method, inspired in the calculation of the CIC [3]. In subsection 3.2 a more accurate technique, called Exact Analytical Method, that allows relaxing some assumptions used in the conventional model is described.

3.1 Approximate Analytical Method

The VP is associated to two components: scheduled and forced outages. Both components can be estimated considering that:

i) The duration of all the outages is equal to the average duration of the outages,

ii) The number of outages is equal to the expected value of outage frequency;

iii) The scheduled outages are associated with the duration and frequency of the state 2 in Figure 2.;

iv) The forced outages are associated with the duration and the frequency of the state 5 in Figure 2.

The considerations (i)-(iv) are only valid when there is little dispersion around the expected values of outage durations. Otherwise, the penalties must be estimated using the Monte Carlo Simulation Method [3], [8].

Based on the considerations (i), (ii) and (iii), it is possible to estimate the VP expected value associated to scheduled outages ($E[VP^s]$):

$$E[VP^s] = \frac{EP}{1440.D}\left(K_s.F_2.D_2.1440.D\right) = EP.K_s.F_{test}.m \tag{10}$$

where D_2 and F_2 are the duration (in months) and the frequency (in occurrences per month) for the state 2 in Figure 2.

On the other hand, the VP expected value concerning forced outages ($E[PV^f]$) is calculated using (i), (ii) and (iv). Therefore, $E[PV^f]$ is:

a) for $D_5 \leq 300$ min:

$$E[PV^f] = \frac{EP}{1440D}\left(K_f.F_5.D_5.1440D\right) = \frac{EP}{1440D}\left(K_f.F_5.r.1440D\right) \tag{11}$$

b) for $D_5 > 300$ min.

$$E[PV^f] = \frac{EP}{1440D}F_5\left[300.K_f + K_s\left(1440.D.D_5 - 300\right)\right] = \frac{EP}{1440D}F_5\left[300.K_f + K_s\left(1440.D.r - 300\right)\right] \tag{12}$$

where:

D_5 and F_5 are the duration (in months) and the frequency (in occurrences per month) for the state 5 in Figure 2.

$$F_5 = P_5 r^{-1} = \frac{\lambda_f\left(1 - F_{test}m\right)}{1 + \lambda_f r}$$

The PV expected value ($E[VP]$) is:

$$E[VP] = E[VP^s] + E[PV^f] \tag{13}$$

In this point, it must be mentioned that the principles used in (13) to calculate the $E[VP]$ are the same used in the estimation of the CIC in distribution networks [3]. The CIC can also be estimated using the expected values of the frequency of failures and restoration times for the load points. Due to this, the estimations of $E[VP]$ and of the CIC are subject to the same restriction: the variations around the average restoration times of restoration and outages (scheduled and forced) should be small.

3.2 Exact Analytical Method

The VP expected values associated to scheduled and forced outages can be defined as:

$$E\left[VP^s\right] = \frac{EP}{1440D}E\left[\sum_{i=1}^{N_s} g\left(DSO_i\right)\right] \tag{14}$$

$$E\left[PV^f\right] = \frac{EP}{1440D}E\left[\sum_{i=1}^{N_f} h\left(DFO_i\right)\right] \tag{15}$$

The expected values in (14) and (15) are actually the expected values of a sum of the random variables, that is, [11]:

$$E[S] = E\left[\sum_{i=1}^{N} X_i\right] = \mu_x E[N] \tag{16}$$

where:

μ_x is the expected value of X_i;

$E[N]$ is the expected value of the whole random variable N.

X_i with i=1,...,N is set of the random variables with identical distributions.

From (16) it follows that $E[VP^s]$ and $E[VP^f]$ are:

$$E\left[VP^s\right] = \frac{EP}{1440D}E\left[N_s\right]E\left[g\left(DSO_i\right)\right] \tag{17}$$

$$E\left[VP^f\right] = \frac{EP}{1440D}E\left[N_f\right]E\left[h\left(DFO_i\right)\right] \tag{18}$$

The expected numbers of scheduled and forced outages can be obtained by calculating the frequency of the test and failure states of the Markov model in Figure 2:

$$E\left[N_s\right] = F_2 = \mu_{21}P_2 = F_{test} \tag{19}$$

$$E\left[N_f\right] = F_5 = \mu_{51}P_5 = \frac{\lambda_f\left(1 - F_{test}m\right)}{1 + \lambda_f r} \tag{20}$$

On the other hand, the expected values of functions $g(DSO_i)$ and $h(DFO_i)$ are associated to the calculation of the expected value of functions of random variables:

$$E\left[\rho(x)\right] = \int_{-\infty}^{+\infty} \rho(x) f_X(x) dx$$

where: $f_X(x)$ is the probability density function of the random variable x and $\rho(x)$ is the function of the random variable x.

Consequently, the probability density functions associated to the durations of the failure and test states must be known to estimate the expected values of functions $g(DSO_i)$ and $h(DFO_i)$. Nevertheless, in a homogeneous Markov chain, the time spent in a given state has exponential distribution with the failure rate equal to the sum of the transition rates that move to another state [12]. Thus, the probability density functions of states of failure (state 5) and test (state 2) are given by:

$$f_5(t) = \frac{1}{r}\exp\left(-t/r\right) \text{ e } f_2(t) = \frac{1}{m}\exp\left(-t/m\right)$$

where $f_2(t)$ and $f_5(t)$ are the probability density functions associated, respectively, to the durations of states 2 and 5 of Figure 2.

Consequently, the expected values of functions $g(DSO_i)$ and $h(DFO_i)$ are:

$$E[g(DSO_i)] = \int_0^{\infty}\left[\frac{K_s t}{1440mD}\right]\exp\left[\frac{-t}{1440mD}\right]dt$$

$$= 1440DK_s m \qquad (21)$$

$$E[h(DFO_i)] = \int_0^{300}\left[\frac{K_f t}{1440rD}\right]\exp\left[\frac{-t}{1440rD}\right]dt +$$

$$+\int_{300}^{\infty}\left\{\frac{[300K_f + K_s(t-300)]}{1440rD}\right\}\exp\left[\frac{-t}{1440rD}\right]dt$$

$$E[h(DFD_i)] = 1440rD\left[K_f + (K_s - K_f)\exp\left(\frac{-5}{24rD}\right)\right] \qquad (22)$$

If (19) and (21) are replaced in (17) the expected penalty associated to the scheduled outages is:

$$E[VP^s] = EPK_s F_{test} m \qquad (23)$$

If (20) and (22) are replaced in (18) the expected penalty associated to non-scheduled outages is:

$$E[VP^f] = EPF_5 r\left[K_f + (K_s - K_f)\exp\left(\frac{-5}{24rD}\right)\right] \qquad (24)$$

Finally, the expression for the estimation of $E[PV]$ is:

$$E[VP] = E[VP^s] + E[VP^f]$$

$$E[VP] = EPF_5 r\left[K_f + (K_s - K_f)\exp\left(\frac{-5}{24rD}\right)\right] +$$

$$+EPK_s F_{test} m \qquad (25)$$

It is important to emphasize that the expressions of $E[VP^s]$ obtained using the approximate (10) and exact 23) analytical methods are identical. However, the expressions of $E[VP^f]$ generated by the approximate (11) and (12) and exact (24) methods are different. The difference is due to the fact that the approximate analytical method estimates $E[VP^f]$ considering that the expected value of the $\rho(x)$ function of a random variable de x is equal to $\rho(\mu_X)$, where μ_X is the expected value of the random variable. The approximation is not valid for linear functions per parts, such as the VP value corresponding to forced outages (see (22)).

4 OPTIMIZATION MODELS USED TO DETERMINE THE MAINTENANCE INTERVAL

In this section, two optimization models for determining the maintenance interval of the protection devices of transmission lines are presented. The first model is based on availability maximization. The second model, which is a contribution of this paper, determines the maintenance interval through the VP expected value minimization.

4.1 Model Based on the Availability Maximization

Maintenance intervals are generally optimized aiming to maximize the equipment availability [2]. In the case of the Markovian model of Figure 2, the availability maximization is associated with the optimization problem:

$$\text{maximize} \qquad P_1 = \frac{1-(F_t m)}{1+\lambda_f r} \qquad (26)$$

where:

$$\lambda_f = \frac{\left(F^0_{stu}e^{\delta_{stu}/F_{test}} + F^0_{ina}e^{\delta_{ina}/F_{test}}\right)}{1 - F_{test}m - r\left(F^0_{stu}e^{\delta_{stu}/F_{test}} + F^0_{ina}e^{\delta_{ina}/F_{test}}\right)}$$

There is only one decision variable in the optimization problem: the frequency of scheduled maintenances F_{test} (the inverse of the maintenance interval). However, it is not possible to obtain an algebraic solution for (26) from the optimality condition, that is, $\frac{dP_1}{dF_{test}} = 0$. The restriction is caused by the presence of exponential terms associated to λ_f. Nevertheless, it is possible to solve (26) using unconstrained optimization algorithms such as: bisection method, Fibonacci Search, Golden Search, Quadratic Interpolation Method and Cubic Interpolation Method. In these methods, the optimal solution is obtained based only on the objective function calculation. However, there is no guarantee that an optimal solution of (26) is an $E[VP]$ local minimum. In other words, there is no guarantee that P_1 maximization is an equivalent optimization problem ("proxy") to the minimization of $E[VP]$.

4.2 Model Based on the VP Expected Value Minimization

In the current scenario of the Brazilian electricity sector it is more convenient for the transmission companies to minimize the $E[VP]$ than to maximize equipment availability. The objective can be achieved through the solution of the optimization problem:

minimize $\quad E[PV]=E[PV^p]+E[PV^o] \qquad (27)$

where:

$$E[VP^s] = EPK_s F_{test} m$$

$$E[VP^f] = EPF_5 r\left[K_f + (K_s - K_f)\exp\left(\frac{-5}{24rD}\right)\right]$$

$$F_5 = \frac{\lambda_f(1-F_{test}m)}{1+\lambda_f r}$$

$$\lambda_f = \frac{\left(F^0_{stu}e^{\delta_{stu}/F_{test}} + F^0_{ina}e^{\delta_{ina}/F_{test}}\right)}{1 - F_{test}m - r\left(F^0_{stu}e^{\delta_{stu}/F_{test}} + F^0_{ina}e^{\delta_{ina}/F_{test}}\right)}$$

The (27) and (26) optimization problems have common characteristics:

i) Both are optimization problems of unrestricted non-linear functions;

ii) the only variable of the decision is the frequency of scheduled maintenances F_{test};

iii) it is not possible to obtain algebraic solutions for both problems due to the exponential terms associated to

λ_f. For example, there are exponential terms in the $E[VP^f]$ expression.
iv) The solution of (27) can be obtained using the same methods used to solve (26).

5 RESULTS

Results obtained with the application of the proposed methods and models to estimate and optimize the VP expected value are presented. The results shown in the next subsections are:
i) In 5.1 there is a comparative study between the approximate and exact analytical methods for the estimation of the VP expected value;
ii) The maintenance intervals obtained with the optimization models described in Section 4 are shown in 5.2. Besides, several sensitivity analysis of the VP expected value in relation to the maintenance interval are shown.

Unfortunately, there were not available data to demonstrate the accuracy of the proposed model regarding to real systems.

5.1 Estimation of the VP Expected Value

The approximate and exact methods described in Subsections 3.1 and 3.2, respectively, are used to estimate the VP expected value. They are compared to those obtained with the Monte Carlo Simulation (MCS) [3], [8]. The MCS is used as a parameter to evaluate the accuracy of the estimates generated by both analytical methods, that is: the analytical method that to produce the estimates closer to those obtained with the MCS will be considered the most accurate one. The comparison is performed considering that:
i) The average duration of test (m) is equal to 21.7490 hours (3.0207×10^{-2} months);
ii) The growth factor (δ_{ina}) and the initial value (F_{ina}^0) of the frequency of inadvertent tripping are equal to 0.01 and 0.03445, respectively;
iii) The growth factor (δ_{stu}) and the initial value (F_{stu}^0) of the frequency of stucks are equal to 0.001 and 0.01867, respectively;
iv) The maintenance interval (F_{test}) is 36 months;
v) the average repair time (r) is variable (from 1 to 20 hours);
vi) The AAR is equal to R$ 25,800,000.00 Brazilian currency (R$). Consequently, the equivalent payment (EP) is equal to R$ 2,150,000.00.
vii) The size of the sample used to estimate PV via MCS was 10,000 months.

The $E[VP]$ estimated by the two analytical methods and by MCS are shown in Figure 3.

From Figure 3, it may be concluded that the estimations of the $E[VP]$ generated by the exact analytical method are closer to the ones obtained via Monte Carlo simulation than those obtained using the approximate analytical method.

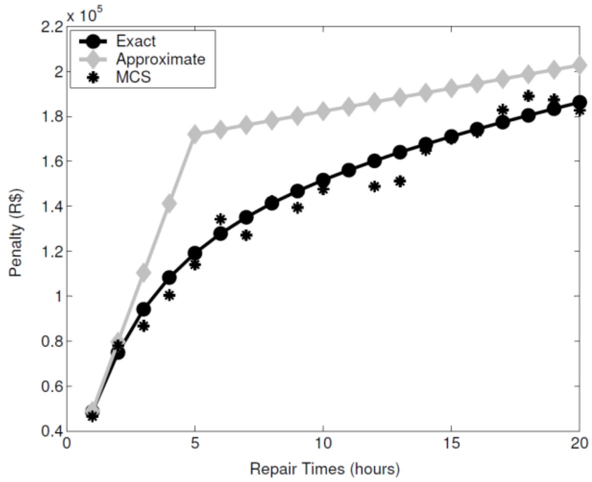

Figure 3: $E[VP]$ calculated by MCS and by the approximate and exact analytical methods

This fact is an indication that the proposed analytical method in this paper is more suitable to estimate the $E[VP]$ than the methods based on the calculation of CIC. The precision of the analytical methods compared to simulation can be more accurately assessed using the relative errors of the estimates of the $E[VP]$ obtained using the analytical methods compared to those generated via simulation. For example, the maximum value of error associated to the approximate analytical method is equal to 50.7287%. On the other hand, the maximum value of error associated to the exact analytical method is only 8.5239%. Furthermore, the average absolute error associated the exact and approximate analytical errors are equal to 20.5925% and 3.9913%, respectively. These results demonstrate quantitatively that the exact analytical method proposed is the most appropriate technique to estimate the VP expected value.

5.2 Determination of the Maintenance Intervals via Optimization Models

The results obtained with two optimization models, (availability maximization and $E[PV]$ minimization) are presented in this section.

The results were obtained for the same conditions defined under Subsection 5.1. The average repair time (r) was fixed in 4,3861 hours ($6{,}0918\times10^{-3}$ months) and the frequency of scheduled maintenances (F_{test}) was determined by the optimization models cited.

Firstly, the optimization problem associated with the availability maximization is solved using the the `fminbnd` function of MATLAB optimization toolbox. The optimum value of the maintenance interval and availability are equal to 50.0497 months and 0.9990, respectively. The optimal solution is shown in Figure 4.

Secondly, the `fminbnd` function is used to minimize the VP expected value. The optimum values for *E[VP]* and maintenance interval are R$ 60,722.1707 and 24.0145 months, respectively.

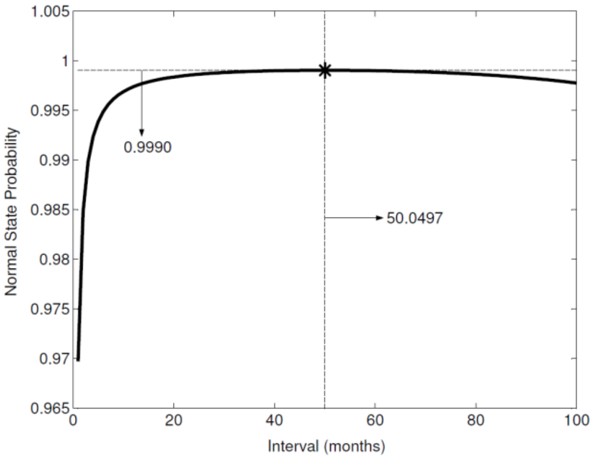

Figure 4: Variation of availability in relation to the maintenance interval (1/Ft).

These values are identified in the dashed straight lines in Figure 5. It is important to observe that the maintenance interval obtained with the maximization of the probability of normal state (vertical dotted line in Figure 5) is greater than the one obtained with the minimization of the $E[VP]$. In other words, the availability maximization results in a suboptimal maintenance interval in relation to the $E[VP]$ minimization solution. For example, the $E[VP]$ obtained with the solution of (26) (93,772.8275 R$) is 35.2455% higher than the $E[VP]$ associated to (27) (60,722.1707 R$).

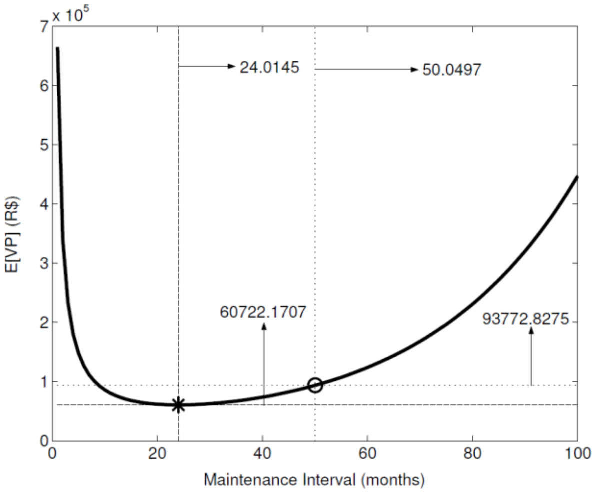

Figure 5: Variation of $E[VP]$ in relation to the maintenance interval ($1/F_{test}$).

6 CONCLUSIONS

A probabilistic model for transmission line protection system incorporating the maintenance interval effect on the failure rate was presented. It was used to optimize the maintenance interval according to two paradigms: maximization of the equipment availability and minimization of the Variable Penalty ($E[VP]$) expected value. The tests carried out demonstrated that the expected value $E[VP]$ minimization is the most suitable paradigm to the characteristics of the Brazilian electrical power supply system because the estimated penalties are lower than those calculated with equipment availability maximization. Additionally, an exact analytical method for estimating the $E[VP]$ was proposed. The estimated values are more accurate than those obtained using the traditional approximations in the probabilistic estimation of customer interruption costs.

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