CONSTRAINING TRANSMISSION/SUBTRANSMISSION REACTIVE POWER FLOWS IN OPF STUDIES

Ángel L. Trigo #1, José L. Martínez #2, Jesús Riquelme #3
Departamento de Ingeniería Eléctrica
Sevilla, Spain
[1] trigoal@us.es; [2]jlmr@esi.us.es; [3] jsantos@us.es

Abstract – In power systems where the transmission and distribution activities are performed by different agents, it is common to include constraints on the reactive power flows in the elements connecting the two systems, normally Transmission/Distribution or Transmission/Subtransmission transformers. This requirement involves the need of introducing new constraints in the mathematical tools used to assist the system operation, traditionally an OPF. The aim of the paper is to study the introduction of a new constraint in the power system operation planning process. Reactive power flow constraints are added to an existing OPF which is later solved using interior-point techniques. This new constraint considers the reactive power flow through the transmission and sub-transmission boundary transformers.

Keywords: Ancillary services; power system operation; reactive power flows.

1 INTRODUCTION

The optimal power flow (OPF) problem has been one of the most widely studied subjects in the power system community since Carpentier first published the concept in 1962 [1]. The objective of this tool is to get an optimal solution for a specific power system objective function. Control variables are determined by the OPF so that a set of operational or physical constraints are satisfied.

Several optimization techniques have been proposed to solve the OPF problem including the gradient method [2], linear programming method [3], [4], the Newton method [5], [6], and the sequential quadratic programming method [7], [8]. Specially adapted to the characteristics of power system optimization problems are the Interior-Point methods [9]. In [10], [11], LP-based interior-point methods were proposed. In [12], an interior-point method was proposed for linear and convex QP and was used to solve power system optimization problems such as the economic dispatch and reactive power planning. Besides, several nonlinear primal-dual interior-point methods for power optimization problems have been proposed in the literature. It is generally accepted that these nonlinear primal-dual methods can efficiently solve nonlinear power system optimization problems [13], [14].

Deregulation and privatization have changed the way power systems are being operated nowadays and pose new challenges to the System Operator (SO) to satisfy the new constraints generated in this process. The OPF, as one of the main mathematical tools used for this purpose, must be adapted to contemplate these new requirements.

A critical task for the system operation is the provision of reactive power and voltage control. This later is especially important to assure a secure system operation, to decrease active power losses, to keep the system capability to deal with perturbations and even to avoid a voltage collapse. In deregulated systems, the reactive power management is a System Operator duty.

The reactive power provision must be satisfied in a local way due to the shortcomings associated to its transport over the system. Moreover, the transport of high amount of reactive power through the lines decreases their capacity to deliver active power. This justifies the necessity to regulate the activities related to the reactive power generation, transmission and consumption. Every power system establishes some limitations to the generation/consumption of reactive power for the generators/loads connected to it, normally expressed in terms of power factor. In deregulated systems, these limitations affect also the subtransmission network managers or distribution system operators, forcing them to fulfil these constraints in the boundary points with the rest of the system. This fact makes them to act such as consumers connected to the transmission network [15].

In this context, the introduction of the new constraints in the already complex power system operation problem becomes necessary. Particularly important is the case of the transmission and sub-transmission boundary transformers, as both are meshed networks.

In this article an optimization tool to assist the power system operation is presented. In this tool the model of the limits on the reactive flows stated before are included. These limits are included as constraints in an OPF which is later solved using interior point techniques.

The structure of the paper is as follows. In paragraph II the problem formulation is presented, detailing the optimization problem to be solved. The objective function and constraints limiting the feasible area are commented here, as well as the variables used to define them. Later, the results obtained in a practical experience with a 24-bus network are presented. The conclusions obtained from the pursued work close this article.

2 PROBLEM FORMULATION

Assuming that the active power generation has been scheduled according to electric market, it is possible to minimize transmission power losses by properly adjusting, within permissible limits, the control parameters at the System Operator's disposal, that is, generator voltage magnitudes, transformer taps and switchable VAR sources. In this process, allowable bounds on certain dependent variables, like load bus voltages, generator reactive powers and, in the particular case studied in this work, the transmission-subtransmission boundary transformers reactive power flows, should be enforced as well.

The following optimization problem is proposed:

$$\begin{aligned} &\min f(X) \\ &s.t. \\ &\qquad h(X) = 0 \\ &\qquad d(X) = 0 \\ &\qquad V^{\min} \le V \le V^{\max} \\ &\qquad T^{\min} \le T \le T^{\max} \\ &\qquad Q^{\min} \le Q \le Q^{\max} \\ &\qquad 0 \le S^2 \le S^{\max} \\ &\qquad K^{\min} \le TAN \le K^{\max} \end{aligned} \quad (1)$$

where:

$X = [\theta, V, T, Q, P_{ij}, Q_{ij}, S^2, TAN]^t$: Probelm variables

$f(X)$: Active power losses (Objective function)

$h(X)$: Network equations

$d(X)$: Definition equations

V: Voltages

T: OLTC taps

Q: reactive power injections

S^2: Tie-lines apparent power flow

TAN: ratios defined as reactive power flow over active power flow in boundary transformers

The last constraint, $K^{\min} \le TAN \le K^{\max}$, is forced in transmission-subtransmission boundary transformers. Usually, the limits on the reactive power flow are established in form of power factor or a percentage of the active power flow. The introduction of this constraint as a power factor should be followed of the inductive or capacitive character, making its formulation more complicated. If the limitation is established as a percentage of the active power flow non-constant limits will appear. The implemented solution in this work has been using of a ratio, defined as reactive power flow over active power flow, as a variable solves this issue. This variable is called $\tan\phi$ in each transformer.

Every generator operates at a constant active power (this situation will not change, since active power generated is not a variable of the problem). So considered reactive power limits are within the range of the generator capability curve for that active power generated.

2.1 Objective function

The objective function to be minimized represents the networks active power losses.

$$f(X) = \sum_{\forall branches} -G_{ij}(V_i^2 + V_j^2 - 2V_iV_j \cos\theta_{ij}) \quad (2)$$

As this function is highly non-linear, it is frequent in the current literature using of linear approximations [16], however the proposed method do not use any approximation, assuming the complete model expressed in (1).

Notice that the transformer taps are included in the correspondent elements of the admittance matrix.

2.2 Network equations

$$\begin{aligned} \Delta P_i &= \sum_{j \ne i} V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) + G_{ii}V_i^2 - P_i = 0 \\ \Delta Q_i &= \sum_{j \ne i} V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) - B_{ii}V_i^2 - Q_i = 0 \end{aligned} \quad (3)$$

where,

$$\begin{aligned} P_i &= P_i^{gen} - P_i^{cons} \\ Q_i &= Q_i^{gen} - Q_i^{cons} \end{aligned} \quad (4)$$

Note that the slack active power remains a problem variable, the same occurs with Q_i^{gen} in the reactive power injection buses.

2.3 Definition equations

It is necessary to define some problem variables to incorporate these ones into the problem. In these equations the exiting relation with the others problem variables is defined.

Active and reactive power flows

$$\begin{aligned} \Delta P_{ij} &= -G_{ij}V_i^2 + V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) - P_{ij} \\ \Delta Q_{ij} &= B_{ii}V_i^2 + V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) - Q_{ij} \end{aligned} \quad (5)$$

Boundary transformer tangent

$$\Delta\tan\phi_{ij} = \tan\phi_{ij} - \frac{Q_{ij}}{P_{ij}} \quad (6)$$

Tie-lines apparent power flow

$$\Delta S_{ij}^2 = P_{ij}^2 + Q_{ij}^2 - S_{ij}^2 \quad (7)$$

3 APPLICATION OF INTERIOR-POINT TECHNIQUES TO THE PROPOSED PROBLEM

The proposed optimization model is solved using interior-point techniques. So, the inequality constraints are converted into equalities ones introducing positive slack variables. The positivity of these variables is assured penalizing the objective function through weighted logarithmic terms, being the penalization factor which tends to zero as the algorithm converges to an optimum.

Thus, the new objective function is:

$$f^*(X, S^M, S^m, \mu) = f(X) - \mu\sum_i(\ln s_i^M + \ln s_i^m) \quad (8)$$

where S^M and S^m are the upper and lower slack vectors, respectively.

Making the aforementioned changes, the problem formulation becomes as stated in (9).

$$\begin{aligned}
&\min f(X) - \mu \sum_i (\ln s_i^M + \ln s_i^m) \\
&s.a. \\
&\quad h(X) = 0 \\
&\quad d(X) = 0 \\
&\quad S_V^M + V - V^{\max} = 0 \\
&\quad S_V^m - V + V^{\min} = 0 \\
&\quad S_t^M + T - T^{\max} = 0 \\
&\quad S_t^m - T + T^{\min} = 0 \\
&\quad S_q^M + Q - Q^{\max} = 0 \\
&\quad S_q^m - Q + Q^{\min} = 0 \\
&\quad S_{S^2}^M + S^2 - S^{\max} = 0 \\
&\quad S_{S^2}^m - S^2 + S^{\min} = 0 \\
&\quad S_{\tan}^M + TAN - K^{\max} = 0 \\
&\quad S_{\tan}^m - TAN + K^{\min} = 0
\end{aligned} \tag{9}$$

where:

- $X = [\theta, V, t, q, P_{ij}, Q_{ij}, S^2, \tan\varphi, s_{X_i}^M, s_{X_i}^m]^t$: Primal variables.
- $G(x) = \begin{bmatrix} h(x) \\ d(x) \end{bmatrix} = 0$: Equality constraints.
- $S^M = [S_V^M, S_t^M, S_q^M, S_{S^2}^M, S_{\tan}^M]$ and $S^m = [S_V^m, S_t^m, S_q^m, S_{S^2}^m, S_{\tan}^m]$: Slack variables. Two slack vectors are needed, one for upper limits and other one for lower limits.

The solution of (9) ensures that all constraints are satisfied obtaining a system feasible working point that minimized the objective function. As the problem has only equality constraints, the Lagrange multipliers method can be used for solving it.

3.1 Lagrangian function

The proposed method consists of building the Lagrangian function associated to (9). Its expression is shown in (10).

$$\begin{aligned}
\mathcal{L} = {} & f(x) - \mu \sum_i (\ln s_i^M + \ln s_i^m) - \sum_{i \neq slack} y_{P_i} \Delta P_i - \sum_i y_{Q_i} \Delta Q_i - \\
& - \sum_{branches} y_{P_{ij}} \Delta P_{ij} - \sum_{branches} y_{Q_{ij}} \Delta Q_{ij} - \sum_{\substack{boundary \\ tranformers}} y_{\tan_{ij}} \Delta \tan\varphi_{ij} - \\
& - \sum_{branches} y_{S_{ij}^2} \Delta S_{ij}^2 + \sum_{X_k \in X_{\lim}} \lambda_k^M (s_k^M + X_k - X_k^{\max}) + \lambda_k^m (s_k^m - X_k + X_k^{\min})
\end{aligned} \tag{10}$$

Where $X_{\lim} = [V, T, Q, S^2, TAN]^t$ and $Y = [y_{P_i}, y_{Q_i}, y_{P_{ij}}, y_{Q_{ij}}, y_{\tan_{ij}}, y_{S_{ij}^2}, \lambda^M, \lambda^m]^t$ are dual variables. These variables are the Lagrange multipliers associated to the system constraints.

3.2 Optimality conditions

The optimality conditions are obtained by differentiating the Lagrangian function for each of the primal variables, and for each of the dual variables. The optimality conditions are not included for clarity.

3.3 Solving the Optimality conditions

The Primal-Dual Interior-Point method consists of iteratively solving the optimality equations reducing at the same time the penalization factor, μ. The optimality equations are solved using the Newton's method due to their non-linear characteristic. The first order derivatives of the Lagrangian function are expressed as $\mathcal{L}|_{x_i}$ and the second order are written as $\mathcal{L}_{x,x}$. The equations that are solved iteratively are shown in (11)-(28).

4 APPLICATION OF INTERIOR-POINT TECHNIQUES TO THE PROPOSED PROBLEM

4.1 IEEE 24-bus system

In this paragraph the results obtained applying the detailed methodology to a 24-bus system are described. The Figure 1 shows the system used for this aim, which corresponds to the "IEEE RTS-96". The cases studied are generated according to the data contained in [17]. Two situations are evaluated:

- Case 1. OPF without introducing the constraint corresponding to the boundary transformers tangent.
- Case 2. OPF contemplating the introduction of the previous constraint.

The main interest of this work remains in the assessment of the power flows through the boundary point between transmission and sub-transmission systems. For the sake of clarity, the studied cases do not take into account the tie-lines overloads.

	Ploss (W)	Decrement (%)
initial	46.51	--
Case 1	42.29	9.08
Case 2	45.10	3.02

Table 1: IEEE 24- Bus System Active Power Losses

Initially, the active power losses, objective of the optimization, are studied. Table I shows the results obtained for each case.

$$
\begin{array}{c}
\begin{array}{c|cccccccccccccccccc|}
 & \theta & V & t & q & P_{ij} & Q_{ij} & S^2 & \tan\varphi & y_{P_i} & y_{Q_i} & y_{P_{ij}} & y_{Q_{ij}} & y_{S_{ij}^2} & y_{\tan_{ij}} & s^M & s^m & \lambda^M & \lambda^m \\
\hline
\theta & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & 0 \\
V & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 1 & -1 \\
t & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 1 & -1 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
P_{ij} & 0 & 0 & 0 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 1 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 \\
Q_{ij} & 0 & 0 & 0 & 0 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & 1 & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 \\
S^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\
\tan\varphi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & -1 \\
y_{P_i} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_{Q_i} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_{P_{ij}} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_{Q_{ij}} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & \mathcal{L}_{x,x} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_{S_{ij}^2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_{\tan_{ij}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
s^M & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda^M & 0 & s^M & 0 \\
s^m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda^m & 0 & s^m \\
\lambda^M & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\lambda^m & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}
\end{array}
\begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta t \\ \Delta q \\ \Delta P_{ij} \\ \Delta Q_{ij} \\ \Delta S^2 \\ \Delta\tan\varphi \\ \Delta y_{P_i} \\ \Delta y_{Q_i} \\ \Delta y_{P_{ij}} \\ \Delta y_{Q_{ij}} \\ \Delta y_{S_{ij}^2} \\ \Delta y_{\tan_{ij}} \\ \Delta s^M \\ \Delta s^m \\ \Delta\lambda^M \\ \Delta\lambda^m \end{bmatrix}
=
\begin{bmatrix} -\mathcal{L}|_{\theta} \\ -\mathcal{L}|_{V} \\ -\mathcal{L}|_{t} \\ -\mathcal{L}|_{q} \\ -\mathcal{L}|_{P_{ij}} \\ -\mathcal{L}|_{Q_{ij}} \\ -\mathcal{L}|_{S_{ij}^2} \\ -\mathcal{L}|_{\tan\varphi_{ij}} \\ -\mathcal{L}|_{y_{P_i}} \\ -\mathcal{L}|_{y_{Q_i}} \\ -\mathcal{L}|_{y_{P_{ij}}} \\ -\mathcal{L}|_{y_{Q_{ij}}} \\ -\mathcal{L}|_{y_{S_{ij}^2}} \\ -\mathcal{L}|_{y_{\tan\varphi_{ij}}} \\ -\mathcal{L}|_{s^M} \\ -\mathcal{L}|_{s^m} \\ -\mathcal{L}|_{\lambda^M} \\ -\mathcal{L}|_{\lambda^m} \end{bmatrix}
\begin{array}{c} (11) \\ (12) \\ (13) \\ (14) \\ (15) \\ (16) \\ (17) \\ (18) \\ (19) \\ (20) \\ (21) \\ (22) \\ (23) \\ (24) \\ (25) \\ (26) \\ (27) \\ (28) \end{array}
$$

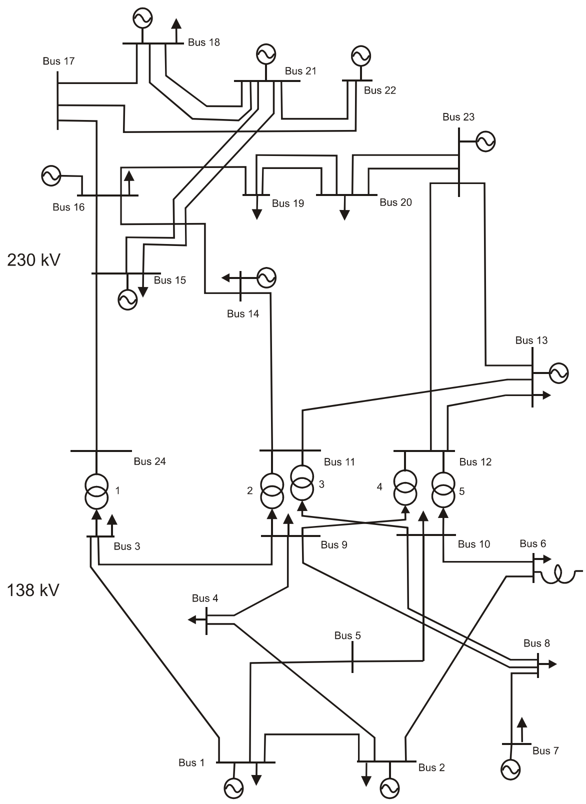

Figure 1: IEEE 24-Bus test system.

As it can be observed in Table I, when the optimization is applied without taking into account the tangent restriction (case 1) the losses are lower that when forcing that restriction (case 2). This is because forcing that restriction involves a more stressed situation. However, still an important reduction can be obtained, of about 3%, in respect to the initial case.

One of the main variables to be kept inside limits is the buses voltages. Figure 2 shows the differences obtained among the three different situations studied. The limits considered are 1.05 p.u. for the upper limit, and 0.95 p.u. for the lower one. It can be observer that all voltages remain inside the specified limits.

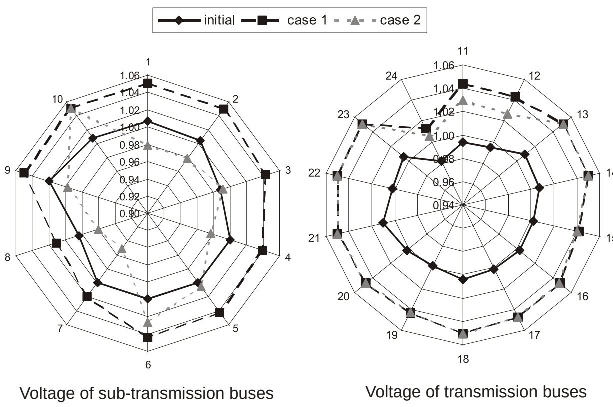

Figure 2: Bus voltages comparison.

From the system operation point of view, it is interesting to know the control variables setting that ensure all constraints fulfilment. Figure 3 compares the voltage setting of the generating units for each studied case.

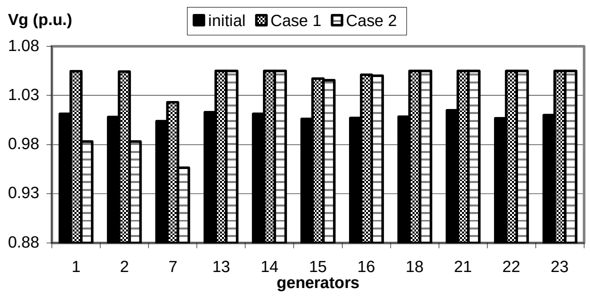

Figure 3: Generator voltage set-point.

Others control variables are the transformer taps. In this work these are considered continuous variables, in case the discrete characteristic should be considered, they should be adjusted later to the closest value. Figure 4 shows the values that these controls must take to reach the objective in each case.

The susceptance connected to bus 6 remains stable on all three cases.

An important dependent variable to be considered is the generated reactive power of generating units.

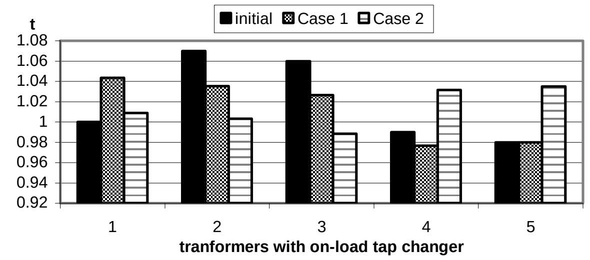

Figure 4: Transformer tap set-point.

Figure 5 indicates the value of these variables on each case studied. It can be noticed that in some cases this variable reaches the upper limit. This represents a problem for future evolutions of the system, since the limited units will not be able to contribute to the reactive support. This problem could be overcome in several ways. A possible alternative is the establishment of an iterative process, establishing an initial reactive power capacity lower than the actual one that ensures a future reactive margin reserve [18].

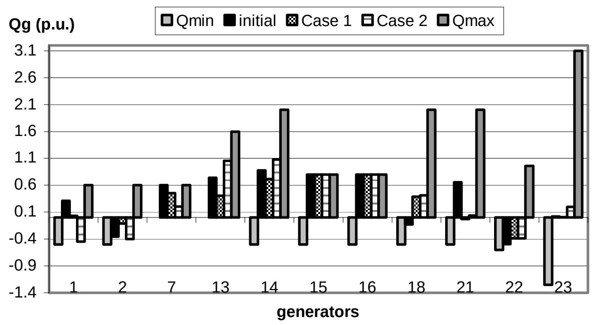

Figure 5: Generated reactive power comparison.

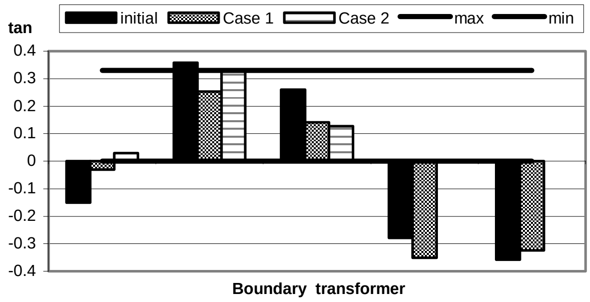

Figure 6: Comparison of tangent value in Boundary transformer.

The Figures 6, 7 and 8 show the magnitudes associated to the transmission and sub-transmission boundary transformers where the reactive power flow towards the sub-transmission system is limited. The limits considered are the most demanding according to the time period of the day. The upper tangent limit is 0.33 and the lower one is 0.0.

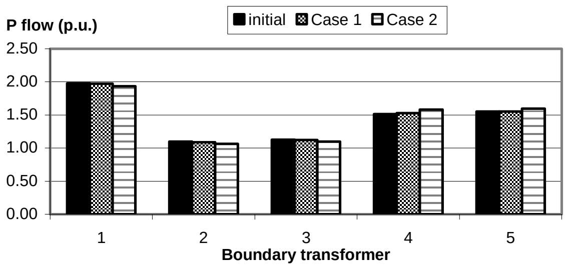

Figure 7: Active power flow through boundary transformers.

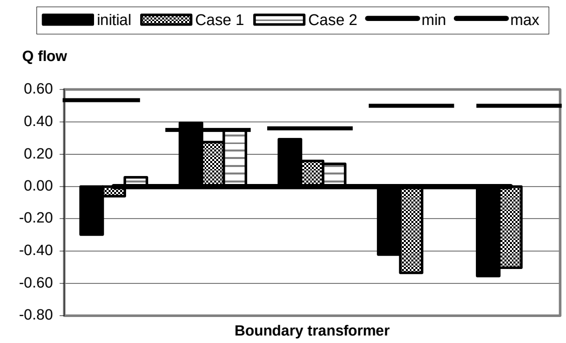

Figure 8: Reactive power flow through boundary transformers.

In Figure 6 the tangent values obtained for each optimization strategy are compared. It can be observed that only in the case in which the tangent restriction is explicitly included in the model this parameter remains inside the established limits. This remarks the importance of the inclusion of the proposed constraint when performing the network planning.

As aforementioned, the transformers active power flows remain stable when applying the different strategies (Figure 7). The contrary occurs with the reactive power flows, which adapt their values to comply with the tangent restriction (Figure 8).

To sum up, it can be stated that including the proposed constraint, the limits on the reactive power flow in boundary transformers can be fulfilled. But, on the other side, forcing this restriction involves a lower improvement in the system's active power losses.

5 CONCLUSIONS

Several countries legislation establishes some kind of limit over the reactive power flow through the transmission and sub-transmission transformers. This fact involves the necessity of introducing some kind of constraint in the mathematical tools used to assist the system operation, traditionally an OPF.

The obtained results convey that the introduction of this new constraint in the OPF formulation makes its solution quite more complicated but, at the same time, guarantee the fulfilment of the limits over the reactive power flows in the boundary transformers.

A logic consequence, shown by the objective function results, is that the fulfilment of this new constraint causes higher system power losses.

The classical constraints are included, apart from the tie-lines overloads, expressed as power terms, and the reactive power flow towards the sub-transmission.

ACKNOWLEDMENT

The authors would like to acknowledge the financial support provided by the Spanish Ministry of Science and Innovation, the Ministry of Foreign Affairs and Cooperation and Junta de Andalucía, under grants ENE 2010-18867, ENE 2007-63306, ENE 2007-66072, A/030124/10, and TEP-5170.

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