

# IMPACT OF MODELING ACCURACY ON INTEGRATED OPTIMIZATION OF A HYDROTHERMAL PORTFOLIO ON SHORT-TERM MARKETS FOR SCHEDULED ENERGY AND SYSTEM RESERVE

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**Abstract** – Due to the liberalization of the European energy market, power generation and trading companies aim for profit maximization instead of cost minimization. Especially for a short-term time horizon this motivates a new modeling of the short-term hydrothermal unit commitment problem as a closed-loop optimization taking into account all relevant technical and market-related restrictions particularly on reserve markets.

This work presents a computational efficient method for modeling both hydraulic and thermal power plants in a mathematically exact approach of mixed-integer quadratic programming. In order to achieve practical results each unit is described in detail by its characteristic constraints. This leads to reasonable power plant schedules especially distinguishing between provision of reserve capacity and supply of reserve energy. In this context the closed-loop formulation of the resulting optimization problem overcomes drawbacks of prior approaches by ensuring that the solution is the guaranteed optimum.

After validating the method's functionality it is applied to a realistic portfolio by optimizing short-term generation and determining the optimal share of reserve and scheduled energy for the considered system. Comparing the different power plant schedules achieved by including, simplifying or even neglecting single optimization constraints in different simulations the additional value of the new optimization method and the applied modeling accuracy can be quantified.

**Keywords:** *mixed-integer quadratic programming, hydrothermal portfolio, short-term markets, modeling accuracy*

## 1 INTRODUCTION

Optimization of a hydrothermal portfolio is a computationally intensive task which has become even more challenging due to the introduction of new markets for system reserve in the recent years. The applied decomposition approach does not guarantee optimality of the solution, which appears acceptable for questions in the medium term time horizon [1].

Developments in the field of hardware and algorithms motivate several new approaches concerning the unit commitment problem [2]. This work uses a closed-loop formulation to optimize a hydrothermal portfolio with variable modeling accuracy at least for a short-term time horizon. Therefore, this paper at first defines the considered system and introduces the applied optimization method. Afterwards, exemplary investigations are conducted for a realistic hydrothermal portfolio. In this context variation of the modeling accuracy illustrates

the model's impact on both solution quality and computation time. High and measurable solution quality is of increasing importance if short-term optimization is conducted directly influencing the generators' real dispatch.

## 2 CONSIDERED SYSTEM

The optimization problems considered in this paper consists of hydro and thermal units as generators on the one hand and markets for scheduled energy and system reserve on the other hand. Figure 1 illustrates the different components.

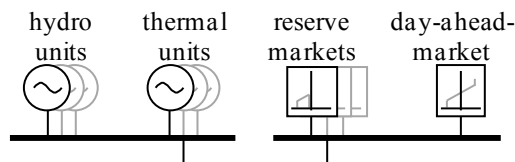


Figure 1: Considered system

### 2.1 Generation

#### 2.1.1 Thermal Units

In terms of optimization, thermal units are characterized by their cost terms including start-up and shut-down costs as well as costs for stationary operation and their power output. Start-up and shut-down cost occur once for each change in the plant's operation mode. According to [3] during normal operation costs  $c_{op}$  are driven by primary energy costs  $c_{PE}$ , emission certificate costs  $c_{CO_2}$ , additional costs  $c_{add}$  and fix costs  $c_{fix,op}$ .

$$c_{op} = [\hat{Q}(P) \cdot (c_{PE} + c_{CO_2} \bar{e}_{PE}) + c_{add} \cdot P + c_{fix,op}] \cdot \Delta t \quad (2.1)$$

In this context  $\bar{e}_{PE}$  stands for specific primary energy consumption and  $\hat{Q}$  describes the unit's quadratic head consumption curve modeling an output-dependent efficiency.

$$\hat{Q}(P) = hc_0 + hc_1 \cdot P + hc_2 \cdot P^2 \quad (2.2)$$

The power output of a thermal unit can be modeled for each time interval as a semi-continuous variable between minimum power output  $P_{min}$  and maximum power output  $P_{max}$  or zero power output. Moreover, a unit's maximal power gradient  $\Delta P_{op}$  represents maximum change in the power output between two time intervals.

$$\begin{aligned} P(t) &\leq P(t-1) + \Delta P_{op} \cdot \Delta t \\ \vee P(t) &\geq P(t-1) - \Delta P_{op} \cdot \Delta t \end{aligned} \quad (2.3)$$

The power gradient can also be used to model maximal positive and negative reserve provision  $P_{res}^{+/-}$  of each unit on all markets in addition to scheduled energy  $P_{se}$ .

$$\begin{aligned} P_{se}(t) + \sum_{reserve} P_{res}^+(t) &\leq p(t-1) + \Delta P_{op} \cdot \Delta t \\ &\leq P_{max} \\ \wedge P_{min} \leq P_{se}(t-1) - \Delta P_{op} \cdot \Delta t &\leq P_{se}(t) - \sum_{reserve} P_{res}^-(t) \end{aligned} \quad (2.4)$$

Changing a thermal unit's mode of operation from stand still to running is called start-up. During this process start-up costs  $c_{su}$  occur, which consist of down-time-dependent costs and fix costs  $c_{fix,su}$ .

$$\begin{aligned} c_{su} &= [Q_{su}(t_{st}) + \dot{Q}(P_{su})\Delta t] \\ &\quad \cdot (c_{PE} + c_{CO_2} e_{PE}) \\ &\quad + c_{fix,su} \end{aligned} \quad (2.5)$$

As  $Q_{su}(t_{st})$  is usually conditioned to the unit's boiler temperature it can be formulated as:

$$Q_{su}(t_{st}) = Q_{su}^{\infty} \cdot \left(1 - e^{-\frac{t_{st}}{\tau}}\right) \quad (2.6)$$

In this context  $\tau$  is named cool down-time coefficient and  $Q_{su}^{\infty}$  cold start energy consumption.

Moreover, minimum up- and down-times can be modeled as constraints of the optimization problem.

### 2.1.2 Hydro Units

Turbines transform potential energy of water into electricity. Especially in combination with storage reservoirs and additional pumps these plants represent an economically efficient possibility to store energy and provide reserve power and energy.

The approach presented in this paper models each hydraulic group as water-based interconnected system which can be described by a continuity equation covering all time intervals [4]. Therefore, the content  $V$  of each reservoir can be described by its inflows  $F_{inf}$ , outgoing flows  $F_{flo}$ , flows through incoming pumps  $F_{pum}$  and outgoing turbines  $F_{tur}$  while  $\beta$  models evaporation.

$$\begin{aligned} V(t-1) - V(t) &= \left[ \sum F^{inf}(t) + \sum F^{pum}(t) - \sum F^{tur}(t) \right. \\ &\quad \left. + \sum F^{flo}(t) \right] \cdot \Delta t - V(t) \\ &\quad \cdot \beta \end{aligned} \quad (2.7)$$

As a reservoir's volume is limited due to geography or other usage of the water the volume can only be varied between a – possibly time dependent – minimum level  $V_{min}(t)$  and a maximum level  $V_{max}(t)$ .

Connections between reservoirs are modeled as waterways which can be specified by a minimum flow  $F_{min}$ , a maximum flow  $F_{max}$ , a flow time  $t_{ft}$  and a loss factor  $\gamma$ . Moreover, a waterway can be defined as pump

or turbine generating or consuming a flow-dependent amount of energy per time interval.

$$P^{tur} = \eta \cdot \rho \cdot g \cdot h \cdot F^{tur} \quad (2.8)$$

$$P^{pum} = \frac{1}{\eta} \cdot \rho \cdot g \cdot h \cdot F^{pum} \quad (2.9)$$

In this context,  $\eta$  stands for the efficiency,  $\rho$  for the density of water,  $g$  for the acceleration of gravity and  $h$  for the head of water. [4] shows that this model can be extended to a level-dependent head of water  $h(V)$  and a level- and flow-dependent efficiency  $\eta(V, F)$ .

In contrast to thermal generation, hydro units can adapt their power output almost instantly. Therefore, requested reserve energy just forms part of the total power output while provision of reserve energy results in an additional flow constraint of the affected waterway.

$$\begin{aligned} P_{min}^{tur/pum}(t) &\leq P_{se}(t) + \sum_{reserve} P_{res}^+(t) \\ &\quad - \sum_{reserve} P_{res}^-(t) \\ &\leq P_{max}^{tur/pum}(t) \end{aligned} \quad (2.10)$$

$$\begin{aligned} F_{min}^{tur/pum}(t) &\leq F_{se}(t) + \sum_{reserve} F_{res}^+(t) \\ &\quad - \sum_{reserve} F_{res}^-(t) \\ &\leq F_{max}^{tur/pum}(t) \end{aligned} \quad (2.11)$$

Moreover, machine concepts may constrain the use of one waterway or even antiparallel waterways. For instance, the coincidental use of a pump and turbine waterway of a pumpturbine is not possible and can be prohibited by the following NAND-composition:

$$\overline{(F_{tur} \neq 0 \wedge F_{pum} \neq 0)} \quad (2.12)$$

Finally, network usage fees can be modeled as additional cost for pump operation.

## 2.2 Short-term Markets

This paper focuses on short-term markets for electrical energy. In the German market area it can be distinguished between an exchanged-based day-ahead spot market and reserve markets with differing trade rules.

### 2.2.1 Market for Scheduled Energy

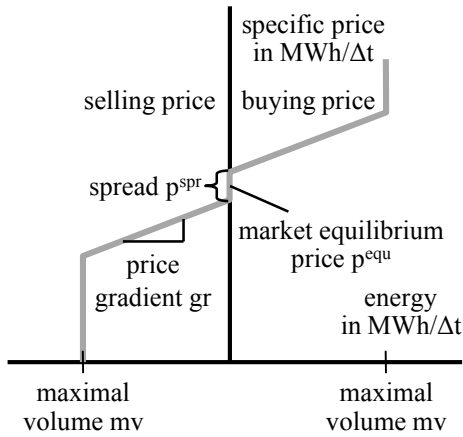
At the German day-ahead spot market electrical energy is traded for all single hours of the next day in single-price auction. This market can be described by a price-sales-function shown in figure 2.

Based on a market's equilibrium price  $p^{equ}$  and a purported price gradient  $gr$  this function models the limited market liquidity while trading costs are represented by a price spread  $p^{spr}$ .

Using this formulation the revenues  $r_{spo}$  generated at the spot market can be easily computed:

$$\begin{aligned} r_{spo} &= \left( \left( p_{spo}^{equ} + \frac{p_{spo}^{spr}}{2} \right) \cdot P_{spo} + gr r_{spo} \right. \\ &\quad \left. \cdot (P_{spo})^2 \right) \cdot \Delta t, \forall P_{spo} \geq 0 \end{aligned} \quad (2.13)$$

$$r_{spo} = \left( \left( p_{spo}^{equ} - \frac{p_{spo}^{spr}}{2} \right) \cdot P_{spo} - g r_{spo} \right) \cdot (P_{spo})^2 \cdot \Delta t, \forall P_{spo} < 0 \quad (2.14)$$



**Figure 2:** Price-sales-function

As power plant owners may face outages after participating in day-ahead trading an exchange-based intraday market for electrical energy has also been established in Germany during the recent years. This trade stage enables offer-based purchase and disposal of energy on an hourly base until 75 minutes before fulfillment. As this new market is based on acceptance and rejection of single bids, it is not part of the optimization problem discussed in this paper.

### 2.2.2 Markets for System Reserve

In contradiction to the above mentioned spot markets, reserve markets apply pay-as-bid auctions for both reserve capacity provision and reserve energy request. Therefore, each bid consists of reserve amount, capacity price and energy price. By considering only capacity prices the low priced bids are accepted for reserve provision at first. Afterwards, accepted bids with the lowest energy prices are used to yield reserve energy if required. Similar to section 2.2.1 this trade stage for reserve power can be modeled by a one-sided price-sales-function applying the same type of equations for each reserve market:

$$r_{res,p} = \left( \left( p_{res,p}^{equ} - \frac{p_{res}^{spr}}{2} \right) \cdot P_{res} - g r_{res} \right) \cdot (P_{res})^2 \cdot \Delta t \quad (2.15)$$

If reserve energy is requested from the power plant owner's point of view additional revenues (for provision of positive reserve energy) or additional costs (for extraction of negative reserve energy) are generated depending on the energy price  $p_{res,e}$  and a request probability  $pr_{res,e}$ :

$$r_{res,e} = pr_{res,e} \cdot p_{res,e} \cdot P_{res} \quad (2.16)$$

On the German market for tertiary reserve, which is considered in section 4, reserve energy must be provided within 15 minutes. Moreover, the trading is proceeded on a daily basis by introducing three products for peak and off-peak periods each with single product periods of four hours.

## 3 OPTIMIZATION METHOD

The goal of the optimization of a hydrothermal portfolio is to maximize the contribution margin  $CM$  of all hydrothermal units  $u$  on all markets  $m$  over all time intervals  $t$  also taking into account the arising generation costs  $c$  applying an hourly time pattern.

$$CM = \max \sum_t \sum_u \sum_m \left\{ \overbrace{P(t, u, m) \cdot p(t, m)}^{r(t,u,m)} - c(t, u) \right\} \quad (3.1)$$

In the following subsections each variable of the equation will be analyzed in detail.

### 3.1 Generation

#### 3.1.1 Thermal Units

Optimization of thermal unit commitment requires – even for a short-term time horizon – a computational efficient formulation of the optimization problem. The method applied in this paper uses the approach discussed in [5]. Basically, this implies a formulation using only one binary variable  $v(t)$  per thermal unit and time interval. In addition, the complexity of the optimization problem is reduced by linearizing the heat consumption curve  $\hat{Q}(P)$  as well as the down-time dependent heat consumption  $Q_{su}(t_{st})$ . In this context, the number of linearization steps offers a trade-off between computation time and solution quality.<sup>1</sup>

In general, the power output of a thermal unit can be modeled in the following way:

$$P_{\min} \cdot v(t) \leq P(t) \leq P_{\max}(t) \vee 0 \leq P_{\max}(t) \leq P_{\max} \cdot v(t) \quad (3.2)$$

In this context,  $P_{\max}(t)$  stands for the maximal power which can be generated by the unit at the current time interval. As variable-efficient implementation of power gradients the required constraints of the optimization problem can be formulated by just applying the binary variable  $v(t)$ .

$$P^{\max}(t) \leq P(t-1) + \Delta P_{op} \cdot v(t-1) + \Delta P_{su} \cdot [v(t) - v(t-1)] + P_{\max} \cdot [1 - v(t)] \quad (3.3)$$

$$P^{\max}(t-1) \leq P_{\max} \cdot v(t) + \Delta P_{su} \cdot [v(t-1) - v(t)] \quad (3.4)$$

<sup>1</sup> Comparative examinations at the Institute for Power Systems and Power Economics at RWTH Aachen University suggest that 4 linearization steps are sufficient to achieve results adequate to an optimization based on quadratic programming.

$$P(t-1) - P(t) \leq \Delta P_{op} v(t) + \Delta P_{su} \cdot [v(t-1) - v(t)] + P_{max} \cdot [1 - v(t-1)] \quad (3.5)$$

It can be distinguished between a power gradient for normal operation  $\Delta P_{op}$  and start-up or shut-down  $\Delta P_{su}$ .  $\Delta P_{op}$  also limits the maximal providable amount of reserve.

Minimum up-/down-times  $t_{min}$  are considered by implementing a checksum of different binary variables of one thermal unit:

$$t_{min} \cdot [v(t-1) - v(t)] \leq \sum_{n=t-t_{min}}^{t-1} v(n) \quad (3.6)$$

This inequation is only fulfilled, if the unit has been in operation for the last  $t_{min}$  time intervals. The formulation for minimum down-times yields vice versa.

### 3.1.2 Hydro Units

The hydro unit commitment problem can be modeled easily by the equations given in section 2.1.2 as semi-continuous problem. However, introducing pump-turbines also causes additional integer decisions. Different specifications of these formulas make it also possible to consider as well pumps, turbines and pump-turbines. The latter can be distinguished by the prohibited coincidental use of its pump and turbine waterway at the same time interval.

For pumps/turbines in operation a minimum flow unequal zero can be given:

$$P^{tur/pum}(t) = 0 \vee P_{min}^{tur/pum}(t) \leq P^{tur/pum}(t) \leq P_{max}^{tur/pum}(t) \quad (3.7)$$

### 3.2 Short-term Markets

The revenues generated at the short-term markets can easily be computed according to section 2.2.1 and 2.2.2. The maximal tradable volume  $mv_m$  of all units on a market  $m$  is considered by the following constraint:

$$\sum_u P_m(t, u) \leq mv_m \quad (3.8)$$

If a price gradient  $gr_m \neq 0$  is taken into account, the market's reaction to the company's trading can be modeled. This transforms the problem from mixed integer linear to mixed integer quadratic programming offering a tradeoff between solution quality and computational recourses.

#### 3.2.1 Market for Scheduled Energy

Participation in the day-ahead spot market leads to the constraint that the amount of traded energy  $P_{se}(t)$  of one time interval must be generated at the same time interval by all hydrothermal units.

#### 3.2.2 Markets for System Reserve

Participation in reserve markets limits the operation of hydrothermal units as the traded positive reserve capacity  $P_{res}^+(t)$  and negative reserve capacity  $P_{res}^-(t)$  must be provided over the entire validity period of the traded product. Moreover, reserve energy can be requested and therefore must be objected. It can be distin-

guished between standalone and pooled reserve provision:

In standalone operation each unit trades reserve on its own while more complex pooling makes it possible to provide traded reserve capacity and energy in the sum of all pooled hydrothermal units. Prior works suggest that this procedure allows additional benefits due to the combination of different generation technologies [6].

## 4 EXEMPLARY INVESTIGATIONS

After introducing the optimization method exemplary investigations will illustrate the practicability of the approach. Therefore, at first a model system is introduced. In order to proof functionality, initially a small optimization problem with traceable results is considered. Subsequently, the model system's complexity grows so the impact of linearization on thermal unit commitment, the impact of reserve market participation on general unit commitment and the evolution of computing time are considered.

### 4.1 Model system

#### 4.1.1 Hydrothermal Units

The hydrothermal model system considered in this paper consists of a modern 850 MW gas power plant (GAP), a hard coal power plant (HCP) with an installed capacity of 750 MW and a pump storage power plant whose pump (PSP) and turbine (PST) capacity is 250 MW. The thermal plants differ concerning the field of generation costs, minimum up-/down-times, efficiency and minimum/maximum power output while the hydro units are assumed to be implemented as pump-turbine. If this reserve market is considered, all units can provide tertiary control reserve. Table 1 sums up the technical data of the considered plants.

	HCP	GAP	PST/P
$P_{max}$ [MW]	750	850	250
$P_{min}$ [MW]	300	340	0
$\Delta P_{op}$ [MW/min]	20	25	250
$\eta$	0.45	0.58	0.90
$\tilde{c}_{PE}$ [€/GJ <sub>th</sub> ]	4.34	7.47	0.00
$\tilde{e}_{PE}$ [t CO <sub>2</sub> /GJ <sub>th</sub> ]	0.09	0.06	0.00

**Table 1:** Input parameters of the hydrothermal generation

In this context,  $\tilde{c}_{PE}$  stands for the specific mean primary energy costs. For emission certificates mean costs of 22.32 €/t CO<sub>2</sub> are assumed while network usage fees for pumping comprise 0.60 €/MWh.

#### 4.1.2 Markets

As spot market prices the exemplary investigations take into account the European Energy Exchange's hourly prices of 2008 (Base: 65.76 €/MWh, Peak: 88.07 €/MWh) [7]. The maximal tradable volume is limited to the considered installed capacities.

For tertiary reserve prices are deduced from the German reserve trade platform in 2008 (mean price for positive reserve: 5.33 €/MW·h, mean price for negative

reserve: 2.61 €/MW·h) [8]. Reserve request is modeled by an artificial signal with a realistic request probability smaller 1 %. This representation implies that the portfolio is participating as generation pool in this market and therefore can provide reserve capacity over the entire product period of four hours in different units. Moreover, reserve is requested on an hourly basis simplifying real request, which is adjusted every quarter of an hour.

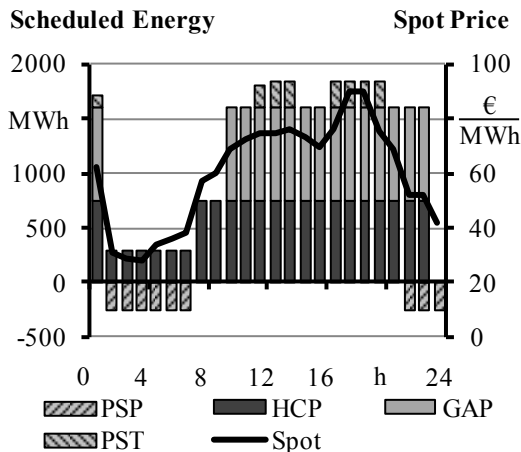
#### 4.2 Functionality

To proof the approach's functionality a simple problem is taken into account, optimizing the hydrothermal dispatch against a spot market only. Figure 4 illustrates the units' schedule. As the gas power plant's marginal costs including emission certificates amount to about 60 €/MWh and the hard coal power plant's marginal costs make up only about 50 €/MWh different schedules can be observed.

Due to the hard coal power plant's minimum up- and down-times it is not shut down after the 1<sup>st</sup> hour but produces energy without earning its marginal costs. Doing so ensures that the power plant can be used in the high-price period starting at 8 a.m. as the additional profits in the following hours exceed the prior losses.

The gas power plant in contrast has higher marginal costs. Therefore, it is used in the 1<sup>st</sup> hour and switched off afterwards. In the 10<sup>th</sup> hour, when the spot price excels the marginal costs, it can be switched on again as the minimum down-time has already been exceeded.

The pump storage power plant pumps in all low price hours and turbines in high price hours as long as the price spread exceeds the revolving efficiency. Due to optimization constraints, the reservoir levels in the end must equal the levels at the beginning. For this reason, the turbine is not always charged to capacity if running.



**Figure 3:** Power plant schedule for spot marketing

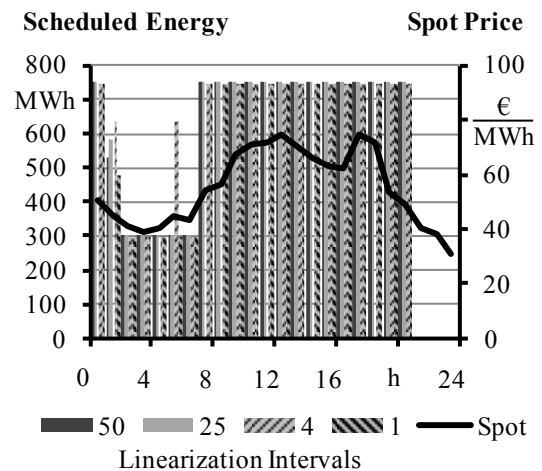
In general, the examination illustrates that the optimization method is suitable for modeling the hydrothermal dispatch.

#### 4.3 Linearization

In order to examine the influence of a different amount of linearization intervals on the optimization problem, four simulations with 50, 25, 4 and 1 linearization intervals have been conducted for the already intro-

duced thermal generation units while the hydro unit is neglected. The time domain comprises six months during which the power plants can participate in the spot market.

Analysis illustrates that the impact of linearization intervals is quiet low. The contribution margin generated by both units in the considered time horizon varies between 79.864 Mio. € (50 intervals) and 79.874 Mio. € (4 intervals) making up a deviation of about 0.6 %. However, figure 4 shows that the impact on the power plant schedule on single days cannot be neglected: As the market price is close to the hard coal power plant's marginal costs in the 1<sup>st</sup> and 5<sup>th</sup> hour linearization in connection with a price gradient influences the schedule in these intervals. Due to the price gradient the hard coal power plant is neither running on maximum nor on minimum power. Instead of this the output ensues from the costs derived from the different linearization results. Especially the schedule deviation only occurring for simulation with 4 linearization intervals in the 5<sup>th</sup> hour illustrates that more linearization intervals not necessarily lead to more accurate results. Anyhow, more intervals make it more improbable that linearization causes unrealistic optimization.



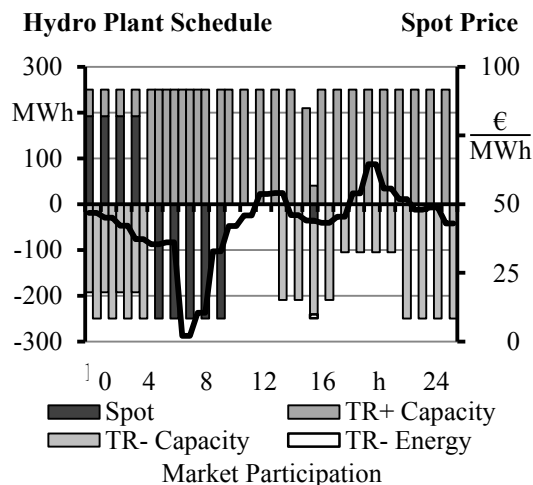
**Figure 4:** Exemplary schedule of the hard coal power plant for different linearization intervals

#### 4.4 Optimal Share of Reserve and Scheduled Energy

In order to evaluate the optimal share of reserve and scheduled energy for the model system an optimization for one month is conducted including spot market and market for tertiary reserve. As for the second market the tradable capacity is assumed to be unlimited, the results identify the optimal share of reserve from the power plant owner's point of view.

Results identify that provision of negative tertiary reserve (TR-) or positive tertiary reserve (TR+) for the considered time horizon and price level is attractive for the pump storage power plant only. This is not only caused by the product periods of four hours which complicate provision in thermal units with minimum up-/down-times but also by the hydro unit's utilization independent efficiency. The traded average capacity comprises 212 MW positive tertiary reserve and

120 MW negative tertiary reserve. In the optimization period 134 MWh of positive and 280 MWh of negative reserve energy are requested.



**Figure 5:** Hydro power plant schedule for turbine (left stack of each time interval) and pump (right stack of each time interval) marketing

Figure 5 illustrates reserve provision in the pump storage power plant for an exemplary day of the optimization problem. In the first half of the day the pump-turbine participates mainly at the spot market. Additionally, free capacities are sold at the reserve market. Although they are implemented as pump-turbine the pump and turbine waterway can participate in the same reserve markets at the same time as reserve energy can be provided at first only from one waterway and afterwards from the other. In the second half of the day the hydro power plant prefers reserve capacity provision to spot marketing as the capacity price in these time intervals makes this market more attractive. Figure 5 shows in the 15<sup>th</sup> time interval the request of negative tertiary reserve energy. Moreover, the usage of both pump and turbine in this interval illustrates that reserve is provided by both waterways as pool in order to meet the product constraints: While the positive reserve capacity provided during this interval in the turbine is 58 MW lower than in the adjoining time intervals, the pump provides this share of reserve as the amount of provided reserve capacity must be the same over the entire product period.

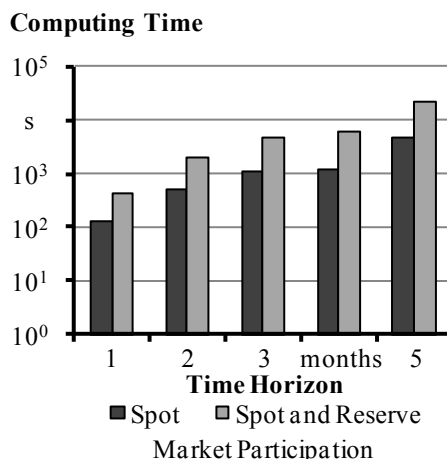
These results illustrate that especially hydro units are suitable for reserve provision in hydrothermal portfolios.

#### 4.5 Computing Time

In order to examine the method's computational efficiency the model system is optimized against the spot market only as well as a combination of spot and tertiary reserve market for different time horizons.

Due to the increasing complexity of the optimization problem it can be expected that the computation requirements increase as well due to the additional reserve market as the longer time horizons. For optimization the commercial solver CPLEX 12.1 has been applied on a

state-of-the-art multi-parallel computer. As stop criterion a maximal gap of 0.001 from the theoretical optimum has been defined.<sup>2</sup>



**Figure 6:** Computing time optimizing the model system on spot or spot and reserve markets for different time horizons

Figure 6 shows the increasing computing time on a logarithmic basis. For both sole spot and combined spot and reserve trading, computing time in general grows exponentially with the optimization's time horizon. For sole spot trading the internal heuristics applied by CPLEX for searching the solution space's single branches prove to be coincidentally very efficient for a four month time horizon. Solving this problem hardly needs more computing time than the optimization comprising three months.

In general, the close-loop approach allows optimization of small hydrothermal portfolios on different short-term markets for up to five month always finding the guaranteed optimum. For longer time horizons decomposition approaches have to be applied in order to achieve results of similar solution quality.

## 5 CONCLUSION

This work presents a computational efficient method for modeling both hydraulic and thermal power plants in a mathematically exact approach of mixed-integer quadratic programming. After defining the considered system, the closed-loop formulation for both thermal and hydro units as well as spot and reserve markets has been introduced. The modeling options identified in this section not only apply to the exemplary system presented in the paper but also enable to model more complex generation portfolios participating in additional markets. Selective investigations illustrate the approach's functionality pointing out that modeling accuracy can be controlled by adjusting the amount of linearization steps for instance showing a tradeoff between solution quality and computational efforts. Further simulations illustrate the positive impact of hydro units on reserve provision

<sup>2</sup> The theoretical optimum is defined as solution, which can be achieved by neglecting all integer decisions.

offering an exemplary procedure for determining the ratio between scheduled energy and reserve in a given portfolio. Finally, the method's potential is evaluated by investigating the development of computing time depending on system complexity.

Recapitulatory, the method presented in this paper allows short-term optimization of realistic hydrothermal portfolios with customizable modeling accuracy and manageable computational efforts.

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