

Discriminatory Pricing among Consumer Classes in Power Market using the Shapley Value

Fan Zhang, Pramode K. Verma and Samuel Cheng
School of Electrical and Computer Engineering
The University of Oklahoma-Tulsa
Tulsa, OK 74135
Email: {fanzhang, pverma, samuel.cheng}@ou.edu

Abstract - A classification of consumers in the power market into different classes from a pricing perspective has been a frequent source of controversy between the energy producer and consumer interest groups since electricity is considered to be a commodity product. Although differences of demand elasticity and variation in costs of service among the different classes justify discriminatory pricing, since it is difficult to allocate transmission cost among different consumer classes, utility companies and utility commissions cannot objectively determine how much the rate differential should be. This paper presents the rules for allocating supply to each class of consumers based on an efficient allocation of resource. Classes of consumers being allocated less resource receive monetary compensation from other classes. The price differential is determined by the inter-class compensations which are based on the outcome of a cooperative game among the user classes and the determination of the Shapley Value of each class.

Keywords - Discriminatory Pricing; Power Market; Shapley Value

1 Introduction

In the power market, the supplier creates the first degree price discrimination by product discrimination. Since electricity is a highly homogeneous product, a producer can only differentiate by offering different degrees of in-corrupibility. The second degree price discrimination can be effected by the producer charging a higher price initially and decreasing price for increasing amounts. In the third degree price discrimination, the one we will investigate in this paper, the supplier charges different consumers different amounts for the same product and manages to do so by providing different tariffs for different consumer classes; for example, residence consumers, business consumers and industrial consumers [1]. As shown in Fig. 1, producers charge the highest price for residence consumer and lowest price for industrial consumers.

Differences in demand elasticity among consumer groups can be viewed as an important reason to study the impact of discriminatory pricing. The demand of the industrial consumers for electricity is more elastic than the aggregate demand of small residential consumers. Particularly for large industrial consumers, electricity should be offered at a price comparable to the price of alternative sources of power. In addition, these consumers also, generally, have the option of constructing their own electric-generating system. Therefore, industrial consumers have a

relatively elastic demand for electricity. Producers would be, generally, interested in restricting sales to the consumers with less elasticity demand and increasing sales to consumers with greater elasticity of demand.

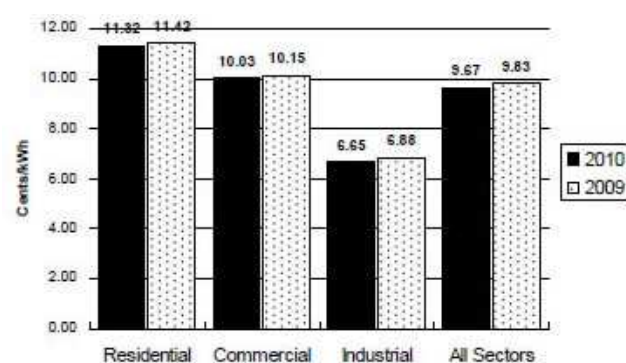


Figure 1: Average Retail Price of Electricity to Ultimate consumers by End-Use Sector, Year-to-Date through June 2010 and 2009, Source: Energy Information Administration [2]

Furthermore, there are also other reasons for price differential among classes of consumers. These are related to the difference in cost of services. For producers, the problem lies in how to distribute fixed costs among different classes of consumers. There have been several proposed strategies for cost sharing of a transmission system, such as the postage stamp method, MW-miles method, and short-run marginal cost (SRMC) [3]. The postage-stamp method is the simplest method of charging for transmission services, irrespective of supply and delivery points. However, research has shown some fatal flaws in this method. One of the most important drawbacks is that the costs are divided evenly among all users independent of the actual cost they cause the system. In other words, light-users of the system would, in fact, be subsidising the others that use the system heavily [4]. The MW-miles method is an attempt to compensate for this shortcoming by charging different transactions in proportion to their usage of the grid. But this method continues to suffer from a failure to encourage economically optimal usage of transmission assets by distinguishing the marginal cost influence on the transmission network brought by each transaction. SRMC pricing has also been extensively studied in relation to transmission services. However, due to the

dominance of the fixed cost component in the provision of transmission services, the approach does not always recover the cost of the transmission system [5].

It costs more per unit of service to serve small and residential consumers compared to commercial and industrial consumers. Producers are also right in their assumptions that consumers with low-load factors should pay a higher price than consumers with high-load factors. Load factor is defined as the ratio of average to maximum consumption for a selected period of time. Although the differences in cost warrant rate differences, it does not determine how much the rate differentials should be.

In this paper, we investigate price differentials among classes of consumers in a game theoretic framework. Each class of consumers has a different willingness to pay for the electricity and the utility function of a class is dependent on this willingness. Producers allocate their supply or the cost of producing the supply to each class so as to maximize all consumers' utilities. Monetary compensations are set up for those classes which have been allocated less resource. And the price difference is set up based on these inter-class compensations. We model this supply allocation problem as a transfer utility game (utilities may be divided among the classes in any way) and then solve it by applying the Shapley Value which will be introduced in Section 3. We consider that the total utility of a coalition is the total utility its members (classes) would have if they had the privilege of being allocated more electricity. The utility share associated with each class corresponds to the Shapley Value of the game.

The Shapley Value has been studied in power system transmission cost allocation among market participants (power plants) [3, 6, 7]. The concept of monetary compensation among agents in a cooperative game was first proposed in [8]. Each agent in [8] has a different unit waiting cost and a queue is organized to minimize total waiting cost. Agents that have lower priority in the queue and therefore longer waiting time receive monetary compensations from agents having smaller waiting time. Reference [9] has interpreted the worth of a coalition of agents in a different manner for the same model as in [8], and derived a different rule. In [10, 11], the queuing problem is studied from a strategic point of view under the assumption that all agents have identical unit waiting cost. In our paper, we borrow the concept of monetary compensation from [8] and propose a fair and efficient way to derive the rate differentials among classes of consumers based on inter-class compensations.

The rest of paper is organized as follows. In Section 2, we present the model. The cooperative game and the Shapley Value are studied in Section 3. In Section 4, we investigate the pricing scheme. An illustrative example is given in Section 5, and Section 6 captures our conclusion.

2 The model

We consider the electricity market with n classes of consumers as shown in Fig. 2. As stated in Section 1, the electricity market normally has three classes of con-

sumers: residential consumers, commercial consumers and industrial consumers. We use n to denote number of classes of consumers in case producers further segment the market. The supply c is allocated among the n classes of consumers and we use c_i to denote the resource allocated to class i . From producers' perspective, we can also assume c as the cost of producing the supply and c_i as the cost allocated to class i consumers.

The set of classes are denoted as $N = \{1, \dots, n\}$. As said in Section 1, different classes of consumers have different elasticity and different willingness to pay. We use w_i to denote class i consumers' willingness to pay for the electricity. If a class of consumers is relatively inelastic to demand, it means that its members are insensitive to the prices and have, therefore, a large willingness to pay for the service. Given a resource allocation $\hat{c} = (c_1, c_2, \dots, c_n)$, the utility of each class i is defined as:

$$u_i(c_i) = w_i \log c_i \quad (1)$$

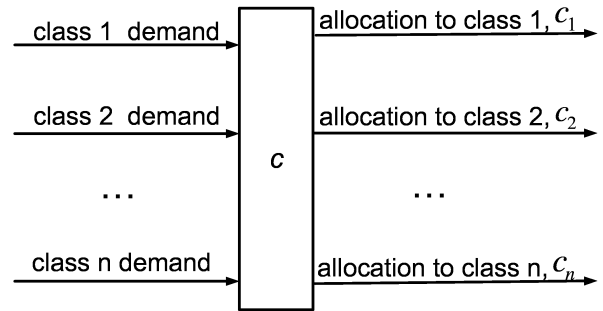


Figure 2: Resource allocation

The total utility of all classes of consumers due to an resource allocation $\hat{c} = (c_1, c_2, \dots, c_n)$ can be written as:

$$u(N) = \sum_{i=1}^n w_i \log c_i \quad (2)$$

We define an allocation as $\psi(\hat{c}, t)$, where $\hat{c} = (c_1, c_2, \dots, c_n)$ is the resource allocated to each class and $t = (t_1, t_2, \dots, t_n)$ is the monetary transfer related to each class. Given a resource allocation \hat{c} and a transfer t , the utility share for class i , φ_i is defined as,

$$\varphi_i = u_i(c_i) + t_i = w_i \log c_i + t_i \quad (3)$$

We define an allocation $\psi(\hat{c}, t)$ to be efficient whenever it maximizes the total utility of all classes (i.e. $u(N)$ is maximized) and no transfer is lost (i.e. $\sum_{i=1}^n t_i = 0$).

We also define that an efficient resource allocation \hat{c} is the one which maximizes the total utility of all classes $u(N)$.

Let's look at (2), the total utility maximization problem becomes:

$$\text{maximize}_{\hat{c}} u(N) = \sum_{i=1}^n w_i \log c_i \quad (4)$$

subject to the following constraint:

$$\sum_{i=1}^n c_i \leq c \quad (5)$$

It can be observed that the $u(N)$ is a strictly concave function over a closed and bounded set defined by (5). Therefore, a unique maximum always exists. We now use Lagrangian multipliers to append constraint to the objective function. Thus, we can rewrite this maximization problem as:

$$\text{maximize}_{\hat{c}} \sum_{i=1}^n w_i \log c_i - \gamma \left(\sum_{i=1}^n c_i - c \right) \quad (6)$$

The necessary and sufficient Karush-Kuhn-Tucker (KKT) Conditions [12] applicable to (6) are given by:

$$\frac{w_i}{c_i} - \gamma = 0 \quad (7)$$

and,

$$\gamma \left(\sum_{i=1}^n c_i - c \right) = 0 \quad (8)$$

And from (7) and (8), we find that $\sum_{i=1}^n c_i - c = 0$ and the efficient resource allocation $\hat{c} = (c_1, c_2, \dots, c_n)$ for each class c_i is proportional to its willingness to pay w_i ,

$$c_i = \frac{w_i c}{\sum_{i=1}^n w_i} \quad (9)$$

So far, we have calculated $u_i(c_i)$ in (3) based on efficient resource allocation \hat{c} . In the next section, we will calculate the utility share φ_i from another perspective: by treating it as a cooperative game and set up the utility share for each class using the Shapley Value.

3 Utility share

Another way of solving the utility share φ_i for each class is by viewing the problem as a cooperative game. As shown in Section 2, each class tries to get more resource to maximize its utility as in (1). Formally speaking, a cooperative game is defined as a game in which the decision makers or agents can conclude a binding agreement as to which outcome will be chosen to exploit the possibility of common interest. Cooperation in the sense of game theory does not mean that either party sacrifices its own interests for the sake of the other, only that each communicates and coordinates its actions for the purpose of furthering its own interests [3].

The game theoretic Shapley Value represents a fair distribution of payoffs. It consistently produces a unique allocation that virtually all researchers consider fair and equitable. This method distributes the total payoff of cooperation based upon the assumption that the payoff of a participant in a *coalition* is determined by the incremental payoff that the participant generates by joining the coalition.

Since the order that participants join a coalition affects the incremental payoff produced, the Shapley Value considers all orderings equally likely and weights all equally. This generates an allocation solution that impartial observers would consider fair and desirable [13]. In this section, we first define the worth of a coalition and then compute the Shapley Value.

We define the worth of a coalition $v(S)$, $S \subseteq N$ as the sum of its members' utility assuming they have the privilege to be allocated more resource in an efficient resource allocation. The privilege will allow them to increase their resource allocation by increasing their w_i . Let's look at the efficient resource allocation as described in (9); the larger the w_i , the more resource will be allocated to class i . When there is a set of classes $N = \{1, 2, \dots, n\}$ with $w = \{w_1, w_2, \dots, w_n\}$ and $w_1 > w_2 > \dots > w_n$, a coalition $S \subseteq N$ means that its members will be given the $|S|$ highest w , that is, $\{w_1, w_2, \dots, w_{|S|}\}$, by its original order ($|S|$ is the length of the set S). At the same time, members in $N \setminus S$ will have the $|N| - |S|$ lowest w $\{w_{|S|+1}, \dots, w_n\}$ by its original order. Therefore, we establish a new $w' = (w'_1, \dots, w'_n)$ when there is a coalition S .

As an example, we assume that there are four different classes of consumers $N = \{1, 2, 3, 4\}$ with $w = \{w_1 = 5, w_2 = 4, w_3 = 2, w_4 = 1\}$. When we consider a coalition $S = \{2, 3\}$, its members class 2 and class 3 will have the two highest w ($|S| = 2$), that is $\{5, 4\}$. By its original impatience order, $w_2 > w_3$, we then have $w'_2 = 5, w'_3 = 4$. Meanwhile, members in the set $N \setminus S = \{1, 4\}$ will have the two lowest w ($|N| - |S| = 4 - 2 = 2$), that is, $\{1, 2\}$ and by their original impatience order, $w_1 > w_4$, we thus have $w'_1 = 2, w'_4 = 1$. Thus, we have established a new $w' = \{w'_1 = 2, w'_2 = 5, w'_3 = 4, w'_4 = 1\}$ when there is a coalition $S = \{2, 3\}$.

After establishing the new w' when there is a coalition S , each class will be allocated resource in an efficient resource allocation manner based on their new impatience w' , and the worth of this coalition S is defined as the sum of its members' utility as follows:

$$v(S) = \sum_{i \in S} w_i \log c'_i \quad (10)$$

where $\hat{c}' = (c'_1, \dots, c'_n)$ is the efficient resource allocation based on their new w' .

The marginal contribution of a class $i \in N$ to a coalition S in v , $i \notin S$ is:

$$v(S \cup \{i\}) - v(S) = \sum_{i \in S \cup \{i\}} w_i \log c''_i - \sum_{i \in S} w_i \log c'_i$$

where $\hat{c}'' = (c''_1, \dots, c''_n)$ is the efficient resource allocation when there is a coalition $S \cup \{i\}$, while $\hat{c}' = (c'_1, \dots, c'_n)$ is the efficient resource allocation when there is a coalition S .

The Shapley Value is defined as a weighted sum of the marginal contribution to all possible coalitions [14]. Let us

recall the definition of the Shapley Value. For all $i \in N$, the payoff (utility share) to class i is given by:

$$SV_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} [v(S \cup \{i\}) - v(S)] \quad (11)$$

As shown before, the term $v(S \cup \{i\}) - v(S)$ calculates the incremental contribution that class i makes to coalition S . This incremental contribution occurs for exactly those orderings in which player i is preceded by $|S|$ other players in $S \cup \{i\}$ and followed by $|N| - |S| - 1$ players not in $|S|$. This means there are exactly $|S|!(|N| - |S| - 1)!$ ordering of interest. $|N|!$ determines the coalition permutations that can be created from the players. Taking together, the expression $\frac{|S|!(|N| - |S| - 1)!}{|N|!}$ is a weighting factor that assigns equal share of the marginal contribution generated to each coalition of interest.

The contribution of the player i is thus weighed and summed for all coalitions where i appears in S . The Shapley Value is that each player i is allocated a value equal to its expected incremental contribution across all possible coalitions which is a fair and desirable for each class. We have thus developed the desirable utility share for each class by using the corresponding Shapley Value.

4 Pricing scheme

Given an efficient resource allocation \hat{c} and a transfer t , the utility share φ_i for class i is equal to $u_i(\hat{c}) + t_i$ as shown in (3) in Section 2. And in Section 3, we calculated the utility share for each class in another way – by modeling this problem as a cooperative game and using the corresponding Shapley Value as the utility share for each class. This means $\varphi_i = SV_i$ and we can then rewrite (3) as follows:

$$SV_i = u_i(\hat{c}) + t_i \quad (12)$$

Now we can calculate the monetary transfer t_i for each class i from (12) as $SV_i - u_i(\hat{c})$. Equation (12) also shows that when the utility share SV_i of class i is larger than its actual utility $u_i(\hat{c})$, class i packets will receive compensations $SV_i - u_i(\hat{c})$ from others because it has been allocated less resource than it should in a fairness manner. On the other hand, if the utility share SV_i of class i is smaller than its actual utility $u_i(\hat{c})$, class i packets will compensate others $SV_i - u_i(\hat{c})$ for it has been allocated more resource than it should be in a fairness way.

Let's now consider $\sum_{i=1}^n t_i$. From (12) we have,

$$\sum_{i=1}^n t_i = \sum_{i=1}^n SV_i - \sum_{i=1}^n u_i(\hat{c})$$

From the Shapley Value's efficiency property, we know $\sum_{i=1}^n SV_i = \text{maximum } u(N)$. We already stated that \hat{c} is an efficient resource allocation and it means that $\sum_{i=1}^n u_i(\hat{c}) = \text{maximum } u(N)$. From those two equations, we observe that $\sum_{i=1}^n t_i = 0$.

We can now state that the allocation $\psi(\hat{c}, t)$ is efficient when \hat{c} is an efficient resource allocation and t_i is calculated using (12).

After getting the transfer t_i for each class based on (12), we are able to define the price difference Δp_{ij} between class i and j as follows:

$$\Delta p_{ij} = \frac{t_j}{c_j} - \frac{t_i}{c_i} \quad (13)$$

Let's now take (12) into (13), and get:

$$\Delta p_{ij} = \frac{SV_j - u_j}{c_j} - \frac{SV_i - u_i}{c_i} \quad (14)$$

where $\hat{c} = (c_1, \dots, c_n)$ is an efficient resource allocation and SV_i is the Shapley Value corresponding to the cooperative game.

We have thus developed the price differentials among classes based on the inter-class compensations. This will provide an important input for producers to set up subsidy-free prices for each class of consumers.

5 An illustrative example

This section presents a numerical example to calculate the price differentials between classes based on the model proposed in this paper.

We assume that there are three classes of consumers in the market: residential consumers, commercial consumers and industrial consumers. A class i of consumers has its w_i ($i = 1, 2, 3$) as its willingness to pay for the service. We assume that $w_1 > w_2 > w_3$. Using (1) and (9), the actual utility for each class is:

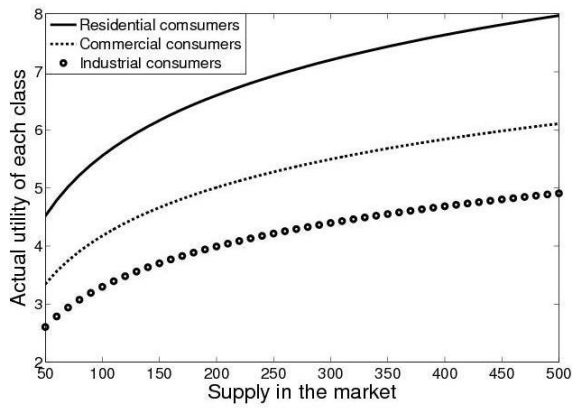
$$u_i = w_i \log \frac{w_i c}{\sum_{i=1}^3 w_i}, i = 1, 2, 3$$

Using (11), the utility share for each class is as follows,

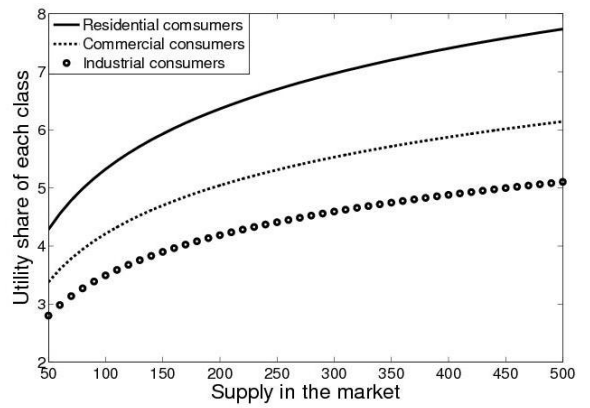
$$\begin{aligned} SV_1 &= \frac{v\{1\}}{3} + \frac{v\{1,2\} - v\{2\}}{6} + \frac{v\{1,3\} - v\{3\}}{6} \\ &\quad + \frac{v\{1,2,3\} - v\{2,3\}}{6} \\ SV_2 &= \frac{v\{2\}}{3} + \frac{v\{2,3\} - v\{3\}}{6} + \frac{v\{1,2\} - v\{1\}}{6} \\ &\quad + \frac{v\{1,2,3\} - v\{1,3\}}{6} \\ SV_3 &= \frac{v\{3\}}{3} + \frac{v\{1,3\} - v\{1\}}{6} + \frac{v\{2,3\} - v\{2\}}{6} \\ &\quad + \frac{v\{1,2,3\} - v\{1,2\}}{6} \end{aligned}$$

As described in Section 3, $v(S)$ is the sum of its members' disutility in an efficient resource allocation assuming that its members have the $|S|$ largest w . Therefore, from (9) and (10), we get:

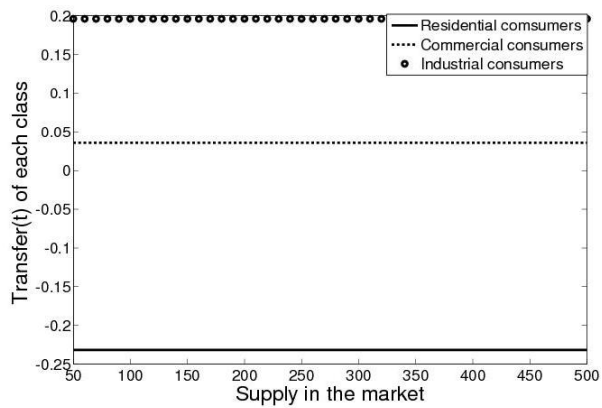
$$\begin{aligned} v\{1\} &= w_1 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} \\ v\{2\} &= w_2 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} \\ v\{3\} &= w_3 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} \\ v\{1, 2\} &= w_1 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + w_2 \log \frac{w_2 c}{\sum_{i=1}^3 w_i} \end{aligned}$$



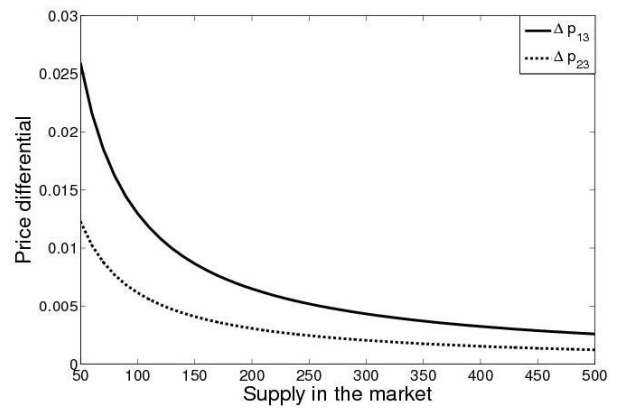
(a)



(b)



(c)



(d)

Figure 3: Profiles $u_i, SV_i, t_i, \Delta p_{13}, \Delta p_{23}, (i = 1, 2, 3)$ against changing c (total supply in the market). (a) Actual utility $u_i, (i = 1, 2, 3)$ for each class against c . (b) Utility share (Shapley Value) $SV_i, (i = 1, 2, 3)$ for each class against c . (c) Transfer $t_i, (i = 1, 2)$ for each class against c . (d) Price difference between the two classes $\Delta p_{13}, \Delta p_{23}$ against c .

6 Conclusion

In this paper, we have investigated the price differentials among different classes of consumers based on the inter-class compensation. In the model proposed in this paper, each class of consumers has a different willingness to pay of the service. We have developed an efficient resource allocation for each class of consumers and then computed monetary transfer for each class based on the utility share of each class which is the corresponding Shapley Value in the cooperative game. The pricing scheme proposed in this paper shows that the price differentials among different classes of consumers decrease when there is more supply in the market.

REFERENCES

- [1] C. Harris, "Electricity Markets: Pricing, Structures and Economics," John Wiley & Sons, Ltd, 2004.
- [2] Energy Information Administration, "Electric Power Monthly," Retrieved in Sep, 2010, http://www.eia.doe.gov/cneaf/electricity/epm/epm_sum.html.
- [3] X. Tan and T. T. Lie, "Application of the Shapley Value on transmission cost allocation in the competitive power market environment," IEE Proceedings - Generation, Transmission and Distribution, Volume: 149 Issue: 1, On page(s): 15-20, Jan, 2002.
- [4] F. PIROVANO, T. EVE, and F. AMANDALY, "Allocation of fixed transmission costs to different network functions," Proceedings of International conference on Electric Utility Deregulation and Restructuring and Power Technologies, London, pp. 3163319, 2000.
- [5] H. M. MERRILL, and B. W. ERICKSON, "Wheeling rates based on marginal cost theory," IEEE Trans. Power Syst., 4, (4), pp. 1445-1451, October 1989,
- [6] G. C. Stamtis and I. Erlich, "Use of cooperative game theory in power system fixed-cost allocation," IEE Proceedings - Generation, Transmission and Distribution, Volume: 151, Issue: 3, On page(s): 401-406, May, 2004.
- [7] J. M. Zolezzi and H. Rudnick, "Transmission Cost Allocation by Cooperative Games and Coalition Formation," IEEE Power Engineering Review, Volume: 22 Issue: 8, On page(s): 64-64, Aug, 2002.
- [8] F. Maniquet, "A Characterization of the Shapley Value in Queueing Problems," Journal of Economic Theory, 109: 90-103, 2003.

$$v\{1, 3\} = w_1 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + w_3 \log \frac{w_2 c}{\sum_{i=1}^3 w_i}$$

$$v\{2, 3\} = w_2 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + w_3 \log \frac{w_2 c}{\sum_{i=1}^3 w_i}$$

$$v\{1, 2, 3\} = w_1 \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + w_2 \log \frac{w_2 c}{\sum_{i=1}^3 w_i} + w_3 \log \frac{w_3 c}{\sum_{i=1}^3 w_i}$$

As defined in (12), we can now calculate the transfer for each class as follows,

$$t_1 = SV_1 - u_1 = \left(-\frac{w_2}{2} - \frac{w_3}{6}\right) \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + \left(\frac{w_2}{2} - \frac{w_3}{6}\right) \log \frac{w_2 c}{\sum_{i=1}^3 w_i} + \frac{w_3}{3} \log \frac{w_3 c}{\sum_{i=1}^3 w_i}$$

$$t_2 = SV_2 - u_2 = \left(\frac{w_2}{2} - \frac{w_3}{6}\right) \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + \left(-\frac{w_2}{2} - \frac{w_3}{6}\right) \log \frac{w_2 c}{\sum_{i=1}^3 w_i} + \frac{w_3}{3} \log \frac{w_3 c}{\sum_{i=1}^3 w_i}$$

$$t_3 = SV_3 - u_3 = \frac{w_3}{3} \log \frac{w_1 c}{\sum_{i=1}^3 w_i} + \frac{w_3}{3} \log \frac{w_2 c}{\sum_{i=1}^3 w_i} - \frac{2w_3}{3} \log \frac{w_3 c}{\sum_{i=1}^3 w_i}$$

Using the definition in (9) (13), the price differentials between classes,

$$\Delta p_{13} = \frac{t_3}{c_3} - \frac{t_1}{c_1} = \frac{t_3}{\sum_{i=1}^3 w_i} - \frac{t_1}{\sum_{i=1}^3 w_i}$$

$$\Delta p_{23} = \frac{t_3}{c_3} - \frac{t_2}{c_2} = \frac{t_3}{\sum_{i=1}^3 w_i} - \frac{t_2}{\sum_{i=1}^3 w_i}$$

For a numerical example, we assume that the willingness to pay of each class is: $w_1 = 1.5, w_2 = 1.2, w_3 = 1$. Fig. 3 depicts $u_i, SV_i, t_i, \Delta p_{13}, \Delta p_{23}, i = 1, 2, 3$ profiles against supply in the market (c). In Fig. 3(a), the actual utility for each class increases when there is more power supply in the market. Since residential consumers have higher willingness to pay and have been allocated more resource, they have higher utility than commercial consumers and industrial consumers. Fig. 3(b) shows the utility share (Shapley Value) for each class against c . Compare Fig. 3(a) and Fig. 3(b), we find that residential consumers have higher actual utility than its utility share and industrial consumers have lower actual utility than its utility share. Fig. 3(c) describes the transfer for each class, specifically, the amount residence consumers should compensate commercial consumers and industrial consumers. In this example, residential consumers will compensate others 0.2319. Commercial consumers and industrial consumers will receive compensation 0.0359 and 0.1959 respectively and the amount for each class of consumers is independent of the supply in the market. As shown in Section 4, the total transfer is equal to 0, $t_1 + t_2 + t_3 = 0$. Fig. 3(d) shows that the price differentials $\Delta p_{13}, \Delta p_{23}$ decrease as the supply in the market increases. We also find that when the market is mature (there is relatively more supply in the market), the price differentials tend to be a constant value.

- [9] Y. Chun, "A Note on Maniquet's Characterization of the Shapley Value in Queueing Problems," Working Paper, Rochester University, 2004.
- [10] H. Moulin, "On Scheduling Fees to Prevent Merging, Splitting and Transferring of Jobs," Working Paper, Rice University, 2004.
- [11] H. Moulin, "Split-proof Probabilistic Scheduling," Working Paper, Rice University, 2004.
- [12] D. P. Bertsekas, "Nonlinear Programming," Athena Scientific, 1999.
- [13] J. R. Boatsman, D. R. Hansen and J. I. Kimbrell, "A rationale and some evidence supporting and alternative to the simple Shapley value," Proc. Univ. of Oklahoma Conf. Cost Allocation, 1981.
- [14] L. S. Shapley, "A Value for n-person Games. In Contributions to the Theory of Games," Annals of Mathematical Studies v. 28, pp. 307-317, Princeton University Press, 1953.