BELLMAN'S DYNAMIC PROGRAMMING METHOD APPLIED TO THE SYNTHESIS OF CONTROL ALGORITHMS FOR THE EXCITATION OF SYNCHRONOUS GENERATORS

N. Zelenokhat O. Zelenokhat S. Lianzberg

Department of electrical power systems

Moscow Power Engineering Institute (Technical University) – Russia

Fax: (+ 7)495-6730340 Phone (+7) 495-4653761 e-mail: zelenokhatni@mpei.ru

Abstract – Recently there's been much attention paid to synchronous generators' excitation algorithms improvement based on the application of microprocessor automatic voltage regulators and thyristor excitation system.

This paper offers a new approach (based on Bellman's derivative equation) to synthesize control algorithms of synchronous generator excitation. The paper provides the functional and Bellman's function as analytical dependence between controlling actions and operating condition parameters in multi-machine electrical power system (EPS). This dependence generally turns to be a non-linear.

Simple linear control algorithms used and applied in automatic voltage regulators (AVR) were developed basing on general solutions. The solution (based on the Bellman's derivative equation) to the problem of synthesis of control algorithms of synchronous generators' excitation under high disturbances in EPS became totally new. The application of synthesized algorithms in microprocessor-based AVR makes the damping of electromechanical transients in EPS very effective.

Keywords: synthesis of control algorithm, Bellman's dynamic programming method, transients in electrical power system.

1. INTRODUCTION

The problem of the creation of efficient synchronous generator AVR, providing the stability and the quality of electromechanic transients damping in EPS required, is connected with determination of AVR structure i.e. control algorithms, and AVR parameters.

The solution of this problem becomes complicated by the behavior of each synchronous generator in complex EPS which is defined by both its own characteristics and characteristics of all other synchronous generators in EPS. It depends on the position of the rotor of each generator i.e. depends on shift angle between rotors of all synchronous machines. However until recently it was rather hard to define technically the mutual position of generator rotors in transients to use it in AVR.

So, in practice simple excitation regulators are in general use still (proportional automatic voltage regulator – AVR). It has simple control algorithm based on the deviation of the generator stator voltage U_g (changing in time) from its setting value U_g^0.

At the same time more complex excitation regulators (power system stabilizer - PSS) and high-speed thyristor excitation systems are applying successfully to provide the stability of synchronous generators and the highest damping properties of transients. The regulator of PSS consists of automatic voltage regulator and additional device - system stabilizer (SS). It provides additional signal to output signal of the voltage regulator for improving damping of EPS.

Automatic voltage regulator is the main part of forced-excitation AVR applied in Russia. There is additional part (system stabilizer) with input signals depend on generator voltage derivative $\mathrm{d}U_g/\mathrm{d}t$ and on deviation of the voltage frequency at stator contacts Δf_{U_g} and its derivative df_{U_g}/dt, as well as on rotor current derivative dI_f/dt.

However PSS has well-known defects, that is why the search of new approaches to control algorithm synthesis of synchronous generators excitation in complex EPS continues. Adaptive AVR, AVR with artificial intelligence elements and the other one are designed [1, 2]. In practice it is very difficult to apply these AVRs because there have to be a complex measuring-converting system and a system that makes up control signal for PSS.

This paper describes one of insufficiently known approaches based on Bellman's method

(method of dynamic programming) [3]. This method allows synthesizing of control algorithm as feedback of mode parameters which define dynamic conditions of EPS.

2. MATHEMATICAL MODEL AIMED SYNTHESYSING THE CONTROL ALGORITHMS

In complex multi-machine EPS containing n synchronous generators the system of the differential equations describes the rotors' motion disregarding damping moments is:

$$M_i d\omega_i /dt = P_{mi} - P_{ei} , \quad (1)$$

(i = 1,..., n),

where $\omega_i = d\delta_i / dt$ – angular velocity of the generator rotor motion (rotation) relatively to reference axis, revolving with constant synchronous velocity ω_0; M_i – mechanical inertia constant (together with turbine); P_{mi} – turbine power; P_{ei} – electromagnetic power, defined by the formula:

$$P_{ei} = E^2_{qi} y_{ii} \sin\alpha_{ii} + \sum_{i\neq j}^{n} E_{qi} E_{qj} y_{ij} \sin(\delta_{ij} - \alpha_{ij}) \quad (2)$$

where $\delta_{ij} = \delta_i - \delta_j$ – the rotor shift angle of the generator i relatively to the rotor of the generator j; y_{ii} – absolute value of own conductivity; y_{ij} – absolute value of mutual conductivity; all other indications correspond to agreed notations [4]. Further, $E_i = E_{qi}$, $E_j = E_{qj}$ is taken.

Synchronous generator electromotive force (EMF) E_{qi} is defined by the solution of the differential equation:

$$E_{qi} = E_{qei} - T_{d0i} \mathrm{d}E'_{qi} / \mathrm{d}t , \quad (3)$$

where E_{qei} – induced (forced) EMF of the generator; E'_{qi} – quadrature-axis transient EMF E'; T_{d0i} – time constant of generator excitation windings.

Let's use the theorem about system inertia centre moving to get the equation characterizing motion of the EPS as a whole with the speed ω_{equ} relatively to synchronous revolving axis [4, 5]:

$$M_{equ} \mathrm{d}\omega_{equ} /\mathrm{d}t = P_{t.equ} - P_{e.equ} , \quad (4)$$

where $P_{t.equ} = \sum_i^n P_{ti}$; $P_{e.equ} = \sum_i^n P_{ei}$;

$\omega_{equ} = \sum_i^n T_i \omega_i$; $T_i = M_i / M_{equ}$; $M_{equ} = \sum_i^n M_i$.

The solution of the equation (4) is a trajectory of the EPS motion as a whole $\delta_{equ}(t)$ i.e. a trajectory of the "equivalent generator" rotor motion with inertia constant M_{equ} and torques applied (the turbine power $P_{t.equ}$ and generator power $P_{e.equ}$).

Got on the base of (1) and (4) differential equation of the generator rotor motion relatively to "equivalent generator" rotor takes form:

$$M_i \mathrm{d}\omega_{i.equ} /\mathrm{d}t = P_{t.i} - P_{e.i} - (P_{t.equ} - P_{e.equ})T_i , \quad (5)$$

where $\omega_{i.equ}$ – a velocity of relative motion of the generator rotor:

$$\omega_{i.equ} = \omega_i - \omega_{equ} . \quad (6)$$

Kinetic energy of a motion or mutual swings of all generators' rotors in the EPS relatively to "equivalent generator" rotor is defined by the expression:

$$W' = \frac{1}{2} \sum_i^n M_i \omega^2_{i.equ} . \quad (7)$$

It's reasonable to present induced (forced) EMF E_{qei}, which is proportional to the voltage of rotor excitation winding u_{fi}, as the sum of two components:

$$E_{qei} = \underset{*}{E_{qei}} + u_i , \quad (8)$$

where $\underset{*}{E_{qei}}$ is not depend on u_i and can be a constant ($\underset{*}{E_{qei}} = \underset{*}{E^0_{qei}}$); u_i – controlling action.

In automatic control theory a lagging and an inertia are usually neglected while control algorithm synthesis. As for AVR it is possible to neglect exciter time constant, amplifier time constant, regulator time constant and time constants of other elements. This approach makes (8) exactly true. Moreover, the last part of (3) has to be compensated by the induction of rotor current negative feedback [5, 6].

Summing the above while control algorithm synthesis it's possible to write:

$$E_{qi} = \underset{*}{E_{qi}} + u_i , \quad (9)$$

where $\underset{*}{E_{qi}} = \underset{*}{E_{qei}}$.

In this case (2) puts on:

$$P_{ei} = (\underset{*}{E_{qi}} + u_i)^2 y_{ii} \sin\alpha_{ii} + \sum_{i\neq j}^{n} (\underset{*}{E_{qi}} + u_i)(\underset{*}{E_{qj}} + u_j) y_{ij} \sin(\delta_{ij} - \alpha_{ij}) \quad (10)$$

Thereby mathematical model can be presented in the form suitable for control algorithm synthesis of synchronous generator excitation in complex EPS.

3. CONTROL ALGORITHM SYNTHESIS BASED ON BELLMAN'S DIFFERENTIAL EQUATION

To apply the Bellman's equation it's necessary to define the functional structure I and Bellman's functions.

Integral criterion characterizing transient damping in complex EPS is reasonable to take as the functional I [5]:

$$I = I_{\omega} = \int_{0}^{T} \sum_{i=1}^{n} M_i \omega_{i.equ}^2 \mathrm{d}t , \tag{11}$$

where doubled kinetic energy of the rotor mutual oscillations is under the integral (7).

The integral criterion (11) reflects oscillations of EPS generators' rotors relatively to their trajectory of the motion as a whole.

Investigations carried out show that the kinetic energy of generators' rotors motion of the EPS relatively to its motion as a whole is reasonable to take as Bellman's function. Therefore, Bellman's function is written as the following:

$$S(\dot{x}) = \frac{1}{2} \sum_{i=1}^{n} k_{i.equ} \omega_{i.equ}^2 , \tag{12}$$

where $k_{i.equ}$ – a coefficient; the vector of coordinates: $x_i = \delta_{i.equ}$, $\dot{x}_i = \omega_{i.equ} = \dfrac{d\delta_{i.equ}}{dt}$,

$\delta_{i.equ} = \delta_i - \delta_{equ}$, $\delta_{equ} = \sum_{i=1}^{n} T_i \delta_i$.

According to (12) Bellman's function does not depend on system coordinates and the time t obviously.

In this case Bellman's equation for control algorithm syntheses in complex EPS is [7]:

$$\min_{u} \left[F(\dot{x}) + \sum_{i=1}^{n} \frac{\partial S(\dot{x})}{\partial \dot{x}_i} f_i(x,u) \right] = 0 , \tag{13}$$

where $F(x)$ – the function under integral I (11); $f(x,u)$ – right part of equation (5) written to the Cauchy form and of the equation $\dfrac{d\delta_{i.equ}}{dt} = f(\dot{x}_i)$; u – controlling action vector.

This equation can be presented as (n+1) equations [5]:

$$F(\dot{x}) + \sum_{i=1}^{n} \frac{\partial S(\dot{x})}{\partial \dot{x}_i} f_i(x,u) = 0 . \tag{14}$$

$$\frac{\partial F(\dot{x})}{\partial u_i} + \sum_{i=1}^{n} \frac{\partial S(\dot{x})}{\partial \dot{x}_i} \cdot \frac{\partial f_i(x,u)}{\partial u_i} = 0 , \tag{15}$$

Carried out investigations show that the using of (14) is enough for control algorithm synthesis. In this case the choice of the coefficient value $k_{i.equ}$ defines not strictly optimal but sub-optimal control.

All optimal values of controlling action u_i, which are a complex functions of all EPS phase coordinates, are defined by the solution of the equations system (15). The realization of these u_i in AVR is connected with difficulties while using modern measuring equipment and information transfer equipment of EPS. At the same time technically it isn't reasonable to install complex PSS on all EPS generators. Therefore it's reasonable to pass from (14) to simplified control algorithms.

In concentrated part of EPS the action of electromagnetic damping moments of synchronous generators are strong enough to damp oscillations of rotors effectively (in most cases) with proportional AVR only. At the same time remote generators rotor oscillations can not be damped out well or it may cause generators to lose the synchronism. First of all PSS is usually used in generators on remote power stations. In this case it's reasonable to take the problem closer to technical realization with real PSS algorithms.

We consider the case when PSS is installed on remote power station i. The equation (14) with taking into account (5),(10),(12) and indications mentioned above turns into:

$$Au_i^2 + Bu_i + C = 0$$

The solution of the equation relatively to u_i is:

$$u_i = (B \pm \sqrt{B^2 - 4AC}\,) / 2A , \tag{16}$$

where

$$A = \sum_{i}^{n} T_i y_{ll} \sin\alpha_{ll} \omega_{i.equ} - y_{ll} \sin\alpha_{ll} \omega_{l.equ}$$

$$B = \frac{1}{2}\{[2E_{l*} y_{ll} \sin\alpha_{ll} + \sum_{j \neq l}^{n} E_{j*} y_{lj} \sin(\delta_{lj} - \alpha_{lj})]\omega_{l.equ} +$$

$$+ \sum_{i \neq l}^{n} E_{i*} y_{il} \sin(\delta_{il} - \alpha_{il})\omega_{i.equ} -$$

$$- \sum_{i}^{n} T_i \omega_{i.equ} [\sum_{\xi \neq 1}^{n} E_{\xi *} y_{\xi l} \sin(\delta_{\xi l} - \alpha_{\xi l})] -$$

$$- \sum_{i}^{n} T_i \omega_{i.equ} [2 \cdot E_l y_{ll} \sin\alpha_{ll} + \sum_{j \neq l}^{n} E_{j*} y_{lj} \sin(\delta_{lj} - \alpha_{lj})]\}$$

$$C = \sum_i^n k_{\omega_{i.equ}} \omega_{i.equ}^2 + \sum_i^n \Delta P_{sp.i} \omega_{i.equ} -$$
$$-\sum_i^n T_i \Delta P_{sp.equ} \omega_{i.equ}$$

$$\Delta \underset{*}{P}_{sp.equ} = \sum_{i \neq l}^n \Delta \underset{*}{P}_{sp.i} ,$$

$$\Delta \underset{*}{P}_{sp.i} = P_{ti} - [\underset{*}{E}_i^2 \, y_{ii} \sin \alpha_{ii} +$$

$$+ \sum_{j \neq i}^n \underset{*}{E}_i \underset{*}{E}_j \, y_{ij} \sin \; \delta_{ij} - \alpha_{ij} \;] -$$ surplus power.

However in this case it is difficult to bring into action control algorithms in AVR installed on power stations because the transfer of information about EPS conditions and circuit changes is required. So there is a need in a simplification of the control algorithm structure of AVR.

In receiving part of concentrated EPS with strict internal connections it's possible not to take into account mutual rotors' motion of all generators, supposing velocity $\omega_{i.equ}$ of all generators i equal to 0 except of generator l in (16). Having not considering all $\Delta \underset{*}{P}_{sp.i}$ the equation (16) was simplified to:

$$u_i = (b \pm \sqrt{b^2 - 4ac} \;) \, / \, 2a \qquad (16')$$

Where

$$a = y_{ii} \sin \alpha_{ii} \, ;$$

$$b = \frac{1}{2} [2 a_i E_{qi} + \sum_{j \neq i}^n E_{qj} y_{ij} \sin(\delta_{ij} - \alpha_{ij})] \, ;$$

$$c = k_{\omega_{i.equ}} \omega_{i.equ} \, .$$

If local load of remote power station i is negligibly small and active resistance isn't taken into account while determine y, we got (16') as:

$$u_i = \frac{k_{\omega_{i.equ}} \omega_{i.equ}}{E_{equ} \, y_{i.equ} \sin \delta_{i.equ}} \, , \qquad (17)$$

And as more simple form:

$$u_i = k_{\omega_{i.equ}} \omega_{i.equ} \, . \qquad (18)$$

The controlling action u_i is transmitted to the PSS of excitation system of synchronous generator instead of angular velocity of generator rotor motion ω_i.

4. PERFORMANCE ANALYSIS OF SYNTHESIZED ALGORITHMS

To illustrate the efficiency of developed algorithms we carry out the research of three-machine EPS (fig.1) using control algorithm (16'). Results of the simulation of three-machine EPS affected by three-phase fault are presented below. The excitation of all *n* generators (*n*= 3) is controlled by means of the next algorithm:

$$E_{qe_i} = E_{qe_i}^0 + \Delta E_{qe_i} \, , \qquad (19)$$

where $\Delta E_{qe_i} = k_{U_i} (U_{g_i} - U_{g_i}^0) + u_i$; $E_{qe_i}^0$ – setting value of E_{qe_i}; U_{g_i} and $U_{g_i}^0$ –generator stator voltage and its setting value; u_i – controlling action formed in the accordance with (16'), (17) or (18).

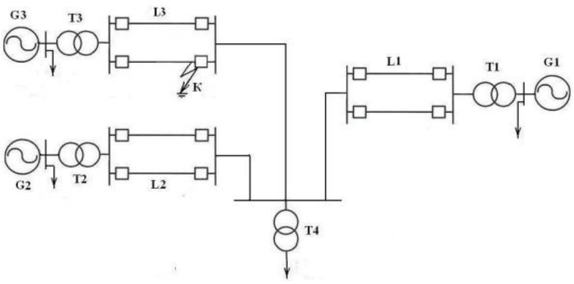

Fig. 1. Three-machine EPS under consideration.

Since a use of microprocessor AVR is provided, exciter time constant and regulator time constant are not taken into account when writing (19). We don't take into account field forcing of generators, but take into consideration the action of automatic speed regulator as:

$$P_{ti} = P_{ti}^0 + k_{ti} \omega_i$$

where P_{ti}^0 – setting value of power P_{ti}; k_{ti} – a coefficient; ω_i – a velocity of generator rotor motion relatively to synchronously revolving axis with velocity ω_0.

To execute calculations we accept quasioptimal values of coefficient $k_{\omega_{i.equ}} = 1{,}5$ for all three generators and control algorithm (16').

Figure 2 presents characteristics of mutual shift angles δ_{12} (small line), δ_{13} (medium line) and δ_{23} (solid line) in dependence of time *t*, and on Fig. 3 – a change of velocities $\omega_{i.equ}$. The analysis of these characteristics shows that EPS transients damping is intensive enough. Small, medium and solid lines characterize G1, G2 and G3 respectively.

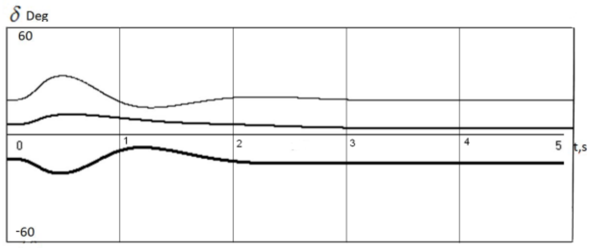

Fig. 2. Mutual rotors angles $\delta_{ij}(t)$ with $k_{\omega_{i.equ}}$ =1,5.

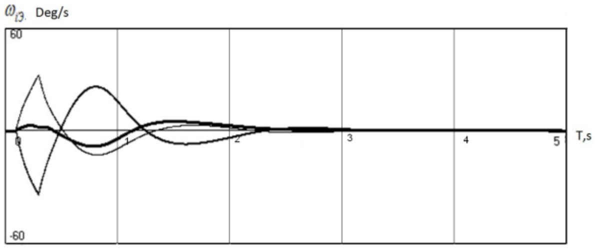

Fig. 3. Characteristics $\omega_{i.equ}(t)$ with $k_{\omega_{i.equ}}$ =1,5.

On Fig. 4 the characteristics $\Delta E_{qe_i}(t)$ are presented which is related to the characteristics on Fig. 2 and Fig. 3. Small, medium and solid lines characterize G1, G2 and G3 respectively.

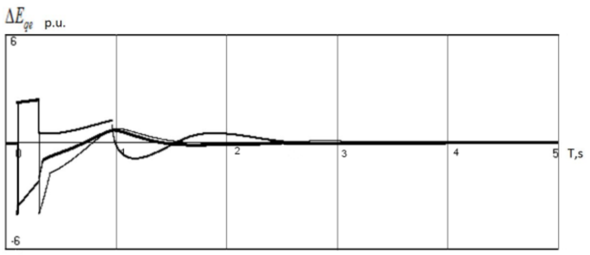

Fig. 4. Characteristics $\Delta E_{qe_i}(t)$ with $k_{\omega_{i.equ}}$ =1,5.

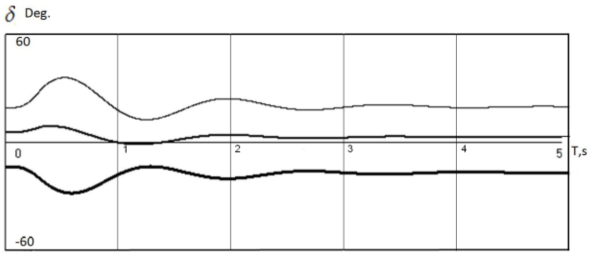

Fig. 5.Mutual rotors angles $\delta_{ij}\ t$ with $k_{\omega_{i.equ}}$ = 400.

Fig. 5 – Fig. 7 illustrate the characteristics of the transients for the same conditions as shown on fig. 2 – fig. 4, but $u_i = k_{\omega_{i.equ}} \omega_{i.equ}$ (18). Coefficient values are accepted as quasioptimal and equal for all generators $k_{\omega_{i.equ}}$ = 400.

The analysis of the characteristics on Fig. 5 - Fig. 7 shows that the use of control algorithm (19) with respect to (18) provides transients damping, but not so intensive.

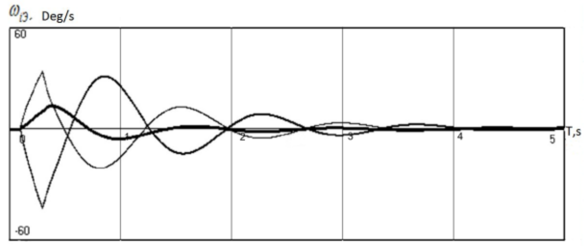

Fig. 6. Characteristics $\omega_{i.equ}(t)$ with $k_{\omega_{i.equ}}$ =400.

The analysis of characteristics $\Delta E_{qe_i}(t)$ on Fig. 4 and on Fig. 8 shows that controlling actions u_i while using (18) are changed with greater velocity than using (16'). Control algorithm (18) provides limited values $u_i^{\min}$ and $u_i^{\max}$ for a long time. The efficiency of control transients is better when using (16'), than (18).

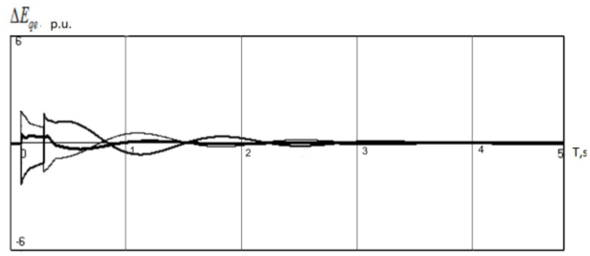

Fig. 7. Characteristics $\Delta E_{qe_i}(t)$ with $k_{\omega_{i.equ}}$ =400.

Fig. 8 – Fig. 10 illustrate the characteristics of the transients for the same conditions as shown on Fig. 2-4 and Fig. 5-7, but no synthesized algorithms are used i.e. $u_i = 0$. The analysis of the characteristics on Fig. 8-10 shows that not using synthesized control algorithms provides poor transients damping.

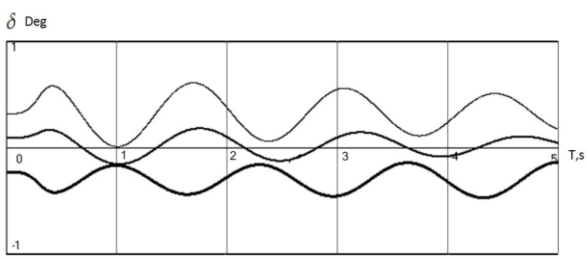

Fig. 8.Mutual rotors angles $\delta_{ij}\ t$ with $u_i = 0$.

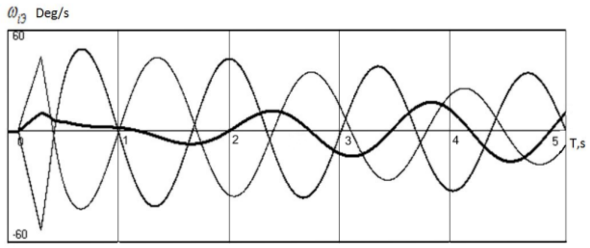

Fig. 9. Characteristics $\omega_{i.equ}(t)$ with $u_i = 0$.

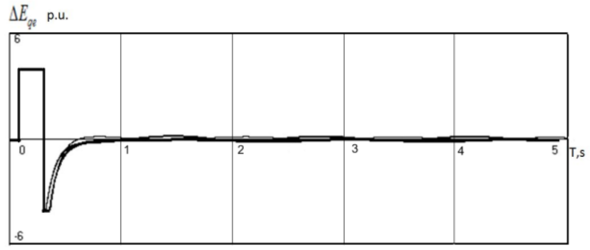

Fig. 10. Characteristics $\Delta E_{qe_i}(t)$ with $u_i = 0$.

Executed calculations confirm the efficiency of synthesized control algorithms using Bellman differential equations.

5. CONCLUSION

The mathematical model of the complex EPS suitable for control algorithm synthesis of generator excitation is designed. This model is based on the presentation of EPS as a whole and on mutual motion of generator rotors. The new approach for the solving the problem of control algorithm synthesis is designed with the use of this model, analytical mechanics statements and the use of Bellman's differential equation. Control algorithms are obtained applying designed approach.

Executed calculations prove the efficiency of designed control algorithms in case of three-phase fault in complex EPS without infinity buses if near optimal value of AVR setup variables are chosen.

It is necessary to use microprocessor AVR and thyristor excitation system for the realization of designed control algorithms.

REFERENCE

[1] Ravi Segal, Avdhesh Sharma and M. L. Kothari. A self-tuning power system stabilizer based on artificial neural network // International Journal of Electrical Power & Energy Systems Volume 26, Issue 6, July 2004.

[2] L. Cong, Y. Wangand D.J. Hill Transient stability and voltage regulation enhancement via coordinated control of generator excitation and SVC// International Journal of Electrical Power & Energy Systems Volume 27, Issue 2, February 2005.

[3] Bellman R. On the application of the theory of dynamic programming to the study of control processes. Proc. of the Symposium of nonlinear circuit analysis. Polytechn. Inst. Of Brooklin. 1956.

[4] V.A.Venikov. Transient Processes in Electrical Power Systems. Mir Publishers, Moscow, 1977.

[5] Zelenokhat N.I. The control system synthesis of transients in complex power system. Elektrichestvo, 1981, No. 9.

[6] V.A. Venikov, N.I. Zelenokhat, Y.G. Shakaryan and L.A. Sukhanov. Application of digital computers to the study of power systems with asynchronized electromechanical frequency converters. Electrical Power & Energy Systems. Vol. 1. No. 2. July 1979.

[7] Guide of automatic control theory. A. A. Krasovsky. – Moscow, "Nauka". 1987.

APPENDIX

Electrical scheme used in the simulation and its parameters:

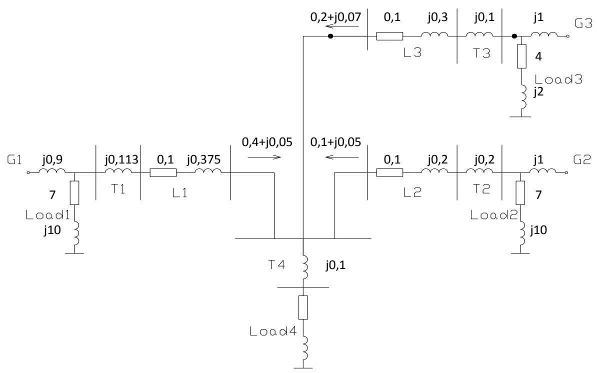

Table 1. Simulation parameters

Node	1	2	3	4
x_d , p.u.	0.9	1	1	
x_T , p.u.	0.113	0.2	0.1	0.1
x_{line} ,p.u.	0.375	0.2	0.3	
r_{line} , p.u.	0.1	0.1	0.1	
r_{load} ,p.u.	7	7	4	
x_{load} ,p.u.	10	10	2	
P, p.u.	0.4	0.1	0.2	
Q, p.u.	0.05	0.05	0.07	
x'_d , p.u.	0.25	0.3	0.3	
T_d , p.u.	3	3	3	
T_J , p.u.	10	9	9	