EFFECT OF TECHNICAL NETWORK CONSTRAINTS ON SINGLE-NODE ELECTRICITY MARKETS

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Abstract - This paper presents a conjectural variation equilibrium model of a single-node electricity market. The distinctive modeling feature introduced in this paper is the effect of congestion between areas on generators' behavior. The results show that if there is a congestion between two areas, generators valued differently the production of each area, and give more importance to the importing area.

Keywords - Conjectural Variation, Market Equilibrium, Network Constraints, Single-node electricity markets.

1 INTRODUCTION

In recent decades, a number of countries have carried out liberalization and deregulation processes in electrical energy systems with the aim of creating competitive electricity markets. A wide range of models have been developed in order to analyze the different market design as well as the strategic behavior of agents in these electricity markets.

Electricity markets can be divided into two ideal groups. In the first group, there are single-price markets. In this kind of markets, power system technical constraints are not taken into account initially and market price is the same for all generators and demands. Then, through additional mechanisms, necessary adjustments are made in order to resolve technical constraints that appear in the system and to remunerate to the implied agents. In the second group, there are nodal-price markets that explicitly consider the constraints posed by the transmission network. Intermediate architectures such as market splitting or market coupling schemes are also very important in practical terms.

In the literature, there are different models to analyze strategic behavior and market equilibrium in both ideal types of electricity markets. However, the models that analyze the single-price markets often obviate the subsequent market mechanism to solve the network constraints [1, 2]. There are also models that study other market architectures as nodal-price markets [3]-[6], market splitting [7, 8] and market coupling [9, 10].

[1, 2] proposed a model to address generation companies' medium-term strategic analysis based on conjectured price-response market equilibrium. [3] developed a Cournot model of an electricity market for describing the effect of transmission constraints on the agent's strategic behavior and the opportunities to influence the set of constrained lines. [4, 5] presented a model where generators can affect the price of transmission services by means of a conjectured supply function model. [6] made a comparison of three different electricity market models with network constraints. This paper showed how the fundamental assumptions of the models can reach different and conflicting results. [7] proposed a conjectured supply function model to compute zonal prices in an electricity market with market splitting. The model internalizes the transmission constraints effect making the conjectured price responses as a function of the congestion state. [8] studied the differences between market splitting and explicit auctions as congestion management methods. [9, 10] analyzed the integration of European electricity markets through a market coupling design.

The distinctive feature between the proposed model and those developed in [3]-[10] is that these models focus on electricity markets with nodal or zonal pricing. However, none of them studies single-price electricity markets. Thus, the main contribution of this paper is to analyze the effect of the transmission network on bidding strategies of generators in a single-node market. The model generalizes a previous model [1] used to find the equilibrium in a single-node market by means of an optimization procedure. Both models can be classified as a conjectural-variation-based equilibrium models. Furthermore, due to the non-convexity and non-linearity of the proposed model, this paper provides an iterative methodology in order to find the optimal solution. The performance of the proposed approach is successfully validated with numerical simulations.

The remainder of this paper is organized as follows. The second section presents the market equilibrium model and describes the proposed solution algorithm. The third section provides and analyzes the numerical results. Finally, the fourth section draws relevant conclusions.

2 MARKET EQUILIBRIUM MODEL

Under the framework of game theory, the market equilibrium is reached at the point where each agent maximizes its own profit, taking into account that the rest of the agents also maximize their profits. This equilibrium point is known as the Nash Equilibrium, which is characterized by the agents not having an incentive to modify their strategic behavior because any deviation entails a decrease in benefits.

Among equilibrium models used to describe an agent's behavior in electricity markets are Cournot models where agents compete in quantities; Bertrand models where agents compete in price; Supply Function Equilibrium models which combines price and quantity competition; and Conjectural Variation models where agents can compete in price and quantity.

2.1 Basic Model

In a medium-term horizon, market equilibrium models based on conjectural variations are useful for representing the agent's strategic behavior. With these models, it is possible to study different kinds of competition, from perfect competition to Cournot oligopoly.

The market equilibrium model proposed in this paper is an extension of the model presented in [1]. This equilibrium model allows different strategic behaviors to be represented by means of a known constant coefficient called the conjectural variation. In the model, the basic assumption is that the conjectural variation θ_i of company i is the change in the electricity market price λ with respect to the change in the production quantity Q_i of the generation company. This value is a non-positive known value, that is:

$$\theta_i = -\frac{\partial \lambda}{\partial Q_i} \geq 0 \tag{1}$$

In the simplest situation, the profit Π_i of the generation company i at the clearing price λ is equal to the revenues minus the costs of the company, thus:

$$\Pi_i = \lambda \cdot Q_i - C_i(Q_i) \tag{2}$$

The equilibrium point is then calculated by expressing the first-order profit-maximization condition for each generation company, which yields:

$$\frac{\partial \Pi_i}{\partial Q_i} = \lambda + Q_i \cdot \frac{\partial \lambda}{\partial Q_i} - \frac{\partial C_i(Q_i)}{\partial Q_i} = 0 \tag{3}$$

Substituting the conjectural variation leads to:

$$\lambda - \theta_i \cdot Q_i = \frac{\partial C_i(Q_i)}{\partial Q_i} \tag{4}$$

In this manner, the market equilibrium is reached when the marginal income equals the marginal cost for each company i.

Furthermore, it is necessary to define the market clearing process. In electric power systems, it is mandatory that the generation and demand are balanced.

$$\sum_{i=1}^{N} Q_i = D \tag{5}$$

It is assumed that there is a relationship between market price and demand. This relation could be expressed as a linear function:

$$D = D_0 - \alpha_0 \cdot \lambda \tag{6}$$

Where D_0 is a constant and α_0 represents the demand slope.

The equilibrium market (3) is therefore defined by (4), (5) and (6). This equilibrium can be calculated as the solution of the quadratic minimization problem:

$$\begin{aligned} \min_{Q_i, D} & \sum_{i=1}^{N} \overline{C_i(Q_i)} - U(D) \\ \text{s.t} \quad & \sum_{i=1}^{N} Q_i = D : \lambda \end{aligned} \tag{7}$$

Where $\overline{C_i(Q_i)}$ is the so-called effective cost function of company i, and $U(D)$ is the utility demand function. It is important to note that the clearing price λ is the dual variable associated to the power balance constraint.

The effective cost function is defined as:

$$\overline{C_i(Q_i)} = C_i(Q_i) + \theta_i \cdot \frac{{Q_i}^2}{2} \tag{8}$$

And the utility demand function is defined as:

$$U(D) = \int_0^D \lambda(D) dD = \frac{1}{\alpha_0} \left(D \cdot D_0 - \frac{D^2}{2} \right) \tag{9}$$

In electricity markets, it is common to assume that the demand is inelastic, i.e., the demand is a known constant. In this situation, the optimization problem does not include the term $U(D)$.

2.2 Congestion between two Areas

In order to study the effect of congestion transmission lines in the model presented previously, it is assumed that there are two areas (nodes) interconnected by a transmission line, with limited transfer capacity, as shown in Figure 1. If the power network is not taken into account in the market clearing process, the resulting flows between the areas B and A may be not feasible, and flows may exceed the maximum capacity. In that case, in order to eliminate overflows, the total generation of exporting area is reduced while the total generation of importing area is incremented. The difference between the real production and the result of the power market will be paid or charged at a certain price. These payments can be viewed as an income or a cost depending on whether the unit increases or reduces its production.

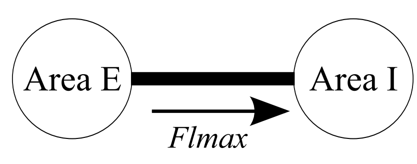

Figure 1: Two-area system

In different systems there are different mechanisms to determine the price of these transactions. The model described in this paper focuses in the mechanism used in Spain. If the unit reduced its production, the difference will be charged at the market price, while if the unit incremented its production the difference will be paid at the agent's offer price in a secondary market of restrictions.

The quantity reduced by the unit depends on the contribution factor to the congestion. This factor expresses the change in flow of the interconnection line that results from increasing the generation of each unit at the exporting area. First, it is reduced the production of the unit with the highest contribution factor and the following reductions will continue in the order of the contributing factors until the congestion disappears. In the case where many units have the same contribution factor the reduction will be proportional to the unit generation.

In the other side, the quantity increased will be established at the lowest cost solution. The minimum cost solution is valued on the bids submitted by the agents in the secondary market of restrictions.

It could be thought that the obvious solution to take into account the congestion is to include a constraint that limits the production in exporting area to ensure that the resulting flow of the market process does not exceed the maximum flow. Although this is the correct approach in a centralized cost-minimization operation, it would be wrong in a market equilibrium problem due to: i) with that constraint, optimality conditions of problem (7) are not equivalent to the market equilibrium conditions, ii) actually, such a constraint would be equivalent to exporting area units bidding at lower prices, and iii) the secondary market of restrictions would not be properly modeled.

Taking into account Figure 1, it is assumed that area B is the exporting area while area A is importing, and the connection between B and A is congested. For a clearing price λ and an offer price in the secondary market of restrictions γ_i^A, the profit Π_i of the generation company i is equal to the revenue in the electricity market, plus the income due to increased generation in A, minus the charge due to reduced generation in B, minus the production costs in each area, thus:

$$\Pi_i = \lambda \cdot (Q_i^A + Q_i^B) + \gamma_i^A \cdot \Delta Q_i^A - \lambda \cdot \Delta Q_i^B - C_i^A(Q_i^A + \Delta Q_i^A) - C_i^B(Q_i^B - \Delta Q_i^B) \quad (10)$$

The restriction market is much more volatile and technically complex than the day-ahead market. It is arguably also subject to a more stringent supervision by the regulator. Therefore, it is made the assumption that the restrictions market price γ_i^A is exogenous data independent of the clearing process.

It can be noted that in a two-area system, increasing the generation of any unit will result in the same change in flow. Hence, all the units in area B have the same contribution factor and the generation reduced by any unit in area B has to be proportional to its own production. Thus, there is a relationship between ΔQ_i^B and Q_i^B as shown in (11). Therefore, the profit is function of Q_i^A, Q_i^B and ΔQ_i^A. f is the so-called reduction factor (12) and Fl (13) is the flow that would result if the final productions were the solution of the market clearing process. The value of Fl is greater than $Flmax$ because the model is describing the case with congestion between areas.

$$\Delta Q_i^B = \frac{Fl - Flmax}{\sum_j Q_j^B} \cdot Q_i^B = f \cdot Q_i^B \quad (11)$$

$$f = \frac{Fl - Flmax}{\sum_j Q_j^B} \quad (12)$$

$$Fl = \sum_j Q_j^B - D^B \quad (13)$$

Although the reduction factor f depends on the production in area B, as shown in (12), it is assumed that f is a given datum by the agents. This assumption can be justified because the agents find extremely difficult to anticipate the outcome of their actions. However, for a given value of f, the market clearing process may result in that agents modify their productions and consequently also change the value of the reduction factor. From that cause, in order to find the true value of f, an iterative process is carried out as described in section 2.3.1.

The equilibrium point is then calculated by expressing the first-order profit-maximization condition for the production in each area for each generation company. This yields to (14) for the production at area A, and to (15) for the production at area B. Equation (16) represents the first-order profit-maximization condition for the increasing production in area A.

$$\frac{\partial \Pi_i}{\partial Q_i^A} = \lambda + \frac{\partial \lambda}{\partial Q_i^A} \cdot (Q_i^A + Q_i^B) - \frac{\partial \lambda}{\partial Q_i^A} \cdot \Delta Q_i^B - \frac{\partial C_i^A(Q_i^A + \Delta Q_i^A)}{\partial Q_i^A} = 0 \quad (14)$$

$$\frac{\partial \Pi_i}{\partial Q_i^B} = \lambda + \frac{\partial \lambda}{\partial Q_i^B} \cdot (Q_i^A + Q_i^B) - \frac{\partial \lambda}{\partial Q_i^B} \cdot \Delta Q_i^B - \lambda \cdot \frac{\partial \Delta Q_i^B}{\partial Q_i^B} - \frac{\partial C_i^B(Q_i^B - \Delta Q_i^B)}{\partial Q_i^B} = 0 \quad (15)$$

$$\frac{\partial \Pi_i}{\partial \Delta Q_i^A} = \gamma_i^A - \frac{\partial C_i^A(Q_i^A + \Delta Q_i^A)}{\partial \Delta Q_i^A} \quad (16)$$

In a single-node market, an agent can affect the price by modifying its production in any area and therefore the agent's conjectural variation is the same in both areas (17).

$$\theta_i = -\frac{\partial \lambda}{\partial Q_i^A} = -\frac{\partial \lambda}{\partial Q_i^B} \geq 0 \quad (17)$$

The equilibrium market equations (14) and (15) can be written as shown in (18) and (19), respectively (see Appendix A for full derivation).

$$\lambda = \theta_i \cdot Q_i^A + \theta_i \cdot (1-f) \cdot Q_i^B + CM_i^A(Q_i^A + \Delta Q_i^A) \quad (18)$$

$$\lambda = \frac{\theta_i}{1-f} \cdot Q_i^A + \theta_i \cdot Q_i^B + CM_i^B\left(Q_i^B \cdot (1-f)\right) \quad (19)$$

According to equations (18) and (19). In the equilibrium market equation for the production at the importing area, the conjectural variation is modified by the factor $(1-f)$ for the production in B. In the same way, in the equilibrium market equation for the production at the exporting area, the conjectural variation is modified by the factor $1/(1-f)$ for the production in A, i.e. in a two-area system with congestion in the interconnection between areas, a generator company evaluates differently the production in each area, and it gives a greater weight to the generation in the importing area.

Finally, in a single-node market, the total generation and demand have to be balanced:

$$\sum_i Q_i^A + \sum_i Q_i^B = D \tag{20}$$

2.3 Equivalent minimization problem

The equilibrium market defined by (18), (19) and (20) can be calculated as the solution of the quadratic minimization problem:

$$\min_{Q_i^A, Q_i^B, \Delta Q_i^A, \Delta Q_i^B} \sum_{i=1}^{N} \overline{C_i(Q_i^A, Q_i^B, \Delta Q_i^A)} \tag{21}$$

s.t.

$$\sum_i Q_i^A + (1-f)^2 \cdot \sum_i Q_i^B = D^* : \lambda \tag{22}$$

$$\Delta Q_i^B = f \cdot Q_i^B \tag{23}$$

$$\sum_i \Delta Q_i^A = Fl - Flmax \tag{24}$$

where, the modified effective cost function $\overline{C_i(Q_i^A, Q_i^B, \Delta Q_i^A)}$ is defined as:

$$\begin{aligned}\overline{C_i(Q_i^A, Q_i^B, \Delta Q_i^A)} &= C_i^A(Q_i^A + \Delta Q_i^A) \\ &+ (1-f) \cdot C_i^B\left((1-f) \cdot Q_i^B\right) \\ &- \gamma_i^A \cdot \Delta Q_i^A \\ &+ \frac{\theta_i}{2} \cdot \left(Q_i^A + (1-f) \cdot Q_i^B\right)^2 \end{aligned} \tag{25}$$

and D^* is defined as:

$$D^* = D + (Fl - Flmax) \cdot (f-2) \tag{26}$$

Constraint (22) is the power balance constraint. However, the original constraint (20) has been modified so that the solution of the minimization problem is equivalent to the market equilibrium equations. It can be note that in the optimal point the factor $(Fl - Flmax) \cdot (f-2)$ is equal to $f \cdot (f-2) \cdot \sum_i Q_i^B$, and the balance between generation and demand is reached. In the same way, Constraints (23) and (24) determine the quantity reduced and incremented in each area, respectively. In the optimal point, the value $(Fl - Flmax)$ is equal to $f \cdot \sum_i Q_i^B$, and therefore the quantity incremented in A is equal to the quantity reduced in B, preserving the balance in the system.

2.3.1 Solution Methodology

The minimization problem (21)-(24) is non-linear and non-convex because f depends on the productions in area B. In order to find the optimal solution of the problem, an iterative methodology is proposed and it is works as follows:

1. Initialize $f = 0$ and $Fl = Flmax$.
2. Solve the minimization problem (21)-(24). This gives a solution of Q_i^A, Q_i^B, ΔQ_i^A, ΔQ_i^B and λ.
3. Update the values f and Fl. If the change of f with respect to the previous iteration is less than an ϵ value, the algorithm stops, otherwise goes to 2.

3 NUMERICAL RESULTS

In order to analyze the effects of congestion between on the model previously presented, a simple case will be used. The model considers 7 generation units owned by 4 generator companies as shown in Table 1. Demand in area A is $D^A = 300$ MW and in area B is $D^B = 100$ MW. In the model, it is assumed that the bids submitted by the agents in the secondary market of restrictions are equal to the variable cost of the unit. The proposed algorithm and the optimization problems were solved with CPLEX 12.1 under GAMS.

GenCo	θ_i [€/MW]	Unit	Area	Variable cost [€/MWh]	Qmax [MW]
G1	0.01	U1	B	42	100
		U4	B	42.5	70
G2	0.02	U2	B	42.5	90
		U5	A	42.9	70
G3	0.05	U3	B	37	110
		U6	A	38.8	70
G4	0.05	U7	A	42.5	60

Table 1: Characteristics of the units

Table 2 presents the results of the reference case. In this case, there is not a congestion between areas. The market clearing price is $\lambda = 44.21$ €/MWh and the flow between areas B and A is $Fl = 265.71$ MW.

GenCo	Unit	Q [MW]
G1	U1	100
	U4	70
G2	U2	85.71
	U5	0
G3	U3	110
	U6	0
G4	U7	34.28

Table 2: Results of the reference case

In next simulations, the flow between areas B and A is limited between 102 MW and 265 MW. Figures 2-6 show the results of these simulations. In these figures, there are three specific zones determining the agents' behavior. The first zone corresponds to a maximum flow, $Flmax$, between 265 MW and 158 MW. The second zone is between a maximum flow of 158 MW and 112 MW. The third zone is between a maximum flow of 112 MW and 102 MW.

Figure 2 presents the productions for each unit. In the first zone, units U2 and U4 are at the exporting area and their production decrease due to the maximum flow decreases. In the other side, the production of unit U6 increases with respect to the case without maximum flow limit. In the second zone, there is an important change in trend. Agent G2 gives more importance to production of its unit in the importing area. Thus, production of unit U5 increases while production of unit U2 decreases. When $Flmax = 133$ MW, it can be noted as the modification of the conjectural variation affects the cost perceived by the agent G2. In that case, production of unit U5 is equal to production of unit U2, i.e. Agent G2 perceives the same *apparent cost* for its both units. In the third zone, there is a similar behavior than in the second one. Agent G3 gives more importance to production of its unit in the importing area. Thus, the production of unit U6 increases while the production of unit U3 decreases.

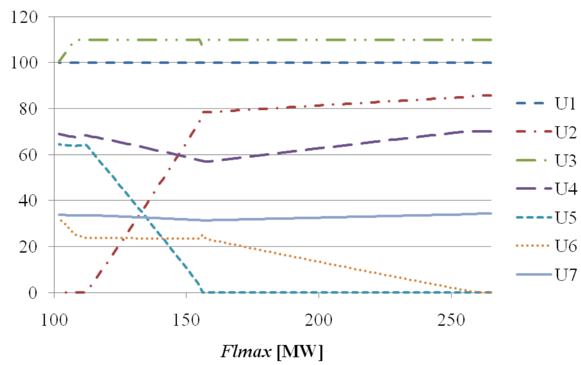

Figure 2: Unit production [MW]

Therefore, the results obtained for the unit productions confirm the behavior of the equilibrium presented in equations (18) and (19). These results show that, when the congestion between areas is considered, the agents give more value to production in the importing area instead than in the exporting area.

Figure 3 presents the production of each generation company. In the first zone, agents G1 and G2 decrease their production while agent G3 increases its production. In the second zone, the reduction rate of G2's production raises and the reduction is compensated by the increment of G1's production. Finally, in the third zone, there are not significant changes in the productions of agents.

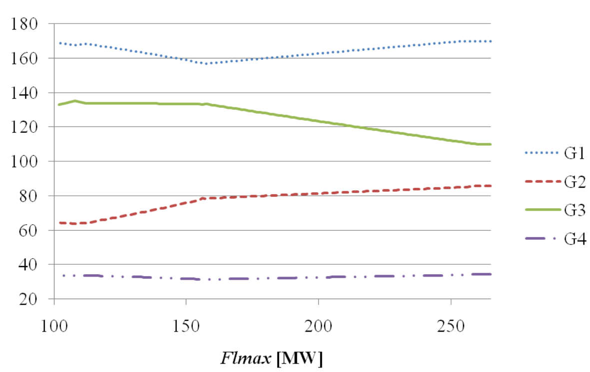

Figure 3: Generation company production [MW]

The market clearing price only changes in the interval [44.07, 44.21] €/MWh, as presented in Figure 4. In the first zone, the market price decreases progressively from 44.21 €/MWh to 44.07 €/MWh. In the second zone, the market price increases until 44.18 €/MWh. Finally, in the third zone, the market price is in the range [44.176, 44.188] €/MWh.

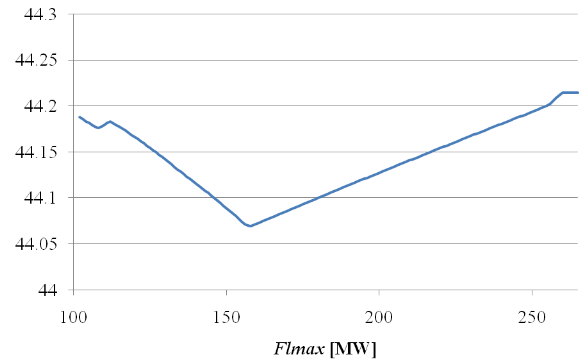

Figure 4: Market clearing price [€/MWh]

Although the productions at the two areas are valued differently when there is a congestion between them, there is not a constraint that explicitly limits the exporting area production and the resulting flow could be greater than the maximum flow. However, the flow calculated with the final productions, $Q_i^B - \Delta Q_i^B$, meets the maximum flow constraint, as shown in Figure 5. In this figure, it can be inferred that the flow decreases due to the production at the importing area increases when there is a congestion.

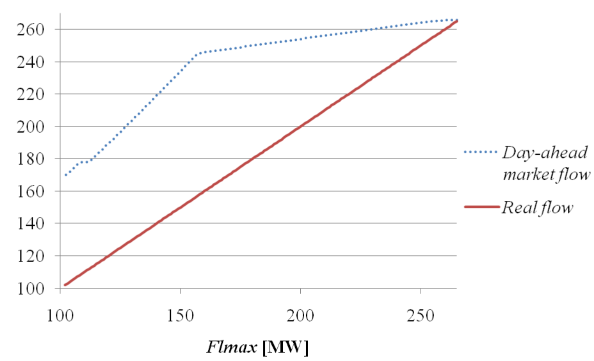

Figure 5: Resulting Flow [MW]

Finally, Figure 6 illustrates the final value of the reduction factor f. In first zone, this value starts in zero because the resulting flow does not exceed the maximum flow. The value of f increases gradually while the maximum flow decreases. There is a trend change in the second zone due to the agents' behavior change and the resulting flow decreases faster (Figure 5).

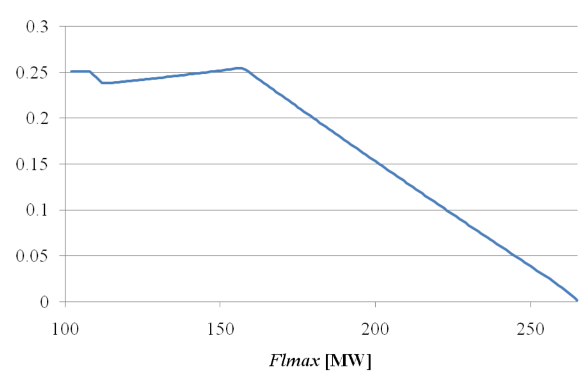

Figure 6: Reduction factor f

4 CONCLUSIONS

This paper has studied the effect of transmission network constraints on a single-node electricity market. The model and the results have shown that generation companies react differently if there is a congestion between two areas. The most important result is that, even if the conjectural variation is the same for each area, the value of the conjectural variation perceived by the agent is modified due to the presence of the congestion. Thus, the production of the importing area is incremented while the production of the exporting area is reduced.

Finally, it is important to say that the research is currently underway to study the agents' behavior when the solution of the electricity market violates reactive power constraints. These cases occur when bus voltages are not within their admissible ranges.

A MARKET EQUILIBRIUM

A.1 Equilibrium equation for production in importing area

The first-order profit-maximization condition is:

$$\frac{\partial \Pi_i}{\partial Q_i^A} = \lambda + \frac{\partial \lambda}{\partial Q_i^A} \cdot (Q_i^A + Q_i^B) - \frac{\partial \lambda}{\partial Q_i^A} \cdot \Delta Q_i^B - \frac{\partial C_i^A(Q_i^A + \Delta Q_i^A)}{\partial Q_i^A} = 0 \quad (27)$$

Substituting $\theta_i = -\partial\lambda/\partial Q_i^A$ and $\Delta Q_i^B = f \cdot Q_i^B$

$$\lambda - \theta_i \cdot (Q_i^A + Q_i^B) + \theta_i \cdot f \cdot Q_i^B - CM_i^A(Q_i^A + \Delta Q_i^A) = 0 \quad (28)$$

Isolating the market clearing price λ

$$\lambda = \theta_i \cdot Q_i^A + \theta_i \cdot (1-f) \cdot Q_i^B + CM_i^A(Q_i^A + \Delta Q_i^A) \quad (29)$$

A.2 Equilibrium equation for production in exporting area

The first-order profit-maximization condition is:

$$\frac{\partial \Pi_i}{\partial Q_i^B} = \lambda + \frac{\partial \lambda}{\partial Q_i^B} \cdot (Q_i^A + Q_i^B) - \frac{\partial \lambda}{\partial Q_i^B} \cdot \Delta Q_i^B - \lambda \cdot \frac{\partial \Delta Q_i^B}{\partial Q_i^B} - \frac{\partial C_i^B(Q_i^B - \Delta Q_i^B)}{\partial Q_i^B} = 0 \quad (30)$$

Substituting $\theta_i = -\partial\lambda/\partial Q_i^B$ and $\Delta Q_i^B = f \cdot Q_i^B$

$$\lambda - \theta_i \cdot (Q_i^A + Q_i^B) + \theta_i \cdot f \cdot Q_i^B - \lambda \cdot f - CM_i^B(Q_i^B - f \cdot Q_i^B) \cdot (1-f) = 0 \quad (31)$$

Isolating the market clearing price λ

$$\lambda \cdot (1-f) = \theta_i \cdot Q_i^A + \theta_i \cdot Q_i^B \cdot (1-f) + CM_i^B\left(Q_i^B \cdot (1-f)\right) \cdot (1-f) \quad (32)$$

Dividing by $(1-f)$

$$\lambda = \frac{\theta_i}{1-f} \cdot Q_i^A + \theta_i \cdot Q_i^B + CM_i^B\left(Q_i^B \cdot (1-f)\right) \quad (33)$$

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