ENERGY AND RESERVE SCHEDULING UNDER AN N-K SECURITY CRITERION VIA ROBUST OPTIMIZATION

Alexandre Street
Pontifical Catholic University
of Rio de Janeiro
Rio de Janeiro, Brazil
street@ele.puc-rio.br

José M. Arroyo
Universidad de Castilla-La Mancha
Ciudad Real, Spain
JoseManuel.Arroyo@uclm.es

Fabrício Oliveira
Pontifical Catholic University
of Rio de Janeiro
Rio de Janeiro, Brazil
fabricio.carlos.oliveira@gmail.com

Abstract – This paper presents a new approach to simultaneously schedule energy and reserve in an electricity market. The proposed model explicitly incorporates an *n-K* security criterion by which power balance is guaranteed under any contingency state comprising the simultaneous loss of up to *K* generation units. Instead of considering all possible contingency states, which would render the problem intractable, a novel method based on robust optimization is proposed. Using the notion of umbrella contingencies, the robust counterpart of the original problem is formulated. The resulting model is a particular instance of bilevel programming which is solved by its transformation to an equivalent single-level mixed-integer programming problem. Unlike previously reported contingency-dependent approaches, the robust model does not depend on the size of the contingency set, thus providing a computationally efficient framework. Simulation results back up these conclusions.

Keywords: Bilevel programming, energy and reserve scheduling, n-K security criterion, robust optimization, umbrella contingencies.

1 INTRODUCTION

Current reliability policy and associated security standards in power systems worldwide mainly focus on events such as the random outage of a single or at most two transmission or generation assets. These standards materialize in the well-known $n-1$ and $n-2$ security criteria which are typically defined as deterministic [1]. Hence, power systems are neither operated nor planned to withstand contingencies comprising multiple simultaneous outages. However, the adequacy of such reliability framework is questionable, as revealed by recent blackouts caused by the coincidence in time of several independent system component outages [2]. As a consequence, researchers are beginning to look at $n-K$ security criteria [3].

Within this context, this paper incorporates the consideration of multiple contingencies in the energy-and-reserve scheduling problem. Such problem plays a central role in power system operation under both centralized and competitive frameworks [1] and, therefore, security is an issue of major concern. Security has been accounted for in the generation scheduling through the definition of several types of reserves by which preventive and corrective actions can be implemented in order to handle outages [4].

From a deterministic viewpoint, reserves have been traditionally modeled in the generation scheduling by imposing different types of pre-specified requirements [5]. The main drawback of these approaches is the dependence on an a priori determination of system-wide or local reserve requirements that may lead to suboptimal or even infeasible solutions once contingencies occur.

In contrast to reserve-constrained approaches, contingency-constrained generation scheduling (CCGS) models [4][6][7] explicitly impose power balance under both normal and contingency states. In [4], contingencies were accounted for by a set of credible generator and line outages and a joint power and reserve scheduling model was presented in a one-period setting. In the same work, a reserve pricing scheme was proposed based on the Lagrange multipliers of the nodal power balance equations. Joint market models for energy and several types of reserves were also proposed in [6], where the beneficial impact of demand-side bidding under $n-1$ and $n-2$ security criteria was analyzed in a single-bus model. In [7], a Benders decomposition approach was proposed for a contingency-constrained model of a joint energy and ancillary services auction.

Current computing capabilities may allow incorporating $n-1$ and $n-2$ security criteria in the CCGS models presented in [4][6][7] for practical power systems. However, the extension to tighter security levels would lead to intractability due to the huge number of contingency states that should be considered. As a consequence, CCGS models only examine a limited set of credible contingencies, which is determined based on experience and engineering judgment.

This paper presents an alternative approach that efficiently incorporates a deterministic $n-K$ security criterion into the CCGS problem. This model is hereinafter referred to as $n-K$ CCGS. Unlike previously reported CCGS models [4][6][7] relying on a reduced set of credible contingencies, we propose a joint energy and reserve dispatch model based on robust optimization that allows considering all combinations of at most K unit outages, i.e., $\sum_{i=1}^{K}\binom{n}{i}$, in a computationally efficient manner.

Robust optimization [8] is an appropriate framework to model optimization problems where the optimal solution must remain feasible for some parameter variations in a given user-defined set (also called "uncertainty set"). In this framework, the unknown parameters (uncertainty) are treated as worst-case deterministic functions of the decision variables, which are set to perform the worst “feasibility damage” in the model for each proposed solution.

Robust optimization has been criticized due to the over-conservatism of the solutions provided. Recent theoretical advances by Bertsimas and Sim [8] allow an easy control of the degree of conservatism with moderate computational effort by controlling the number of coefficients that may change in each constraint of the problem. The main advantage of this technique is that robust counterparts do not increase in complexity compared to their original formulation. Such work has paved the way for an enormous number of applications in the optimization field due to its intuitive interpretation, easy implementation, and independence of any subjective probability specification process.

The $n-K$ CCGS model presented here belongs to the class of problems suitable for robust optimization, where the parameters allowed to vary represent the generation unit availability under the contingency states. The robust counterpart of the original $n-K$ CCGS problem is first constructed. This problem is modeled as a worst-case bilevel programming problem [9] wherein contingency states are characterized as decision variables. Using recent findings from robust optimization [8], the $n-K$ CCGS bilevel program is subsequently transformed into an equivalent single-level mixed-integer programming (MIP) problem. The main advantage of the proposed solution is that the dimension of the resulting MIP problem does not depend on the security level defined by parameter K, thereby allowing an efficient solution by off-the-shelf branch-and-cut software.

The main contributions of this paper are:

1. The system operator is provided with a generation scheduling tool that accounts for tighter security levels than the traditional $n-1$ and $n-2$ security criteria.
2. A novel $n-K$ CCGS model is formulated and implemented based on robust optimization. This methodology is effective in attaining globally optimal solutions with moderate computational effort.
3. The performance of the proposed approach is successfully validated with numerical simulations.

The remainder of this paper is organized as follows. In Section 2, the formulation of the $n-K$ CCGS problem is presented. Section 3 describes the robust optimization approach. Section 4 provides and discusses results from several case studies. Finally, some relevant conclusions are drawn in Section 5.

2 PROBLEM FORMULATION

The contingency-constrained generation scheduling problem determines the optimal generation schedule and reserve allocation so that the power demand is supplied under both normal and contingency states. Here we propose the explicit consideration of an $n-K$ security criterion by which all combinations of up to K unit outages are modeled. For expository purposes, we use a single-period scheduling model without network considerations such as congestion or losses. This model is simpler to describe and analyze, yet bringing out the main features of the robust optimization approach. An extended model including a multiperiod setting and non-spinning reserve is available in [10].

Based on the models presented in [4] and [6], the $n-K$ CCGS problem can be formulated as:

$$\underset{p_i, p_i^k, r_i, v_i}{\text{Minimize}} \sum_{i \in N} \left(C_i^P(p_i, v_i) + C_i^R r_i \right) \quad (1.1)$$

subject to:

$$\sum_{i \in N} p_i = D \quad (1.2)$$

$$\sum_{i \in N} p_i^k = D \qquad \forall\, k \in \mathcal{C} \quad (1.3)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i \qquad \forall\, i \in N \quad (1.4)$$

$$\underline{P}_i A_i^k v_i \le p_i^k \le \overline{P}_i A_i^k v_i \qquad \forall\, i \in N, \forall\, k \in \mathcal{C} \quad (1.5)$$

$$p_i^k \le A_i^k (p_i + r_i) \qquad \forall\, i \in N, \forall\, k \in \mathcal{C} \quad (1.6)$$

$$p_i + r_i \le \overline{P}_i v_i \qquad \forall\, i \in N \quad (1.7)$$

$$0 \le r_i \le \overline{R}_i v_i \qquad \forall\, i \in N \quad (1.8)$$

$$v_i \in \{0,1\} \qquad \forall\, i \in N, \quad (1.9)$$

where p_i is the power output of generator i in the pre-contingency state; p_i^k is the power output of generator i under contingency k; r_i is the up-spinning reserve provided by generator i; v_i is a binary variable that is equal to 1 if generator i is scheduled, being 0 otherwise; N is the set of generator indexes; $C_i^P(\cdot)$ is the energy cost function offered by generator i; C_i^R is the cost rate offered by generator i to provide up-spinning reserve; D is the system demand; $\mathcal{C}$ is the set of contingency indexes; $\underline{P}_i$ is the minimum power output of generator i; $\overline{P}_i$ is the capacity of generator i; A_i^k is a constant equal to 1 if unit i is available under contingency state k, being 0 otherwise; and $\overline{R}_i$ is the upper bound for the reserve contribution of generator i.

Parameters A_i^k are used to characterize contingency states. Thus, the $n-K$ security criterion is enforced by considering all contingency states such that

$$\sum_{i \in N} A_i^k \ge n - K \qquad \forall\, k \in \mathcal{C}, \quad (2)$$

where n is the number of generation units and K is the number of unavailable generators.

The objective function to be minimized (1.1) consists of the sum of the offered cost functions for generating energy plus the cost of all up-spinning reserves offered by the generators. It should be noted that the loss of any set of generation units does not require the reduction of the production of any remaining available unit to keep the power balance. Therefore, up-spinning reserves are the only reserve products included in the model. Moreover, as done in [4], the cost of the corrective actions in the event of a contingency, that is, the actual use of the reserve, is not included in the objective function.

Constraints (1.2) and (1.3) represent the power balance equations under the pre-contingency and contingency states, respectively. Constraints (1.4) and (1.5) set the generation limits for the pre-contingency and contingency states, respectively. Note that the vector of on/off variables, $\boldsymbol{v}$, is the same under both states, indicating that only those generators that are scheduled on in the pre-contingency state can participate in corrective actions during contingency states. Constraints (1.6) and (1.7) relate the up-spinning reserve contributions to the power levels produced under the pre-contingency and contingency states. Constraints (1.8) provide the bounds for the up-spinning reserve contributions. Finally, the binary nature of variables v_i is expressed in (1.9).

Since the redispatch cost is not included in the objective function, problem (1) can be equivalently reformulated by dropping variables p_i^k. As can be noted, by summing over i in (1.6),

$$\sum_{i\in N} p_i^k \le \sum_{i\in N} A_i^k (p_i + r_i) \qquad \forall\, k \in \mathcal{C}, \tag{3}$$

and introducing (1.3) in (3) yields:

$$\sum_{i\in N} A_i^k (p_i + r_i) \ge D \qquad \forall\, k \in \mathcal{C}. \tag{4}$$

Thus, by replacing constraints (1.3), (1.5), and (1.6) in the original model by (4), post-contingency variables p_i^k are removed. For the sake of clarity, the equivalent model is stated below:

$$\underset{p_i, r_i, v_i}{\text{Minimize}} \sum_{i\in N} \left(C_i^P(p_i, v_i) + \ C_i^R r_i\right) \tag{5.1}$$

subject to:

$$\sum_{i\in N} p_i = D \tag{5.2}$$

$$\sum_{i\in N} A_i^k (p_i + r_i) \ge D \qquad \forall\, k \in \mathcal{C} \tag{5.3}$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i \qquad \forall\, i \in N \tag{5.4}$$

$$p_i + r_i \le \overline{P}_i v_i \qquad \forall\, i \in N \tag{5.5}$$

$$0 \le r_i \le \overline{R}_i v_i \qquad \forall\, i \in N \tag{5.6}$$

$$v_i \in \{0,1\} \qquad \forall\, i \in N. \tag{5.7}$$

In spite of the absence of post-contingency variables, it is worth mentioning that the equivalent model implicitly guarantees a feasible post-contingency schedule. The post-contingency power output cannot violate the minimum bound since only up-spinning reserve contributions from scheduled units are considered. Additionally, constraints (5.5) guarantee that the post-contingency power output does not exceed the generating capacity. Finally, constraints (5.3) ensure the power balance for all contingencies analyzed.

Problem (1) and its equivalent (5) both explicitly model the $n-K$ security criterion by considering all $\sum_{i=1}^{K}\binom{n}{i}$ combinations of unit outages in (1.3) or (5.3). For realistic power systems comprising hundreds of units, this number of contingency states may become prohibitive and render the problem essentially intractable even for low values of K. The approach presented next addresses such issue while keeping the modeling accuracy.

3 ROBUST OPTIMIZATION APPROACH

Problem (5) can be viewed as a particular instance of robust optimization [8] in which the parameters allowed to vary are parameters A_i^k representing the availability of generation units under each contingency state. Based on this fact, we propose a robust optimization approach to solve the $n-K$ CCGS problem (5), where K is identified as the robustness parameter used to adjust the conservatism level. First, the contingency-dependent model (5) is equivalently reformulated as a robust bilevel counterpart. The resulting robust formulation embeds all contingencies associated with the $n-K$ security criterion but does not depend on the size of the contingency set. Using recent findings from robust optimization, the resulting bilevel program is subsequently transformed into an equivalent single-level MIP problem suitable for commercially available software.

3.1 Robust Bilevel Counterpart

The contingency dependence of problem (5) is introduced in (5.3), which can be referred to as the complicating constraints. These constraints require that the sum of the pre-contingency power outputs and up-spinning reserve contributions of all available units be greater than or equal to the system demand for each contingency state. Since this requirement must hold for all contingencies $k \in \mathcal{C}$, it is sufficient to guarantee that it holds for the worst case, i.e., the contingencies with the tightest left-hand side of (5.3). These contingencies are also known as the umbrella contingencies [11]. Therefore, constraints (5.3) can be expressed in a compact way as:

$$d^{wc*}(\boldsymbol{p}, \boldsymbol{r}) \ge D \tag{6}$$

$$d^{wc*}(\boldsymbol{p}, \boldsymbol{r}) = \min_{k\in\mathcal{C}} \left\{ \sum_{i\in N} A_i^k (p_i + r_i) \right\}, \tag{7}$$

where $d^{wc*}(\boldsymbol{p}, \boldsymbol{r})$ denotes the maximum power that can be supplied under the worst-case contingency for a given pair of vectors of scheduled power and up-spinning reserve $\boldsymbol{p} = [p_1, \dots, p_n]^T$ and $\boldsymbol{r} = [r_1, \dots, r_n]^T$.

The minimum function in (7) can be formulated as an optimization problem by defining a new vector of decision variables $\boldsymbol{a} = [a_1, \dots, a_n]^T$ associated with the worst-case contingency. Hence, a_i is a binary variable which is equal to 0 if generator i is unavailable in the worst-contingency state, being 1 otherwise.

Therefore, the $n-K$ CCGS problem can be restated as the following bilevel programming problem [9]:

$$\underset{p_i, r_i, v_i}{\text{Minimize}} \sum_{i \in N} (C_i^P(p_i, v_i) + C_i^R r_i) \quad (8.1)$$

subject to:

$$\sum_{i \in N} p_i = D \quad (8.2)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i \quad \forall\, i \in N \quad (8.3)$$

$$p_i + r_i \le \overline{P}_i v_i \quad \forall\, i \in N \quad (8.4)$$

$$0 \le r_i \le \overline{R}_i v_i \quad \forall\, i \in N \quad (8.5)$$

$$v_i \in \{0,1\} \quad \forall\, i \in N \quad (8.6)$$

$$d^{wc*}(\boldsymbol{p}, \boldsymbol{r}) \ge D \quad (8.7)$$

$$d^{wc*}(\boldsymbol{p}, \boldsymbol{r}) = \min_{a_i} \sum_{i \in N} a_i(p_i + r_i) \quad (8.8)$$

subject to:

$$\sum_{i \in N} a_i \ge n - K \quad : y \quad (8.9)$$

$$0 \le a_i \le 1 \quad : z_i \quad \forall\, i \in N. \quad (8.10)$$

The robust bilevel counterpart (8) for the $n-K$ CCGS problem comprises an upper-level problem (8.1)-(8.7), associated with the system operator, and a lower-level problem (8.8)-(8.10), corresponding to the worst-case contingency. The upper-level problem consists in the determination of the generation unit schedule, including both pre-contingency power outputs and up-spinning reserve contributions, where the unavailability of generation units in the worst-case contingency results from the solution to the lower-level problem. Upper-level decision variables are vectors $\boldsymbol{p}$, $\boldsymbol{r}$, and $\boldsymbol{v}$, whereas $\boldsymbol{a}$ is the lower-level decision vector. The dual variables associated with (8.9) and (8.10) are y and z_i, respectively.

The upper-level objective function (8.1) is identical to (5.1). Analogously, upper-level constraints (8.2)-(8.6) are identical to (5.2), (5.4)-(5.7), respectively. Constraint (8.7) imposes that the generation capability under the worst-case contingency is greater than or equal to the system demand, i.e., power balance is guaranteed under all contingencies.

The lower-level objective function (8.8) represents the total post-contingency power output that can be supplied by generation units under any combination of available units. Such availability is modeled by the lower-level decision variables a_i. Therefore, minimizing this objective function leads to the worst-case contingency.

Constraint (8.9) enforces the $n-K$ security criterion. Finally, constraints (8.10) set the upper and lower bounds for variables a_i. It should be noted that constraints (8.9)-(8.10) have a unimodular matrix structure (see proposition 3.2 in [12]), which guarantees that, for integer values of K, the lower-level problem (8.8)-(8.10) always provides integer (binary) optimal solutions for vector $\boldsymbol{a}$. In other words, vectors composed of the unit availability parameters, $\boldsymbol{A}^k = [A_1^k, \dots, A_n^k]^T \;\forall\, k \in \mathcal{C}$, are the vertexes of the polyhedral set defined by constraints (8.9) and (8.10). In Fig. 1, such polyhedron and the set $\{\boldsymbol{A}^k\}_{k \in \mathcal{C}}$ are both illustrated for the case of a three-unit system with $K = 1$. Circles represent the vertexes and the dashed volume, above the two-sum plane, is the so-called polyhedral uncertainty set, in robust optimization nomenclature. This set comprises all data variations in the unit availability space, $\boldsymbol{a} \in [0,1]^n$, under which the system demand is met even if a single unit happens to fail. Note that such uncertainty set also contains fractional (non-binary) availability vectors for which the system demand is also met (points inside the dashed volume). However, for any feasible upper-level scheduling decision such points will never characterize the worst-case contingency.

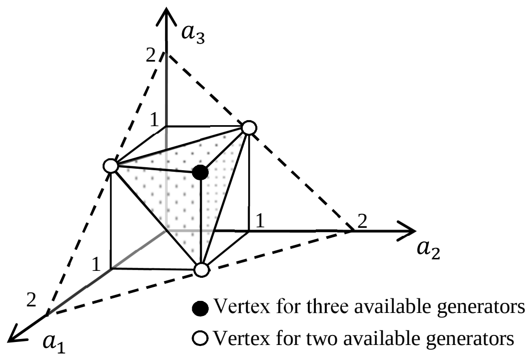

Figure 1: Unit availability set for $n = 3$ and $K = 1$.

Besides its intrinsic complexity due to the two levels of optimization, problem (8) is mixed integer (containing both continuous and binary variables) and nonlinear due to the products $a_i p_i$ and $a_i r_i$ in (8.8).

3.2 Robust Single-Level Counterpart

Based on the robust optimization approach presented in [8], an efficient single-level equivalent formulation is provided for the bilevel problem (8).

Note that the only requirement on the lower-level optimization problem (8.8)-(8.10) imposed at the upper level is that its optimal objective function, $d^{wc*}(\boldsymbol{p}, \boldsymbol{r})$, be at least D. Thus, this requirement is also met by imposing that a lower bound for $d^{wc*}(\boldsymbol{p}, \boldsymbol{r})$, given by the dual objective function of the lower-level problem, be greater than or equal to D. Therefore, the procedure to derive the final single-level robust counterpart is summarized as follows:

1. replace $d^{wc*}(\boldsymbol{p}, \boldsymbol{r})$ in (8.7) by the dual objective function of the lower-level optimization problem (8.8)-(8.10);
2. replace the lower-level optimization problem (8.8)-(8.10) by its dual feasibility constraints.

Step 2 guarantees that the dual objective function of the lower-level problem at step 1 provides a lower bound for $d^{wc*}(\boldsymbol{p}, \boldsymbol{r})$. The interested reader is referred to [8] for a detailed proof of the transformation. For the sake of completeness, the equivalent single-level MIP model for the robust bilevel $n-K$ CCGS problem (8) is as follows:

$$\underset{p_i,r_i,v_i,y,z_i}{\text{Minimize}} \sum_{i \in N} (C_i^P(p_i, v_i) + C_i^R r_i) \quad (9.1)$$

subject to:

$$\sum_{i \in N} p_i = D \quad (9.2)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i \qquad \forall\, i \in N \quad (9.3)$$

$$p_i + r_i \le \overline{P}_i v_i \qquad \forall\, i \in N \quad (9.4)$$

$$0 \le r_i \le \overline{R}_i v_i \qquad \forall\, i \in N \quad (9.5)$$

$$v_i \in \{0,1\} \qquad \forall\, i \in N \quad (9.6)$$

$$(n-K)y - \sum_{i \in N} z_i \ge D \quad (9.7)$$

$$y - z_i \le p_i + r_i \qquad \forall\, i \in N \quad (9.8)$$

$$z_i \ge 0 \qquad \forall\, i \in N \quad (9.9)$$

$$y \ge 0. \quad (9.10)$$

Note that expressions (9.1)-(9.6) are identical to (8.1)-(8.6). Constraint (9.7) corresponds to (8.7). Finally, constraints (9.8)-(9.10) are the dual feasibility constraints of the lower-level problem (8.8)-(8.10).

Table 1 compares the size of the two formulations for the $n-K$ CCGS problem addressed in this paper, namely the proposed robust equivalent (9) and the original contingency-dependent model (1). Model sizes are expressed in terms of the number of constraints excluding variable bounds, the number of continuous variables, and the number of binary variables. As can be seen, even for the simplest case of $K=1$, i.e., n contingency states ($|\mathcal{C}|=n$), the number of constraints and continuous variables of the robust model grows linearly with the number of generation units whereas this increase is quadratic for the contingency-dependent model. This comparison reveals the theoretical superiority of the proposed robust approach over the contingency-dependent model. In the next section, such advantage is shown in terms of significant computational time savings when solving a set of realistic case studies.

Model	# Constraints	# Continuous variables	# Binary variables
Contingency-dependent	$3n\|\mathcal{C}\| + \|\mathcal{C}\| + 4n + 1$	$2n + n\|\mathcal{C}\|$	n
Robust	$5n+2$	$3n+1$	n

$|\mathcal{C}| = \sum_{i=1}^{K} \binom{n}{i}$.

Table 1: Size comparison.

4 CASE STUDIES

Results from several case studies are presented in this Section. For didactical purposes, the robust formulation has been first applied to an illustrative example comprising three generation units. In order to assess the practical applicability of the proposed robust model and the influence of the problem size on its computational performance, a ten-unit system has been replicated to analyze systems including up to 100 generation units. For the sake of simplicity, generators offer linear cost functions of the form $C_i^P(p_i, v_i) = C_i^f v_i + C_i^l p_i$. Therefore, the robust single-level counterpart is a mixed-integer linear programming problem. The model has been implemented on a Pentium E8400, 3 GHz processor with 2 GB of RAM using Xpress-MP7.0 under MOSEL.

4.1 Three-Unit Example

Data for the generators are given in Table 2. The system demand is equal to 50 MW.

Unit	$\underline{P}_i$ (MW)	$\overline{P}_i$ (MW)	$\overline{R}_i$ (MW)	C_i^f ($)	C_i^l ($/MW)	C_i^R ($/MW)
1	10	100	100	300	10	1
2	10	100	100	200	20	2
3	10	100	100	150	30	3

Table 2: Generator data for the three-unit system.

For $K=0$, i.e., without imposing any security criterion, the optimal value of the objective function is $800. At the optimal solution, the cheapest generator 1 is the only unit scheduled, thereby supplying the whole demand, and no spinning reserve contribution is required.

Due to the small size of this problem (eight possible generation schedules), it can be solved by enumeration of the scheduling variables $\boldsymbol{v}$. Table 3 provides the results associated with the feasible schedules for $K=1$ and $K=2$. For $K>2$ the problem is infeasible.

	$K=1$				$K=2$
$\boldsymbol{v}$	$[1\ 1\ 0]^T$	$[1\ 0\ 1]^T$	$[0\ 1\ 1]^T$	$[1\ 1\ 1]^T$	$[1\ 1\ 1]^T$
p_1/r_1 (MW)	40/10	40/10	0/0	30/0	30/20
p_2/r_2 (MW)	10/40	0/0	40/10	10/20	10/40
p_3/r_3 (MW)	0/0	10/40	10/40	10/10	10/40
Cost ($)	1190	1280	1590	1520	1670

Table 3: Results for the feasible schedules of the three-unit system.

For $K=1$, there are only four feasible schedules comprising the commitment of at least two generation units. The optimal solution requires the commitment of the expensive generator 2. This generator operates at its minimum power output of 10 MW, also providing 40 MW of up-spinning reserve. In addition, preventive security requires that the power output of the cheapest generator 1 be reduced with respect to the unconstrained case. The optimal value of the objective function for this security-constrained case is $1190, which is considerably higher than the $800 that it costs to operate the system without security.

For $K=2$, there is only one feasible solution. This optimal solution requires the commitment of all available generation units. Similar to the previous case, the most expensive generators (units 2 and 3) produce the minimum power output of 10 MW while providing 40 MW of up-spinning reserve. Generator 1 reduces its output down to 30 MW and increases its up-spinning reserve contribution up to 20 MW. As expected, the higher level of conservatism yields an increase in the optimal value of the objective function, which is equal to $1670.

The robust optimization model (9) was applied to this illustrative example for $K = 1$ and $K = 2$. The optimal solutions were attained in 0.2 s.

4.2 *Real-Size Case Studies*

The proposed robust formulation has been applied to solve several real-size case studies built on a base test system comprising ten generators. The data for the generators of the base test system can be found in Table 4. The system demand for the base test system is equal to 700 MW. Nine additional case studies have been generated by replicating the original test system and scaling the system demand accordingly.

Unit	$\underline{P}_i$ (MW)	$\overline{P}_i$ (MW)	$\overline{R}_i$ (MW)	C_i^f ($)	C_i^l ($/MW)	C_i^R ($/MW)
1	150	455	455	2550	16.19	1.62
2	150	455	455	2550	17.26	1.73
3	70	180	130	1300	16.60	1.66
4	70	180	130	1300	16.50	1.65
5	50	165	162	1620	19.70	1.97
6	30	90	80	800	22.26	2.23
7	40	85	85	850	27.74	2.77
8	20	60	55	550	25.92	2.59
9	20	60	55	550	27.27	2.73
10	20	60	55	550	27.79	2.78

Table 4: Generator data for the ten-unit system.

In these case studies, the execution of Xpress was stopped when the value of the objective function was within 0.1% of the optimal solution, which is a reasonable choice in terms of solution accuracy. In addition, a time limit of 1000 s was set.

Table 5 provides information on costs attained by the robust model for different values of the security parameter K ranging between 0 and 5. The second column lists the costs for the unconstrained case ($K = 0$). Columns 3-7 show the percent cost increase over the unconstrained case when security is accounted for. Note that the base case problem with ten units is infeasible for $K = 4$ and $K = 5$.

# Units	Cost without security ($)	Cost increase due to security (%)				
		K				
		1	2	3	4	5
10	16433.4	19.7	45.0	63.0	Infeasible	Infeasible
20	31693.4	10.2	21.9	36.9	50.2	59.5
30	47311.8	7.0	14.1	22.0	30.5	41.6
40	62620.2	5.3	10.7	16.1	23.1	29.0
50	78152.1	4.3	8.5	12.8	18.0	22.9
60	93684.1	3.6	7.1	10.7	14.8	18.9
70	108992.0	3.1	6.1	9.2	12.2	16.0
80	124524.0	2.7	5.4	8.0	10.7	13.8
90	140056.0	2.4	4.8	7.1	9.5	12.3
100	155365.0	2.1	4.3	6.4	8.6	10.7

Table 5: Impact of security on cost for real-size systems.

It is remarkable that for each security level, the percent cost increase experiences a significant reduction as the system size grows. As an example, for $K = 3$, the cost increase due to security is reduced from 63.0% down to 6.4%. Furthermore, for each test system, higher values of the security parameter K yield higher cost increases, as expected. However, it is worth mentioning that for real-size systems comprising over 60 units this cost increase is moderate, reaching values between 16.0% and 10.7% for $K = 5$. Both results suggest that tighter security levels than currently used $n - 1$ and $n - 2$ criteria might be used in real-size systems without drastically increasing the total cost.

The relationship between the total cost and the system size can be expressed through a linear regression for each security level K. Table 6 lists the angular and linear coefficients of such regression for values of K ranging between 0 and 5. The angular coefficient represents the increase in cost due to security (additional $ per additional security level K). It should be noted that the angular coefficient varies slightly with K. This result means that, despite the increase in fixed cost due to the schedule of extra units, tighter security levels do not significantly change the rate with which the total cost varies with the system size. As a consequence, the unitary cost, defined as the total cost divided by the number of generation units, decreases with the system size for all security levels. This result is depicted in Fig. 2.

	K					
	0	1	2	3	4	5
Angular ($/security)	1545.9	1546.7	1544.2	1532.2	1518.3	1514.8
Linear ($)	843.2	4118.7	7649.2	11921.7	16510.9	20586.8

Table 6: Coefficients of the linear regression for cost vs. system size.

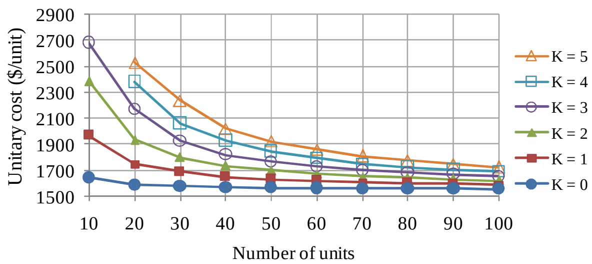

Figure 2: Optimal unitary cost vs. system size.

Table 7 presents the computing times required by the proposed robust model, denoted by R, for all test systems and values of K up to 5. Symbols “I” and “OM” represent “Infeasible” and “Out of memory”, respectively. The computational performance of the robust approach is assessed through the comparison with the contingency-dependent formulation (1), referred to as CD. For the sake of tractability of problem (1), the set of contingencies in CD only includes those with exactly K units simultaneously out of service, i.e., the cardinality of $\mathcal{C}$ is equal to $\binom{n}{K}$ rather than $\sum_{i=1}^{K}\binom{n}{i}$. Note that this reduced contingency set covers all contingencies with fewer unavailable generators and hence optimality is not affected. As can be seen, the robust approach is able to find a solution satisfying the pre-specified optimality tolerance with little computational effort for all feasible case studies. In contrast, the contingency-dependent model requires much larger computing times for most cases, particularly for large-scale systems.

Moreover, for large numbers of units and more conservative security criteria, the contingency-dependent model is unable to find a near optimal solution within the time limit and even leads to intractable problems for which not enough memory is available. These results clearly back the superiority of the robust model over the contingency-dependent formulation from a computational viewpoint.

# Units	K									
	1		2		3		4		5	
	R	CD	R	CD	R	CD	R	CD	R	CD
10	0.1	0.1	0.1	0.5	0.1	0.1	I	I	I	I
20	0.1	0.3	0.1	1.3	0.1	1.3	0.1	13.9	0.1	146.9
30	0.9	1.0	0.5	1.7	0.9	20.3	0.1	314.6	0.3	1000.0
40	1.4	2.0	1.1	8.1	0.4	30.0	0.6	1000.0	0.2	OM
50	3.0	2.5	3.1	22.6	0.3	96.9	9.5	OM	1.4	OM
60	11.3	14.8	6.4	97.5	2.5	704.3	68.7	OM	4.1	OM
70	12.2	14.8	25.7	390.5	8.3	1000.0	0.8	OM	65.8	OM
80	6.6	9.7	10.1	136.8	3.1	1000.0	0.7	OM	108.7	OM
90	6.7	11.2	1.7	1000.0	73.2	OM	6.0	OM	195.1	OM
100	18.7	38.9	33.7	1000.0	21.2	OM	3.3	OM	0.8	OM

Table 7: Comparison of computing times (s) for real-size systems.

5 CONCLUSIONS

This paper presents a robust optimization approach for the contingency-constrained generation scheduling problem with an $n-K$ security criterion. As a major contribution of this paper, the model described allows system operators to schedule energy and reserve while explicitly considering all combinations of up to K generation unit outages. The original contingency-dependent model is first formulated as a robust bilevel counterpart. The resulting bilevel program is subsequently transformed into an equivalent single-level mixed-integer program that is efficiently solved using available commercial branch-and-cut software.

Numerical results show that the proposed robust model outperforms the contingency-dependent formulation since significant cost reductions are achieved within moderate computing times. Moreover, the robust model can handle problems that are essentially intractable for the contingency-dependent model.

Research is currently underway to consider a network-constrained system. Finally, further research will also be devoted to pricing energy and reserve under the $n-K$ security criterion and to assessing the tradeoff between cost and security.

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REFERENCES

[1] A. J. Wood and B. F. Wollenberg, “Power Generation, Operation, and Control”, 2nd ed., New York, Wiley, 1996, ISBN 0-471-58699-4

[2] G. Andersson, P. Donalek, R. Farmer, N. Hatziargyriou, I. Kamwa, P. Kundur, N. Martins, J. Paserba, P. Pourbeik, J. Sanchez-Gasca, R. Schulz, A. Stankovic, C. Taylor and V. Vittal, “Causes of the 2003 Major Grid Blackouts in North America and Europe, and Recommended Means to Improve System Dynamic Performance”, IEEE Transactions on Power Systems, vol. 20, no. 4, pp 1922-1928, November 2005

[3] F. Fonteneau-Belmudes, D. Ernst and L. Wehenkel, “A Rare Event Approach to Build Security Analisys Tools when $N-k$ $(k>1)$ Analyses Are Needed (as they Are in Large Scale Power Systems)”, 2009 IEEE PowerTech Proceedings, pp 1-8, June-July 2009

[4] J. M. Arroyo and F. D. Galiana, “Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets”, IEEE Transactions on Power Systems, vol. 20, no. 2, pp 634-643, May 2005

[5] M. Shahidehpour, W. F. Tinney and Y. Fu, “Impact of Security on Power Systems Operation”, Proceedings of the IEEE, vol. 93, no. 11, pp 2013-2025, November 2005

[6] J. Wang, N. E. Redondo and F. D. Galiana, “Demand-Side Reserve Offers in Joint Energy/Reserve Electricity Markets”, IEEE Transactions on Power Systems, vol. 18, no. 4, pp 1300-1306, November 2003

[7] J. Wang, M. Shahidehpour and Z. Li, “Contingency-Constrained Reserve Requirements in Joint Energy and Ancillary Services Auction”, IEEE Transactions on Power Systems, vol. 24, no. 3, pp 1457-1468, August 2009

[8] D. Bertsimas and M. Sim, “The Price of Robustness”, Operations Research, vol. 52, no. 1, pp 35-53, January-February 2004

[9] S. Dempe, “Foundations of Bilevel Programming”, Norwell, Kluwer Academic Publishers, 2002, ISBN 1-4020-0631-4

[10] A. Street, F. Oliveira and J. M. Arroyo, “Contingency-Constrained Unit Commitment with $n-K$ Security Criterion: A Robust Optimization Approach”, IEEE Transactions on Power Systems, in press, 2011

[11] F. Bouffard, F. D. Galiana and J. M. Arroyo, “Umbrella Contingencies in Security-Constrained Optimal Power Flow”, 15th PSCC Proceedings, Session 21, Paper 4, pp 1-7, August 2005

[12] L. Wolsey, “Integer Programming”, New York, Wiley Inter-Science, 1998, ISBN 0-471-28366-5