PAYMENT MINIMIZATION IN A COMBINED ENERGY MARKET THROUGH A BILEVEL LINEAR MODEL

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Abstract – A combined market model of energy and reserve services operating under Payment Minimization (PM) is formulated through a bilevel linear model. The combined market model considers the simultaneous presence of long term bilateral contracts, short term pool load and reserve services in the system operation. The model has several important characteristics such as considering the system transmission network, generation and transmission capacity limits as well as multi-period constraints. The obtained payments, revenues, costs and prices of services allow establishing comparisons between the PM model and the usual Bid Cost Minimization (BCM) model. These comparisons allows observing the behavior of the system operation and economic indexes in several scenarios that include different degrees of participation of bilateral contracts on total load, and several degrees of the system reserve requirements. The model is tested in a 5-Bus system and in a 30-Bus system.

Keywords: bilevel linear optimization, pool market, bilateral contracts, ancillary reserve services.

1 NOMENCLATURE

For each generator *i* and demand *j* participating in the integrated market we define the following notation. For the sake of simplicity[1] and without loss of generality, only one generator is considered at each bus *i* and only one load is considered at bus *j*.

Generators Bid Parameters in ($/h):

$C_{g\,i}(p^{p}_{g\,i})$	Pool Energy bid cost component
$C^{RU}_{g\,i}(ru_{g\,i})$	Regulation up reserve bid cost component
$C^{RD}_{g\,i}(rd_{g\,i})$	Regulation down reserve bid cost component
$C^{SR}_{g\,i}(sr_{g\,i})$	Spinning reserve bid cost component
$C^{NS}_{g\,i}(ns_{g\,i})$	Non spinning reserve bid cost component
$C^{RC}_{g\,i}(rc_{g\,i})$	Complementary reserve bid cost component
$s_{g\,i}$	Start up cost of unit i

Variables:

$p^{p}_{g\,i}$	Awarded level of Active Pool Generation (MW)
$ru_{g\,i}$	Awarded level of regulation up reserve (MW)
$rd_{g\,i}$	Awarded level of regulation down reserve (MW)
$sr_{g\,i}$	Awarded level of spinning reserve (MW)
$ns_{g\,i}$	Awarded level of non spinning reserve (MW)
$rc_{g\,i}$	Awarded level of complementary reserve (MW)
$p_{i\,j}$	Active power flow in transmission line connecting bus *i* and bus *j*
$u_{g\,i}$	On/off Binary variable of unit *i*

Constant Parameters:

$p^{b}_{g\,i}$	Bilateral Active power generation level at bus *i* (MW)
$p^{\min}_{g\,i}, p^{\max}_{g\,i}$	active power capacity limits of generator *i*
$y_{i\,j}$	Susceptance of the transmission line connecting *i* and *j*
$p^{\max}_{i\,j}$	Maximum transmission capacity limit of transmission line connecting bus *i* and bus *j*
$p_{d\,j}$	Active power demand level at bus *j* (MW)
$ru^{\max}_{i}, sr^{\max}_{i}$	Maximum bid capacity limit parameter for reg-up, and spinning. The same notation is followed for maximum bid capacity limit of the other reserve services.
$p^{b}_{d\,j}$	Bilateral Active power demand level at bus *j* (MW)
$RP^{RU}_{g\,i}, RP^{RD}_{g\,i}$	ramp rate limit for providing up and down reserve
$RP^{SNS}_{g\,i}$	ramp rate limit[2] for $sr_{g\,i}$ and $ns_{g\,i}$

2 INTRODUCTION

In recent years several approaches concerned with energy pricing have been suggested in restructured electricity markets for obtaining prices that truly reflect the use of services. From the energy consumer side, prices and resulting payments should reflect the real cost of the service and from the energy producer side prices and consequent revenues should compensate investments. One of the most widely followed approaches in the electricity market industry is the use of bid cost minimization (BCM) auctions which defines awarded bids and associated prices of energy. This approach has several advantages such as its simplicity for using in practical purposes and the fact that the obtained prices reflect the cost of producing one additional MW. Nevertheless, one disadvantage is the fact that the final bid cost obtained in the auction is not the same as

[1] Additional indexes are required with more generators and/or loads in each bus.

[2] For simplicity and without loss of generality the same ramp limit is considered for these two services.

the payment cost made during the settlement process. As a result, the total payment cost could be significantly higher than the minimized bid auction cost [1,2]. Other disadvantage is the fact that the market participants´ bid costs do not always fully reflect production costs. Because of this inconsistence models concerned with the payment minimization instead of the bid cost minimization have been developed recently [1,2]. Moreover, electricity markets in general work with two types of markets. One is the long term bilateral contract market and the other is the short term pool market (which normally uses bid cost minimization auctions). These two types of markets coexist in different proportions depending on the structure of a particular system and the behavior of agents. There is also a possibility of a third market aimed for the procurement of ancillary services [3,4,6]. In some systems this last market operates in an independent manner and therefore not necessarily reducing the cost of the combined use of services. On the contrary, others consider the co-optimization of the ancillary service procurement with the simultaneous presence of other two markets convenient for the purpose of reducing bid costs reflecting the real use of the system services [4]. The proposed approach is a payment minimization model formulated through a bilevel linear model with several important characteristics. First, the mixed integer linear version of the payment minimization model allows using available efficient commercial solvers even in large systems and with several time intervals; the model is also able to incorporate a combined market operation with physical bilateral contracts (not only financial) and pool transactions as well as the procurement of the ancillary services of reserve of several types; the model allows obtaining economical indexes of several energy services such as prices, generators revenues, and load payments in several operation conditions considering capacity limits of generation units and transmission lines; and the model allows to use discrete and/or continuous bids costs. Several important results are presented based on simulations in a five bus system and in the IEEE 30-Bus system. In all simulated cases a comparison between bid cost minimization (BCM) and payment minimization (PM) is presented.

The structure of presentation is as follows: section 3 describes the PM model. Section 4 introduces the methodology. Section 5 shows numerical examples and section 6 provides some relevant conclusions.

3 COST MINIMIZATION MODEL

Initially, the classical short term bid cost minimization of the combined market (pool/bilateral/reserve) [3, 4] is described in detail. The corresponding payment minimization model is further formulated in subsection 3.8.

3.1 Pool and Reserve Auction

The short term combined Pool and reserve auction determine awarded energy and reserve bids for a specific period of time (in order to simplify the notation the index t related to the period of time with a fixed load level is omitted in the formulation). In this market a merit order list based on low bid costs is obtained for energy and reserve services by minimizing the following objective function.

$$Minimize \quad J_b = C_{energy} + C_{reserve} \tag{1}$$

Where, $C_{energy} = \sum_i C_i\,(p^p_{g\,i}) + s_{g\,i} u_{g\,i}$

$$C_{reserve} = \{ \sum_i C_i^{RU}(ru_{g\,i}) + \sum_i C_i^{SR}(sr_{g\,i}) + \sum_i C_i^{NS}(ns_{g\,i}) + \sum_i C_i^{RC}(rc_{g\,i}) + \sum_i C_i^{RD}(rd_{g\,i}) \}$$

Bid[3] cost functions can be considered as continuous quadratic or piece-wise linear functions.

3.2 Transmission Network

From the ISO point of view, awarded bids and firm bilateral contracts must observe operation constraints imposed by the transmission system. Therefore, feasible solutions should belong to the set defined by constraints (2) to (5) for all buses i in the transmission system. Long-term physical bilateral contracts and the pool demand are known quantities. Each demand j is considered to have two energy components $p_{d\,j} = p^p_{d\,j} + p^b_{d\,j}$, and each generator i has also two energy components $p_{g\,i} = p^p_{g\,i} + p^b_{g\,i}$. The network linear load flow equations are represented by (2). Constraints (3) force the transmission lines active power flow to operate within limits (thermal or stability limits); constraints (4) define the operation range of generators; constraint (5) defines the bus voltage phase reference angle. Vector δ represents bus voltage phase angles with dimension equal to the total number of buses n.

$$p_{g\,i} - p_{d\,i} = \sum_j y_{ij}(\delta_i - \delta_j), \quad \forall i \quad \rightarrow \lambda_i \tag{2}$$

$$-p^{\max}_{i\,j} \le y_{ij}(\delta_i - \delta_j) \le p^{\max}_{i\,j} \quad , \quad \forall i,j \tag{3}$$

$$u_{g\,i}\, p^{\min}_{g\,i} \le p_{g\,i} \le p^{\max}_{g\,i}\, u_{g\,i} \,, \quad \forall i,j \tag{4}$$

$$\delta_{ref} = 0 \tag{5}$$

The set defined by (2) – (5) represents the security region of the power system in the space of generation levels (one alternative form of formulating this problem is by using the Power Transmission Distribution Factors-PTDF). It is assumed in this model that the market for voltage support is obtained by specific agreements between regulator and suppliers, which is not part of the short term auction model. Lambdas in (2) are Lagrange multipliers associated with each constraint and represent nodal prices (or LMP) of injected active power.

3.3 Reserve Market characteristics

Generators agents can bid in five kinds of reserve services: Regulation up, Regulation Down (for compen-

[3] In this paper, generator cost is understood to mean bid cost.

sating possible frequency deviations), Spinning, Non-Spinning, and Replacement (for compensating possible energy unbalance). The speed response defines the quality of each service, i.e., faster response reserves are graded as higher quality or higher value. In order to open possibilities for reducing costs, it is also permitted the possibility of substitution among reserve services. The substitution consists in allowing services with better quality and lower cost to replace services with lower quality and higher cost [4,5]. Hence, a feasible bid selection in the reserve market auction, which also avoids price reversal between reserve services, should belong to the set described by constraints (6) to (12).

$$R^{RU} \le \sum_{i=1}^{n} ru_{g\,i} \quad \to \lambda^{RU} \tag{6}$$

$$R^{RU} + R^{SR} \le \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} \quad \to \lambda^{SR} \tag{7}$$

$$\begin{aligned} & R^{RU} + R^{SR} + R^{NS} \le \\ & \qquad \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} + \sum_{i=1}^{n} ns_{g\,i} \to \lambda^{NS} \end{aligned} \tag{8}$$

$$R^{RC} \le \sum_{i=1}^{n} rc_{g\,i} \quad \to \lambda^{RC} \tag{9}$$

$$R^{RD} \le \sum_{i=1}^{n} rd_{g\,i} \quad \to \lambda^{RD} \tag{10}$$

$$ru_{g\,i} \ge 0, sr_{g\,i} \ge 0, ns_{g\,i} \ge 0, rc_{g\,i} \ge 0, rd_{g\,i} \ge 0 \tag{11}$$

$$\begin{aligned} & ru_{g\,i} \le ru_i^{\max} u_{g\,i},\ sr_{g\,i} \le sr_i^{\max} u_{g\,i},\ ns_{g\,i} \le ns_i^{\max} u_{g\,i}, \\ & rc_{g\,i} \le rc_i^{\max} u_{g\,i},\ rd_{g\,i} \le rd_i^{\max} u_{g\,i} \end{aligned} \tag{12}$$

Where, R^{RU}, R^{SR}, R^{NS}, R^{RC} and R^{RD} (in MW) are the estimated required system amounts of each reserve service during a specific period of time (for instance, one hour). These quantities are considered known and defined by the ISO before the market auction occurs. Lambda variables are Lagrange multipliers associated with each constraint. Upper limits in (12) are physical limits associated to generators ramp rates and they are part of the information of the agents reserve bids.

3.4 Long Term Bilateral Contracts

Private long term bilateral contracts are considered as firm physical (not financial) contracts which are authorized and implemented by the ISO who takes into consideration the security conditions of the transmission network. In a compact form, bilateral contracts are grouped in a **T** matrix where each coefficient $T_{i\,j}$ represents the MW traded between generator at bus *i* and load at bus *j*. Therefore, the total amount of contracts supplied by generator *i* is,

$$p_{g\,i}^{b} = \sum_{j=1}^{n} T_{i\,j} \tag{13}$$

In addition, the total amount of contracts supplying the demand at bus *j* is,

$$p_{d\,j}^{b} = \sum_{i=1}^{n} T_{i\,j} \tag{14}$$

In this model is supposed that firm long-term bilateral contracts are already in place at the moment of performing one auction and they have priority in relation to the pool dispatch in terms of allocating generation capacity [3]. Because of this, the already committed capacity to bilateral contracts imposes a constraint on the lower generation limit for generators participating in the pool market as shown in (15),

$$p_{g\,i}^{b} \le p_{g\,i} \le p_{g\,i}^{\max} \tag{15}$$

Obviously, generation units dispatching long-term bilateral contracts have their binary variables $u_{g\,i}$ (on/off) equal to one (turned on).

3.5 Reserve Availability of Generators Capacity

In the combined market, besides meeting bilateral and pool loads, each generator *i* can also participate in the reserve market by bidding several reserve services. The awarded reserve bids should respect the operational capacity limits of each generator as described in (16) and (17).

$$p_{g\,i} + ru_{g\,i} + sr_{g\,i} + ns_{g\,i} \le p_{g\,i}^{\max} u_{g\,i} \tag{16}$$

$$-rd_{g\,i} + p_{g\,i} \ge p_{g\,i}^{b} \tag{17}$$

It is worth noting that because of the substitution, if the system in a particular scenario cleared its needs for up regulation and spinning reserve on the basis of lower bid prices and it is still looking for meeting non-spinning reserve requirements, then the optimization not only considers the non-spinning bid costs but also considers lower price bids of spinning still available. Therefore, a particular spinning reserve bid can be accepted on the basis of lower cost for supplying the non-spinning requirement. In case there is no possibility of substitution because of the operation conditions, the generators only biding for non-spinning reserve have their pool generation, regulation up and spinning levels at zero in constraint (16).

3.6 Inter-temporal constraints

An important consideration is the capacity of response due to the technology adopted by each generator *i* characterized by the corresponding ramp rate limit. As suggested in [6] and considering (4) and (16), the ramping up limitations are

$$\begin{aligned} & p_{g\,i\,t} \le p_{g\,i(t-1)} + RP_{gi}^{RU} u_{g\,i(t-1)} + RP_{gi}^{SNS}(u_{g\,i\,t} - u_{g\,i(t-1)}) + p_{gi}^{\max}(1-u_{g\,i\,t}) \\ & t = 2,\ldots,T \end{aligned} \tag{18}$$

$$\begin{aligned} & p_{g\,i\,t} \le p_{g\,i0} + RP_{gi}^{RU} u_{g\,i0} + RP_{gi}^{SNS}(u_{g\,i\,t} - u_{g\,i0}) + p_{gi}^{\max}(1-u_{g\,i\,t}) \\ & t = 1. \end{aligned} \tag{19}$$

Furthermore, the ramping down limitations have similar linear constraints. Start-up costs are considered as well in the inter-temporal constraints in a similar way by using additional binary variables.

3.7 Marginal Price of Reserve Services

The Lagrangean function of the optimization problem described in (1) to (18) allows obtaining expressions for the market clearing prices of services based on the Lagrange Multipliers. These prices represent the sensitivity of cost in terms of incremental perturbations of reserve requirements as shown in (20) to (22).

$$\frac{\partial L}{\partial R^{RU}} = \lambda^{RU} + \lambda^{SR} + \lambda^{NS} = MCP_{RU} \tag{20}$$

$$\frac{\partial L}{\partial R^{SR}} = \lambda^{SR} + \lambda^{NS} = MCP_{SR} \tag{21}$$

$$\frac{\partial L}{\partial R^{NS}} = \lambda^{NS} = MCP_{NS} \tag{22}$$

Similar definitions are obtained for *RC* and *RD* reserve services. Since Lagrange multipliers are positive, this formulation avoids reversal prices among services. In other words, $MCP_{RU} \geq MCP_{SR} \geq MCP_{NS}$ as discussed in [4] and [5].

4 PAYMENT MINIMIZATION MODEL

In the PM auction, the total pool energy payment and reserve service payment are minimized according to the bilevel optimization problem defined in (23) in a particular time period *t*.

First level:

$$Minimize\, J_p = \pi_{energy} + \pi_{reserve} \tag{23}$$

Subject to:

Second Level:

$$Minimize\, J_c = C_{energy} + C_{reserve}$$

Subject to: *Constraints (2) – (19)*

Where,

$$\pi_{energy} = \sum_i \lambda_i \; p^p_{d\,i} \tag{24}$$

$$\pi_{reserve} = \pi_{RU} + \pi_{SR} + \pi_{NS} + \pi_{RC} + \pi_{RD} \tag{25}$$

and,

$$\pi_{RU} = MCP_{RU} \; R^{RU} \tag{26}$$

$$\pi_{SR} = MCP_{SR} \; R^{SR} \tag{27}$$

$$\pi_{NS} = MCP_{NS} \; R^{NS} \tag{28}$$

(π_{RC} and π_{RD} follow similar definitions).

In the first level, the objective function (23) consists in minimizing payments of energy and reserve. The estimated pool load and reserve requirements are considered known. The optimization variables in this level are nodal prices and marginal clearing prices of reserve services. The first level problem is subject to solving the second level optimization problem (1) – (19) which is the original bid cost minimization problem (BCM). It is worth noting that prices of services (composed of dual Variables) in (24)-(28) are obtained when solving the BCM problem. Therefore, solving the PM problem requires the inseparable structure of the BCM problem. The formulation of the problem through a bilevel model is appropriate for solving the PM problem through a linear formulation as shown in next section. Because of the possibility of generator units get lower revenues than their bid costs in the PM formulation, it is necessary to guarantee that prices of services are greater or equal to the incremental cost of the units participating in the combined market. Therefore, the following constraints (29) and (30) are added to the first level PM problem in the pool market.

$$\lambda_i \geq IC_{g\,i}\, u_{g\,i} \quad , \quad \forall_i \tag{29}$$

Where, $IC_{g\,i}$ is the incremental bid cost of unit i. In the reserve market the price of services should be greater or equal to the incremental reserve cost of units as shown in (30).

$$MCP_{reserve} \geq ICR_{g\,i}\; u_{g\,i} \quad , \quad \forall_i \tag{30}$$

The formulation presented characterizes a bilevel linear problem with continuous and binary variables.

5 METHODOLOGY

The strategy in solving a bilevel linear problem is transforming the original problem into a one level standard linear problem which is easy to solve. The transformation is made by incorporating the optimality necessary conditions of the second level problem into the body of constraints of the original first level problem [7]. The resulting set of constraints should include the complementary slackness of inequality constraints. This last requirement introduces non-linearity to the set of equations because of the presence of the product of variables. To overcome this difficulty, equivalent linear equations are introduced to substitute the non-linear constraints. This procedure increases the number of linear constraints and binary variables. The resulting one level optimization problem is a mixed integer linear problem. In order to formulate the equivalent one level linear problem, it is necessary to observe the primal and dual characteristics of the problem. First, primal variables are $\{p^p_{g\,i}\}$, $\{\delta_i\}$, $\{ru_{g\,i}\}$, $\{sr_{g\,i}\}$, $\{ns_{g\,i}\}$, $\{rc_{g\,i}\}$, $\{rd_{g\,i}\}$. The primal feasibility set of solutions is defined by the set of constraints (2) – (18). The dual feasibility is obtained by ensuring non negative values of dual variables (31). The first six dual variables are associated to lower and upper limits of constraints (3), (4) and (16), respectively. The last five dual variables are associated to the reserve requirements constraints (6), (7), (8), (9) and (10).

$$\begin{aligned} &\beta^{\min}_{g\,i}, \beta^{\max}_{g\,i}, \gamma^{\min}_{i\,j}, \gamma^{\max}_{i\,j}, \sigma^{\min}_{g\,i}, \sigma^{\max}_{g\,i}, \\ &\lambda^{RU}, \lambda^{SR}, \lambda^{NS}, \lambda^{RC}, \lambda^{RD} \geq 0 \quad , \forall i, j \end{aligned} \tag{31}$$

The complementary slackness related to the necessary conditions is considered through the following set of constraints.

$$\gamma^{\max}_{i\,j}\,(y_{ij}(\delta_i - \delta_j) - p^{\max}_{i\,j}) = 0, \quad \forall i, j \tag{32}$$

$$\beta_i^{\min}(\, p_{gi}^{\min} - p_{g\,i}\,) = 0, \quad \forall i \tag{33}$$

$$\beta_i^{\max}(\, p_{g\,i} - p_{g\,i}^{\max}\,) = 0, \quad \forall i \tag{34}$$

$$\sigma_i^{\min}(p_{g\,i}^{b} - p_{g\,i} + ru_{g\,i} + sr_{g\,i} + ns_{g\,i}) = 0, \quad \forall i \tag{35}$$

$$\sigma_i^{\max}(p_{g\,i} + ru_{g\,i} + sr_{g\,i} + ns_{g\,i} - p_{g\,i}^{\max}) = 0, \quad \forall i \tag{36}$$

The complementary slackness of reserve requirements for regulation up and spinning reserve are described in (37) and (38). The other reserve requirements follow similar equations.

$$\lambda^{RU}(R^{RU} - \sum_{i=1}^{n} ru_{g\,i}) = 0 \tag{37}$$

$$\lambda^{SR}(R^{RU} + R^{SR} - \sum_{i=1}^{n} ru_{g\,i} - \sum_{i=1}^{n} sr_{g\,i}) = 0 \tag{38}$$

The set of constraints (32) – (38) are nonlinear equations due to the presence of product of variables. Nevertheless, these equations can be represented through equivalent linear equations by introducing additional binary variables as shown in the case of (33). As suggested in [8], this constraint can be replaced by the following two linear equations.

$$p_{g\,i} - p_{g\,i}^{\max} \le M_{p\,gi}\, u_{g\,i}^{\max}, \qquad \forall i \tag{39}$$

$$\beta_i^{\max} \le M_{p\,gi}(1 - u_{g\,i}^{\max}\,), \quad \forall i \tag{40}$$

Where, $u_{g\,i}^{\max} \in \{0,1\}$ and $M_{p\,gi}$ is a large positive number (for instance, one order higher than the largest cost coefficient). In practical terms, the choice of this large number can be calibrated without difficulty even in large systems and still preserving stability [8]. After substituting the original non-linear complementary slackness equations for the equivalent linear equations, the resulting one level problem is a mixed binary linear optimization problem easy to solve through efficient existing solvers [8].

6 REVENUES AND PAYMENTS

This section presents how revenues and payments are calculated for generators and loads participating in the combined market after finishing the short term auction.

<u>*Bilateral Contracts*</u>

Because bilateral contracts are negotiated in private, their prices are not available. Nevertheless, we adopt as a price for these contracts the corresponding incremental costs of generators i.e., $IC_{g\,i}^{b} = dC_{g\,i}(P_{g\,i}^{b})\big/dP_{g\,i}$.

Revenues: The revenue of generator *i* due to bilateral contracts is,

$$c_{g\,i}^{b} = IC_{g\,i}^{b} \sum_{i=1}^{n} T_{i\,j} \tag{41}$$

Payments: Load *j* pays the supplied bilateral contracts according to (42),

$$c_{d\,i}^{b} = IC_{g\,i}^{b} \sum_{i=1}^{n} T_{i\,j} \tag{42}$$

Bilateral contracts should pay for congestion management according to (43),

$$c_{i\,j}^{bc} = \sum_{i=1}^{n} (\lambda_i - \lambda_j)\, T_{i\,j} \tag{43}$$

This amount could be split among contract parties in a 50/50 basis or other proportion; we adopt a split of 50/50. The corresponding payment of generator *i* for all its bilateral contracts congestion is,

$$c_{g\,i}^{bc} = (1/2) \sum_{j=1}^{n} c_{i\,j}^{bc} \tag{44}$$

Similarly, the payment of the bilateral load *j* is,

$$c_{d\,j}^{bc} = (1/2) \sum_{i=1}^{n} c_{i\,j}^{bc} \tag{45}$$

<u>*Pool and Reserve*</u>

Revenues: Generator *i* has revenues for supplying pool demand, bilateral congestion management as well as providing reserve capacity as shown in (46) to (48).

$$\hat{c}_{g\,i}^{p} = \lambda_i \cdot p_{g\,i}^{p} \tag{46}$$

$$c_{g\,i}^{RU} = MCP_{RU} \cdot ru_{g\,i} \tag{47}$$

$$c_{g\,i}^{SR} = MCP_{SR} \cdot sr_{g\,i} \tag{48}$$

Revenues obtained from other reserve services follow similar calculations.

Payments: Load *j* has payments related to the use of pool demand, and reserve services according to (49) to (51). In the case of reserve services, load *j* pays in a pro-rate manner as shown in (50) and (51).

$$\hat{c}_{d\,j}^{p} = \lambda_j\, p_{d\,j}^{p} \tag{49}$$

$$c_{d\,j}^{SR} = MCP_{SR} \cdot (\sum_{i=1}^{n} sr_{g\,i}) \cdot (p_{d\,j}^{p} + p_{d\,j}^{b}) / \, p_{d\,Total} \tag{50}$$

$$c_{d\,j}^{RU} = MCP_{RU} \cdot (\sum_{i=1}^{n} ru_{g\,i}) \cdot (p_{d\,j}^{p} + p_{d\,j}^{b}) / \, p_{d\,Total} \tag{51}$$

Where, $P_{d\,Total}$ is the total system load including bilateral and pool demand. Load payments related to the other services are obtained in a similar way.

7 GENERATORS AND LOADS PORTFOLIOS

The portfolio revenue of generator *i* is obtained by using (52). This portfolio is composed basically by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{gi} = c_{gi}^{b} + c_{gi}^{p} + c_{gi}^{R} \tag{52}$$

Where,

$$c_{gi}^{R} = c_{gi}^{RU} + c_{gi}^{RD} + c_{gi}^{SR} + c_{gi}^{NS} + c_{gi}^{RC} \tag{53}$$

$$c_{gi}^{p} = \hat{c}_{g\,i}^{p} - c_{g\,i}^{bc} \tag{54}$$

Likewise, the portfolio payment of load *j* is composed by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{d\,j} = c_{d\,j}^{b} + c_{d\,j}^{p} + c_{d\,j}^{R} \tag{55}$$

Where,

$$c_{dj}^{R} = c_{dj}^{RU} + c_{dj}^{RD} + c_{dj}^{SR} + c_{dj}^{NS} + c_{dj}^{RC} \quad (56)$$

$$c_{dj}^{p} = \hat{c}_{dj}^{p} + c_{dj}^{bc} \quad (57)$$

8 NUMERICAL EXAMPLES

For the sake of simplicity and for not extending too much the paper it is considered in the following examples only one period of time, only two types of reserve, the same bid strategy for both PM and BCM and the specific individual economic values obtained for loads and generators such as revenues, costs and payments are not shown. Unless stated differently, in most of the cases energy bids are the same as reserve bids (first, second and third cases), and no start up costs (first and second cases).

8.1 5-Bus System

A 5-bus system described in tables 1 and 2 is considered in the following examples. Table 1 shows generation bid costs, capacity and loads. Table 2 shows the transmission system data. In this first example total load of 600 MW is 70% supplied by bilateral contracts (420MW) according to the following bilateral contracts: generator 1 supplies load 3 with T_{13} = 210 and load 5 with T_{15} = 210. The remaining 30% of load is supplied with pool generation (180 MW with p^{P}_{d3}=60 and p^{P}_{d5}=120). Based on records, the system estimates the requirement of the following reserve services: R^{RU}=80 MW and R^{SR}=10 MW.

Bus	Bid_{gi} (\$/MWh)	p_{gi}^{min} (MW)	p_{gi}^{max} (MW)	p_{dj} (MW)
1	10	60	450	0
2	20	15	210	0
3	-	-	-	270
4	40	20	480	0
5	60	10	300	330

Table 1: Generators Bid cost, Generators and load data

line	1 - 2	1 - 5	2 - 5	2 - 3	4 - 3	4 - 5
y_{ij} (S)	156.3	33.7	32.9	35.6	92.6	33.7
p_{ij}^{max} (MW)	300	300	150	300	300	300

Table 2: Transmission lines with maximum flow capacity.

Table 3 shows results of the combined market with the BCM and PM approaches. Results show that transmission congestion occurs under BCM in line 2-5. However, this situation is not observed under PM and, as a consequence, nodal prices are uniform and equal to 40\$/MWh. The full capacity of the cheap generator 1 is used for supplying the bilateral and pool load in both approaches. Under PM only generators 1 and 4 are participating in the energy market with 30 and 150 MW. The obtained PM dispatch reduces the load payment by also avoiding transmission congestion. The allocated reserve capacity is the same under both approaches with MCP_{RU}=20 (\$/MWh). Under PM generator 2 is in operation but only available for participating in the reserve market. Its reserve bid cost is awarded in the reserve market for providing its capacity to the system.

	BCM				PM			
	p_{gi}^{p}	λ_i	sr_{gi}	ru_{gi}	p_{gi}^{p}	λ_i	sr_{gi}	ru_{gi}
1	30.0	26,3	0	0	30	40	0	0
2	118.2	20	10	80	0	40	10	80
3	0	34.5	0	0	0	40	0	0
4	31.8	40	0	0	150	40	0	0
5	0	55.3	0	0	0	40	0	0

Table 3: Allocated levels of Pool generation and Reserve in (MW). Nodal prices in (\$/MWh).

BCM			PM		
Revenue	Payment	Costs	Revenue	Payment	Costs
6,025	10,298	5,736	8,800	8,800	8,100

Table 4: Pool and Reserve revenues, payments and costs(\$/h)

Table 4 shows the economic impact of the different allocated levels in the short term market. Under PM, total revenue is higher (46%), total payment is lower (14.6%) and total cost is higher (41.2%). Total profit of generators is 289 (\$/h) under BCM and 700 (\$/h) under PM. Other simulations considering the capacity increase of the transmission line 2-5 to 300 MW produce the same results in both approaches independent of the presence of the same size of bilateral contracts.

Tables 5 and 6 show results of the same system but in this case operating without bilateral contracts. The full load (600 MW with P_{d3}=200 and P_{d5}=400) is supplied only by pool generation and the system requires the same level of reserve services as before. Results show again transmission congestion in line 2-5 under BCM but not under PM. It can be observed in table 5 that the PM dispatch avoids transmission congestion in order to minimizing total payment of loads (at buses 3 and 5) and for this purpose the cheapest unit 1 is not participating in the energy market due to its position in the network. The system congestion revenues are therefore reduced in benefit of reducing load payments. The PM dispatch essentially reduces the bigger load 5 payment in order to minimizing the total load payment. Prices of reserve services are now MCP_{RU}=20 (\$/MWh) under BCM and MCP_{RU}=10 under PM; however, the reserve capacity is allocated in different units. Under PM generator 1 is in operation but only available for providing reserve capacity in the reserve market. Table 6 shows that under PM total revenue is higher (34%), total payment is lower (18.9%) and total cost is higher (81.7%). Total profit of generators is 7,114 (\$/h) under BCM and 4,100 (\$/h) under PM. By comparing table 3 with table 5, it can be observed the impact on the production levels of generators due to the bilateral contracts supplied by the cheap generator 1 in the first case.

	BCM				PM			
	p^p_{gi}	λ_i	sr_{gi}	ru_{gi}	p^p_{gi}	λ_i	sr_{gi}	ru_{gi}
1	450.0	26.3	0	0	0	40	10	80
2	45.3	20	10	80	210	40	0	0
3	0	34.4	0	0	0	40	0	0
4	104.7	40	0	0	390	40	0	0
5	0	55.3	0	0	0	40	0	0

Table 5: Allocated levels of Pool generation and Reserve in (MW). Nodal prices in ($/MWh).

BCM			PM		
Revenue	Payment	Costs	Revenue	Payment	Costs
18,507	30,594	11,393	24,800	24,800	20,700

Table 6: Pool and Reserve revenues, payments and costs($/h)

Additional observations were performed considering the previous two cases with each generator reserve bid cost for spinning reserve being the same as the corresponding energy bid, but with the reserve bid cost for regulation up being half of the corresponding energy bid (i.e., Bid^{ru}_{gi}=(0.5)Bid_{gi}). Results show that the substitution between reserve services is verified. The spinning reserve requirement is fully supplied by a cheap regulation up bid of a generator with availability of capacity in both approaches. In the first analog case (with bilateral contracts) generator 2 supplies the required reserve at a price of MCP_{RU}=10 ($/MWh) in both approaches. In the second analog case, under BCM the result is the same as the mentioned before. Under PM generator 1 supplies the full reserve requirement at a price of MCP_{RU}=5 ($/MWh). In both cases, total payment is lower under PM.

The third example considers the same transmission network topology as in the previous cases. Tables 7 and 8 show a case with different bid cost, demand, start up cost of units and transmission lines capacities.

Bus	Bid_{gi} ($/MWh)	p_{gi}^{min} (MW)	p_{gi}^{max} (MW)	P_{dj} (MW)	S_g ($)
1	10	60	800	0	-
2	15	15	210	0	1,500
3	-	-	-	60	-
4	30	20	280	340	-
5	45	10	200	600	-

Table 7: Generators Bid cost, Generators and load data

line	1 - 2	1 - 5	2 - 5	2 - 3	4 - 3	4 - 5
y_{ij}(S)	156.3	33.7	32.9	35.6	92.6	33.7
p_{ij}^{max}	500	400	300	290	400	400

Table 8: Transmission connections and maximum flow capacity of lines.

In this example total load of 1000 MW is 80% supplied by bilateral contracts (800MW) according to the following contracts: T_{13}=40, T_{14}=240 and T_{15}=520. The remaining 20% is supplied by pool generation (200 MW with P^p_{d3}=20, P^p_{d4}=100 and P^p_{d5}=80). The system is not requiring reserve services.

Table 9 shows results of the combined energy market. Since the full capacity of generator 1 is used for supplying bilateral contracts (see table 7), this generator is not participating in the pool dispatch in both approaches. Transmission congestion occurs under BCM in transmission line 2-3. However, under PM congestion is not verified and nodal prices are uniform and equal to 30$/MWh. Under PM only generator at bus 4 is dispatched with 200 MW different from the allocated pool energy levels under BCM. This dispatch avoids paying the start up cost of generator 2 reducing the total payment of the loads at buses 3, 4 and 5, and also avoiding transmission congestion.

Table 10 shows the economic impact of the different allocated levels. Under PM total revenue is higher (16.4%), total payment is lower (11.8%) and total cost is higher (16.4%). Total pool profit of generators is nil in both approaches. Loads pay more under BCM to compensate the start- up cost of the committed unit 2. Other examples like in the IEEE 30-bus system considered in next example and in [1] show that payments under BCM are always higher than in PM when there are units with high bid prices but low startup costs competing with units with low bid prices and high startup costs.

	BCM		PM	
	p^p_{gi}	λ_i	p^p_{gi}	λ_i
1	0	15.95	0	30
2	156.37	15	0	30
3	0	33.50	0	30
4	43.64	30	200	30
5	0	20.36	0	30

Table 9: Generation levels (MW) and nodal prices ($/MWh)

BCM			PM		
Revenue	Payment	Costs	Revenue	Payment	Costs
5154.8	6798.8	5154.8	6000	6000	6000

Table 10: Pool and Reserve revenues, payments and costs($/h)

8.2 IEEE 30-Bus System

The last two cases are related to the IEEE 30-Bus system [5] with a total load of 532.4 MW. The system is requiring the following levels of reserve services: R^{RU}= 80 MW and R^{SR}= 10 MW. Half of the total load (266MW) is supplied through the following bilateral contracts of generators in MW: P^b_{g1}=25, P^b_{g2}=25, P^b_{g8}=41.2, P^b_{g11}=45, P^b_{g13}=95, P^b_{g15}=25, P^b_{g30}=10. Table 11 shows generators' energy bid data and capacity. Generators minimum capacity levels are considered 0 MW. Each generator reserve bid for spinning reserve is the same as the corresponding energy bid and the reserve bid for regulation up is half the corresponding energy bid (i.e., Bid^{ru}_{gi}=(0.5)Bid_{gi}).

Bus	$Bid_{g\,i}$ \$/MWh	S_g \$	$p_{g\,i}^{max}$ MW	Bus	$Bid_{g\,i}$ \$/MWh	S_g (\$)	$p_{g\,i}^{max}$ MW
1	15	-	100	13	30	-	330
2	27	-	80	15	27	30	60
5	19.5	2000	100	24	60	-	60
8	19.5	-	200	30	75	-	20
11	57	-	120	-	-	-	-

Table 11: IEEE 30 Bus system. Generators Energy Bids and capacity data.

The first case consists in a normal operation without contingencies. Results show that there is no transmission congestion under both approaches. The pool dispatch under BCM has a uniform nodal price of 57 $/MWh with the following pool dispatch in MW: p^p_{g1}=75, p^p_{g8}=158.8, p^p_{g11}=22.4 and p^p_{g13}=10. These generators do not have start up costs but generator 11 has one of the highest bid costs (57 $/MWh). Under BCM, both the spinning reserve and regulation up requirements are fully supplied by the available lower regulation up reserve bids of generator 2 (with 55MW) and generator 13 (with 35MW). The reserve clearing price is MCP_{RU}=15 ($/MWh). The cheap generators 1 and 8 have their full capacity allocated to the bilateral and pool energy market. On the other hand, the pool dispatch under PM has a lower uniform price of 19.5 $/MWh and the following pool dispatch in MW: p^p_{g1}=75, p^p_{g5}=32.4, p^p_{g8}=158.8. Generator 5 has a start up cost of 2000 ($). Under PM, both reserve requirements are supplied by the accepted regulation up reserve bid of generator 2 (22.4 MW) and generator 5 (67.6 MW). The reserve market clearing price is lower: MCP_{RU}=13.5 ($/MWh). The cheap generator 1 capacity is fully used for supplying contracts and pool market. The available units with low reserve bid are units 2 and 5, in spite of the fact that unit 5 has a high start up cost. Table 11 shows a summary of total economic results obtained with both approaches. As can be observed, if compared with the BCM approach, under PM total revenue is lower (49.1%), total payment is lower (49.12%) and total cost is higher (10.6%). Under BCM the total profit of generators is 9457 ($/h) and under PM is 591 ($/h), a reduction of 93.7%.

BCM ($/h)			PM ($/h)		
Revenue	Payment	Costs	Revenue	Payment	Costs
16,523	16,523	7,066	8,406	8,406	7,815

Table 11: IEEE 30 Bus system with 50% bilateral. Total pool revenues, payments and costs without contingency.

The second case observed in this system considers an unexpected contingency in line 14 connecting bus 9 to bus 10. The capacity of this line is reduced from 70MW to 54MW. The remaining operating conditions are the same as in the previous case. Under BCM, results show transmission congestion during this contingency with nodal prices assuming values between 30 and 66.4 ($/MWh) and the following pool dispatch in MW: p^p_{g2}=55, p^p_{g8}=158.8, p^p_{g13}=10, p^p_{g15}=23 and p^p_{g24}=19. This last unit has one of the higher bid costs and unit 15 has a start up cost of 30 ($). The cheap unit 1 is not used for supplying the pool demand. Under BCM, both the spinning reserve and regulation up requirements are fully supplied by the available lower regulation up reserve bids of generator 1 (with 75MW) and generator 13 (with 15 MW). The reserve clearing price is MCP_{RU}=15 ($/MWh). The full capacity of the cheap unit 1 is used in the bilateral and reserve markets. Because unit 5 is not committed the reserve requirements need to be supplied also with the not cheap unit 13. On the other hand, results show that transmission congestion does not occur in the pool dispatch under PM. It is obtained a lower and uniform price of 19.5 $/MWh and the following pool dispatch in MW: p^p_{g1}=75, p^p_{g5}=53.44, p^p_{g8}=137.76. Generator 5 has the highest start up cost of 2000 ($). The capacity of the cheap unit 1 is fully used in the energy market (25MW in contracts and 75 MW in pool dispatch). Under PM, both reserve requirements are supplied by the accepted regulation up reserve bid of available units: generator 2 (55 MW) and generator 5 (46 MW). The clearing price in the reserve market is MCP_{RU}=13.5 ($/MWh) which is lower than the one obtained under BCM. Table 12 shows a summary of total economic results obtained with both approaches. As can be observed, if compared with the BCM approach, under PM total revenue is lower (48.5%), total payment is lower (48.5%) and total cost is higher (7.7%). Under BCM the total profit of generators is 8331 ($/h) and under PM is 512 ($/h), a reduction of 93.8%.

BCM ($/h)			PM ($/h)		
Revenue	Payment	Costs	Revenue	Payment	Costs
15,808	16,614	7,477	8,562	8,562	8,050

Table 12: IEEE 30 Bus system with 50% bilateral. Total pool revenues, payments and costs with contingency.

Since the main objective of this study is to compare the BCM and PM approaches through the bilevel model, results with only one load period of time are considered enough here. The extension of this model considering a multi-period horizon creates an optimization problem of larger size but still conserving the characteristic of a mixed linear optimization problem.

9 CONCLUSIONS

In this paper is presented a model of a combined market of energy (pool and bilateral contracts) and reserve (with ancillary reserve services) where the short term market, involving pool energy and reserve services, is cleared through a payment minimization auction. The suggested formulation of the model results in a bilevel linear integer optimization problem which has several important characteristics such as considering the transmission network flows; the transmission and generation capacities; the possibility of substitution among reserve services; start up costs and the inter-temporal ramp constraints of generation units. The model allows obtaining prices of energy and services reflecting the natural intrinsic interaction between services when they are considered in a co-optimized process. If assuming that agents in both BCM and PM auctions follow the same bid strategy, results show that total payments under PM are always equal or lower than under BCM. If there is no transmission congestion and without startup costs, both approaches produce the same results. The presence of physical firm bilateral contracts can produce or increase the possibility of transmission congestion due to their size and/or distribution in the transmission network. When transmission congestion occurs, results show that the load payment (including energy and reserve) under PM is lower than under BCM. The same result is obtained when units with low bid cost and high start up cost participate in the energy and reserve market. The importance in developing PM auctions for combined energy markets without using extremely complex models is one of the main concerns of market participants. This importance is more evident if the market participant´s bid costs do not fully reflect production costs. Evidence of this fact has been reported in real systems. Moreover, due to the fact that the suggested bilevel linear mixed integer approach can be solved like a regular mixed binary linear problem, its use is appealing to be used in large systems considering the current efficient standard solvers for this type of optimization problems.

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