

# DAMPING OF LOCAL MODE OSCILLATIONS IN A POWER SYSTEM BY DELAYED FEEDBACK CONTROL

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**Abstract** – This paper presents a delayed feedback controller (DFC) for damping local mode oscillations in power systems. The proposed controller uses the generator rotor angular speed signal as the sole input. The output of the controller is combined into the automatic voltage regulator (AVR) as the stabilizing signal. Stability regions depending on the DFC time delay have been determined by the cluster treatment of characteristic roots method. Time domain simulations in a single-machine infinite-busbar power system are presented to demonstrate the effectiveness of the proposed controller in damping local mode oscillations. Moreover, comparison of DFC with a generic power system stabilizer is discussed. The impact of DFC on the terminal voltage regulating function of AVR is also investigated.

**Keywords:** Power system stability, damping controller, low frequency oscillation

## 1 INTRODUCTION

Electromechanical oscillations in electric power systems have been widely studied over the decades [1, 2]. Local mode oscillations involve a single generator or a generating plant connected to a large power system. Inter-area oscillations, on the other hand, occur in power systems if a number of generators in one area swing against generators in another area. In many power systems constrained by stability, the limiting factor is not the first swing but the damping of system oscillations [3].

The techniques for damping power system oscillations include the power system stabilizer (PSS) [4], high-speed governor control [5] and flexible ac transmission system (FACTS) devices [6-8]. PSS is the most widely used device for damping low frequency oscillations [9]. If tuned correctly, PSS contributes to positive damping by generating an electric torque in phase with the generator rotor speed.

Delayed feedback control [10] is a simple and efficient method to stabilize both unstable periodic orbits (UPO) embedded in the strange attractors of chaotic systems [11] and unstable steady states [12]. Also known as Time Delay Auto-Synchronization (TDAS), this control scheme makes use of the current state of a system and its state  $\tau$ -time unit in the past to generate a control signal. Successful implementations of TDAS algorithm are reported in diverse systems including mechanical pendulums [13], helicopter rotor blades [14], a cardiac system [15], trajectory tracking [16],

absorption of mechanical vibrations [17], and suppression of subsynchronous resonance [18].

In this paper, a novel controller based on the delayed feedback control theory has been proposed to improve damping in a single-machine infinite-busbar (SMIB) power system. Generator rotor angular speed is used as the sole input of the controller. The output of the controller is combined into the AVR as the stabilizing signal.

In order to identify stable regions in the parameter domain of the DFC time delay, we employ the methodology proposed in [19]. The cluster treatment of characteristic roots (CTCR) is a mathematical paradigm which can be used to find stability intervals in time delayed linear time invariant systems. The method involves determining all possible purely imaginary characteristic roots for any positive delay by applying Rekasius substitution. The number of such roots is finite and each of these roots is created by periodically distributed time delays which are infinitely many. Liu and Jiang [20] applied the CTCR method to analyze the stability of a power system considering the feedback transmission delay.

This paper is organized as follows: the mathematical model of the SMIB power system is given in Section II. DFC and brief overview of the CTCR method is given and stability pockets for the nonlinear model with DFC are determined in Section III. Finally, DFC effectiveness for damping local mode oscillations and its comparison with a generic PSS are discussed in Section IV.

## 2 MODEL DESCRIPTION

### 2.1 SMIB Power System

The SMIB power system used in this study is shown in Fig. 1. The nonlinear state equations representing the dynamics of the model can be derived using direct and quadrature d-q axes and Park's transformation [21]. The effect of damper windings is included. Governor dynamics and saturation effects are neglected.

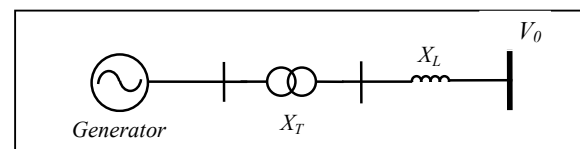


Figure 1: SIMB power system

State equations:

$$-(X_d + X_E) \frac{di_d}{dt} + X_{md} \frac{di_f}{dt} + X_{md} \frac{di_D}{dt} = w_b [(w_r X_q + X_E) i_q + R_a i_d + w_r X_{mq} i_Q + V_0 \sin \delta_r] \quad (1)$$

$$-(X_q + X_E) \frac{di_q}{dt} + X_{mq} \frac{di_Q}{dt} = w_b [(w_r X_d + X_E) i_d - w_r X_{md} i_f + R_a i_q + w_r X_{md} i_D + V_0 \cos \delta_r] \quad (2)$$

$$-X_{md} \frac{di_d}{dt} + X_f \frac{di_f}{dt} + X_{md} \frac{di_D}{dt} = w_b \left[ R_f i_f + \frac{R_f E_{fd}}{X_{md}} \right] \quad (3)$$

$$-X_{md} \frac{di_d}{dt} + X_{md} \frac{di_f}{dt} + X_D \frac{di_D}{dt} = -w_b R_D i_D \quad (4)$$

$$-X_{mq} \frac{di_q}{dt} + X_Q \frac{di_Q}{dt} = -w_b R_Q i_Q \quad (5)$$

where  $X_E = X_L + X_T$ .

Generator:

$$\frac{dw_r}{dt} = \frac{1}{2H} [T_m - T_e - D(w_r - 1)] \quad (6)$$

$$\frac{d\delta_r}{dt} = w_b (w_r - 1) \quad (7)$$

Static Exciter (IEEE ST1A [2]):

$$\frac{dV_C}{dt} = \frac{1}{T_R} [V_t - V_C] \quad (8)$$

$$\frac{dE_{fd}}{dt} = \frac{1}{T_A} [K_A V_{ref} - K_A V_C - E_{fd}] \quad (9)$$

In (6),  $T_e$  represents the electromechanical torque and it is expressed as

$$T_e = (X_q - X_d) i_d i_q + X_{md} i_{fd} + X_{mq} i_Q i_D + X_{md} i_D i_Q \quad (10)$$

$V_t$  in (8) is the generator terminal voltage. Neglecting the transients, it can be expressed as:

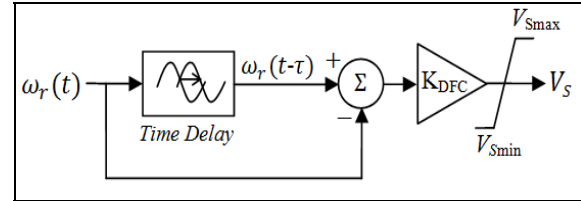
$$V_t = \sqrt{(X_q i_q - r_a i_d)^2 + (X_{md} i_{fd} - R_a i_q - X_d i_d)^2} \quad (11)$$

Using (1-11), one can obtain the set of ODEs of the nonlinear model in the state space representation form. There are 9 state variables:  $i_d, i_q, i_{fd}, i_Q, i_D, w_r, \delta_r, V_C$  and  $E_{fd}$ . The numerical values of the parameters used in the model are given in the appendix.

MATLAB has been used to solve the set of ODEs representing the dynamic system under study. The Simulink model including the embedded M-file codes has been developed to evaluate the performance of the proposed DFC.

### 3 DELAYED FEEDBACK CONTROLLER

The delayed feedback controller consists of time delay, subtraction and gain operators. Fig. 2 shows the block diagram of the DFC. The synchronous generator rotor angular speed ( $w_r$ ), an accessible state variable, is the sole input signal to the DFC block. The output signal ( $V_S$ ) is obtained by subtracting the  $\tau$  time delayed value of  $w_r$  from its current value and then multiplying the result by a gain ( $K_{DFC}$ ).  $V_S$  is then combined into the AVR as the stabilizing signal to improve damping of local mode oscillation. The output limiter is used to prevent blocking the voltage regulation function of the AVR.



**Figure 2:** Block diagram of the Delayed Feedback Controller

It is important to note that determining stability regions of a dynamic system in the domain of time delays is a particularly complex task. In this study, the method of cluster treatment of characteristic roots (CTCR) has been employed to assess the stability depending on time delay.

#### 3.1 CTCR Method for Stability Analysis

Consider a general class of time delayed linear time invariant (LTI) system, which has a form:

$$\dot{x} = Ax + Bx(t - \tau) \quad x \in \mathfrak{R}^{nx1}; A, B \in \mathfrak{R}^{nxn} \quad (12)$$

The time delay in (12) injects exponential transcendentality to the characteristic equation, which is given by:

$$\text{Det}(sI - A - B e^{-s\tau}) = 0 \text{ with } \tau > 0 \quad (13)$$

We write the generic form of (13) as follows:

$$\text{CE}(s, \tau) = \sum_{k=0}^n a_k(s) e^{-k\tau} = 0 \quad (14)$$

The linear system in (12) is asymptotically stable if all the characteristic roots of (14) are on the left half of the complex plane. Since there are infinitely many characteristic roots to be examined depending on time delay, this is clearly a complex task.

For a simplification in (14), we deploy the Rekasius substitution [22], which is given by

$$e^{-k\tau} = \frac{1 - Ts}{1 + Ts} \quad \tau \in \mathfrak{R}^+, T \in \mathfrak{R} \quad (15)$$

and defined only for  $s = wi$ ,  $w \in \mathfrak{R}$ . This is an exact substitution with the mapping condition of

$$\tau = \frac{2}{w} [\tan^{-1}(wT) \pm \ell\pi] \text{ for } \ell = 0, 1, 2, \dots, \infty \quad (16)$$

The substitution of (15) into (14) and multiplication by  $(1+Ts)^n$  results in a rational polynomial:

$$\text{CE}(s, \tau) = \sum_{k=0}^n a_k(s) (1+Ts)^{n-k} (1-Ts)^k = 0 \quad (17)$$

Sorting the terms in (17) to obtain a polynomial of  $s$ :

$$\text{CE}(s, \tau) = \sum_{k=0}^{2n} b_k(T) s^k = 0 \quad (18)$$

The number of sign changes in the first column elements (NS) of the Routh table of (18) for a range of  $T$  is obtained. NS indicates the number of unstable roots. For each  $T$  value at which NS changes by 2, there exists one pair of purely imaginary roots ( $\pm iw_{ck}$ ) which are determined by solving (18) for  $T_{c1}$ ,  $T_{c2}$ , ...,  $T_{cm}$ . The case for  $T=0$  is ignored. For each  $T_{ck}$  (or the corresponding  $w_{ck}$ ), there are infinitely many time delays.

The invariant expression for the Root tendency (RT) which is used to cumulatively calculate the number of unstable roots is:

$$\text{RT} \Big|_{\substack{s=wi \\ \tau=T\ell}} = \text{sgn} \left[ \text{Re} \left( \frac{\sum_{j=0}^n a_j j s e^{-j\tau}}{\sum_{j=0}^n \left[ \frac{da_j}{ds} - ja_j \tau \right] e^{-j\tau}} \right) \Big|_{\substack{s=wi \\ \tau=T\ell}} \right] \quad (19)$$

$k=1, \dots, m \quad \ell=0, 1, \dots, \infty$

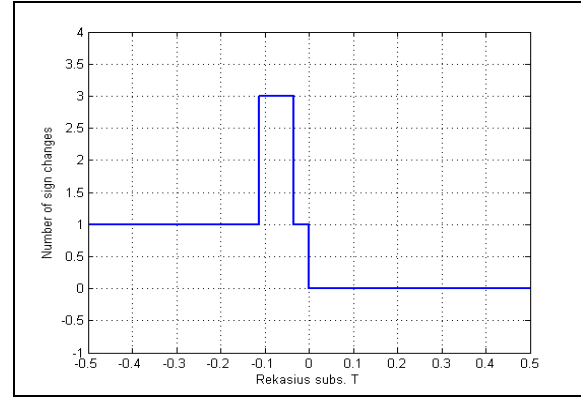
Finally, the expression to determine the number of unstable roots ( $NU$ ) as an explicit function of  $\tau$  is given by

$$NU(\tau) = NU(0) + \sum_{k=1}^m \Gamma \left( \frac{\tau - \tau_{k0}}{\Delta\tau} \right) U(\tau, \tau_{k0}) RT_k \quad (20)$$

where  $NU(0)$  is the number of unstable roots when  $\tau=0$ ,  $\Delta\tau$  is the periodicity (i.e.  $2\pi/w_{ck}$ ),  $\Gamma(x)$  is the ceiling function returning the smallest integer greater than or equal to  $x$  and  $U(\tau, \tau_{k0})$  is the step function defined in  $\tau$  with step taking place at  $\tau_{k0}$ .

### 3.2 Stability Analysis of the Model with DFC

By employing the CTCR method, the stability regions depending on the time delay value have been evaluated for the operating condition:  $P_e=0.798$ ,  $Q_e=0.1297$  and  $V_f=0.9909$  p.u. Fig. 3 shows the number of sign changes with  $T$ .



**Figure 3:** The number of sign changes (NS)

In essence, the system under study can have purely imaginary roots only at two frequencies: 8.708 and 10.466 rad/s. Using (19), root tendencies have been calculated for these frequencies as -1 and 1 respectively. Table-1 gives the stability regions in the parametric domain of time delays for  $K_{DFC}=20$ . It reveals from Table-1 that setting the time delay to a value less than 0.4352 s will not result in instability in the dynamic system. Similarly, a time delay value between 0.6525 s and 1.0356s ensures stable operation.

$\tau$ (sec)	RT	Stable/Unstable	$NU(\tau)$	$\nu$ (rad/s)	$T$
0		Stable	0		
0.4352	1	Unstable	2	10.466	-0.1119
0.6525	-1	Stable	0	8.7078	-0.0356
1.0356	1	Unstable	2	10.466	-0.1119
1.3740	-1	Stable	0	8.7078	-0.0356
⋮	⋮	⋮	⋮	⋮	⋮

**Table 1:** Stability regions (shaded) for  $K_{DFC}=20$

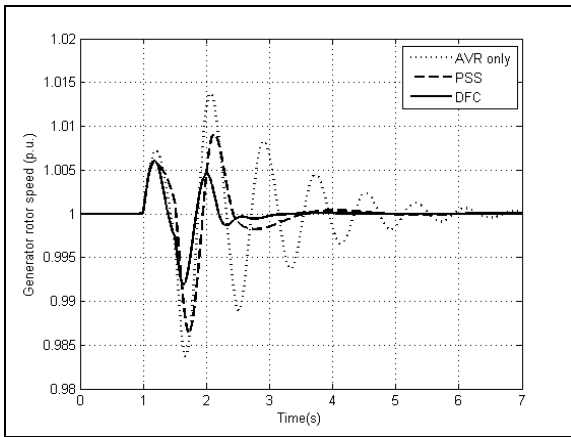
#### 4 PERFORMANCE EVALUATION OF DFC

In this section, we present evaluation of damping performance of the DFC by comparison to a generic PSS [2], which has the following transfer function:

$$p(s) = \frac{40s}{1+2s} \left( \frac{1+0.05s}{1+0.02s} \right) \left( \frac{1+3s}{1+5.4s} \right) \quad (21)$$

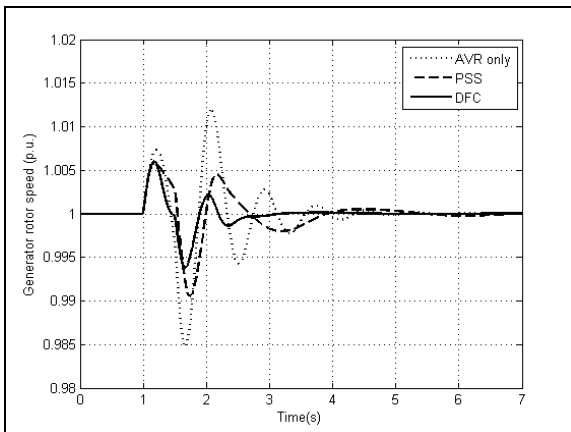
The PSS output limiter settings are  $\pm 0.15$ . Time domain simulations in Matlab-Simulink have been performed by applying a disturbance in the form of sudden torque change on the generator rotor.

It is evident from Fig. 4 that both PSS and DFC provide additional damping for the local mode oscillations. With  $\tau=0.20$  s and  $K_{DFC}=20$ , damping contribution by DFC is clearly more significant than the damping improvement achieved by generic PSS.



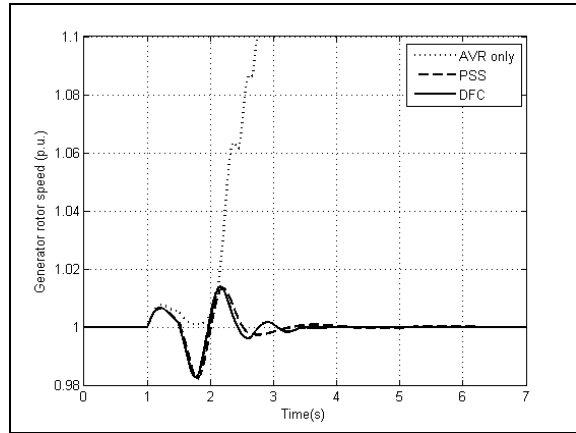
**Figure 4:** Generator rotor speed ( $P_e=0.8$ ,  $Q_e=0.13$ ,  $V_t=0.99$  p.u.)

Effectiveness of DFC has been evaluated for different operating points without changing the controller parameters. Fig.5 shows the generator rotor speed response for  $P_e=0.5989$  and  $Q_e=0.0665$  p.u. The damping achieved by DFC in this case is significantly better than the PSS.



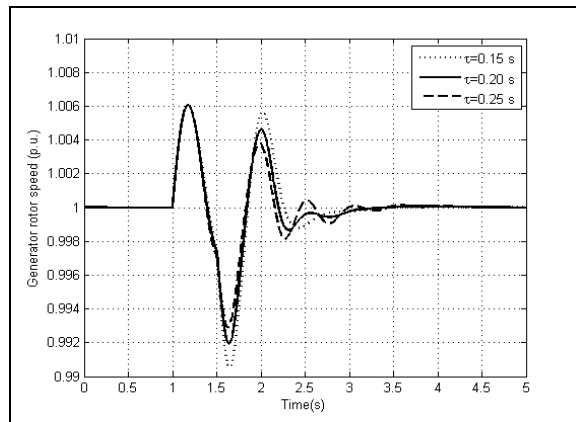
**Figure 5:** Generator rotor speed ( $P_e=0.6$ ,  $Q_e=0.067$ ,  $V_t=0.993$  p.u.)

Fig. 6 depicts a case with negative reactive power, at which stability is lost at AVR only case. On the other hand, DFC and PSS prevent loss of stability.



**Figure 6:** Generator rotor speed ( $P_e=0.78$ ,  $Q_e=-0.048$ ,  $V_t=0.89$  p.u.)

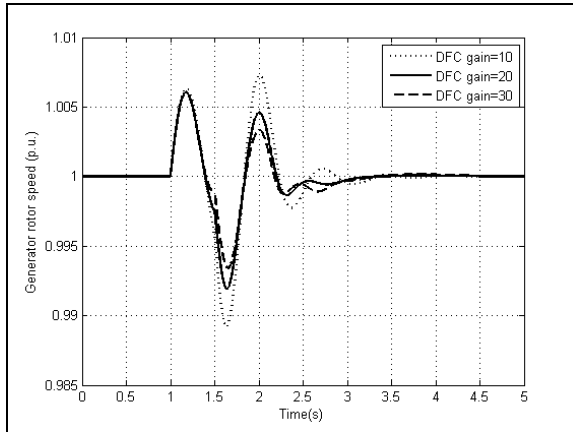
Moreover, the effectiveness of the DFC has been investigated for different time delay and gain parameters. Fig. 7 and Fig. 8 show the response of generator rotor speed for various time delay values and DFC gains, respectively. It is interesting to observe that there exists a consistency between the controller parameters and the damping performance.



**Figure 7:** Generator rotor speed with DFC ( $P_e=0.6$ ,  $Q_e=0.067$  p.u.)

It is important to note that the DFC has only two parameters that require tuning. A practical approach is to determine the first stability pocket by CTCR method for a preset  $K_{DFC}$  and selecting a time delay of about half the stability limit. Alternatively, dynamic responses can be evaluated for different values of DFC parameters.

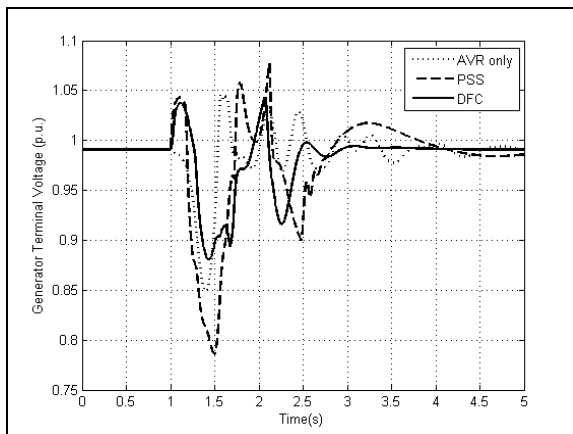
On the other hand, PSS tuning is mostly a complex task due to the control blocks included. A generic PSS consisting of one wash-out block and lead-lag compensators have six parameters to set and the correct tuning of them is required for the optimum performance. The task is particularly complex with the dual-input and multi-band PSSs.



**Figure 8:** Generator rotor speed with DFC ( $P_e=0.6$ ,  $Q_e=0.067$  p.u.)

In addition to providing better damping performance than the generic PSS, DFC does not limit the voltage regulating function of AVR as it is mostly the case with PSS. In essence, the tradeoff between transient stability provided by a high-gain fast response AVR and oscillation stability provided by a well-tuned PSS [23] can be eliminated by DFC.

Fig. 9 shows the generator terminal voltage for the cases AVR only, PSS and DFC. The voltage drop following the disturbance is the smallest with DFC and the voltage fluctuations disappear much faster. It is evident from Fig. 9 that DFC also contributes to the terminal voltage regulation.



**Figure 9:** Generator rotor speed with DFC ( $P_e=0.6$ ,  $Q_e=0.067$  p.u.)

## 5 CONCLUSION

In this paper, a delayed feedback controller is proposed to provide additional damping for local mode oscillations as an alternative to PSS. The merits of the DFC involve the significant improvement in damping, less tuning requirement with only two parameters and more effective regulation of the generator terminal voltage following a disturbance. Time domain simulations demonstrate the effectiveness of the proposed

controller for which the stability intervals depending on the controller time delay have been determined by CTCR method. It is found that the controller parameters set as optimum values remain effective at various operating points.

Future work should focus on performance evaluation of the DFC in a multi-machine power system. In addition, damping of inter-area oscillations by DFC should be investigated.

## APPENDIX

### A. Per-Unit Base Values

$$P_b = 555 \text{ MVA}, V_b = 24 \text{ kV}, \omega_b = 376.99 \text{ rad/s}$$

### B. Synchronous Generator

$$X_d = 1.81 \text{ p.u.}, X_q = 1.76 \text{ p.u.}, X_l = 0.15 \text{ p.u.}$$

$$R_a = 0.003 \text{ p.u.}, X'_d = 0.3 \text{ p.u.}, X'_q = 0.65 \text{ p.u.}$$

$$X''_d = 0.25 \text{ p.u.}, X''_q = 0.25 \text{ p.u.}$$

$$T'_{d0} = 8.0 \text{ s}, T'_{q0} = 1.0 \text{ s},$$

$$T''_{d0} = 0.03 \text{ s}, T''_{q0} = 0.07 \text{ s}$$

$$H = 3.5, D = 0$$

### C. Electrical network

$$X_T = 0.15 \text{ p.u.}, X_L = 0.30 \text{ p.u.}$$

### D. Excitation system (IEEE ST1A)

$$K_A = 210, T_A = 0.01 \text{ s}, T_R = 0.15 \text{ s}$$

$$E_{Fmax} = 7.0 \text{ p.u.}, E_{Fmin} = -7.0 \text{ p.u.}$$

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