

CdTe THIN-FILM SOLAR MODULE MODELING USING A NON-LINEAR REGRESSION APPROACH

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Abstract - This paper describes the application of a Non-Linear Regression technique to model CdTe Thin-Film Solar Modules. From the proposed approach, relevant non-linear relations between environmental and electrical variables are given, minimizing the number of fitting parameters in comparison with previous approaches.

The proposed non-linear model can be also applied to check the range of electrical power generation, according to the expected datasheet values provided by the manufacturers. This fact avoids the necessity of removing and analyzing the solar modules in a laboratory environment. A comparison with previous approaches is included in the present paper as well, discussing the relevant advantages of the proposed technique with respect to published models.

Keywords - CdTe Thin-Film Solar Modules, Non-linear Regression

1 INTRODUCTION

WITH the decreasing of traditional fossil energy sources, combined with pollutant emission reduction agreements, most of developed countries have established special feed in tariff systems and tax reductions for grid-connected Photovoltaic (PV) plants. This electric generation thus has the prospective to play an important role towards a sustainable energy system in the 21st century [1]. Indeed, in 2010 large scale PV plants with cumulative power more than 3 GWp (preliminary data) were connected to the grid. Market leader was Germany with more than 1 GWp new power capacity installed closely followed by Italy and the Czech Republic, [2].

During the last years, the international market of both thin-film and organic PV modules has been increasing considerably [3], mainly due to their simple and low-cost manufacturing process. In addition, physical structure as well as composition of thin-film modules—different from crystalline modules—offer relevant properties and a proper behavior with regard to Temperature (T), Solar Irradiation (G) and partial shadow parameters. In fact, both PV technologies are quite different from the effect of operating temperature on the device output performance. Specifically, and depending on the environmental conditions, thin-film cells can achieve a performance similar or even higher than crystalline modules [4] [5]. Under real situations, the production processes in terms of through-

put and yield can differ significantly and may offset the advantage of a higher performance [6].

Regarding the increase in the production capacity of PV modules and advances in power electronics, it is promoting a continuous cost decreasing of PV kWp [7]. Indeed, and taking into account the expected increase of the energy price, grid parity cost can be reached in a short-term horizon. In this scenario, PV Plant site location should be studied in depth, since their economic efficiency highly depends on the meteorological conditions and the performance of the selected PV modules. It is then necessary to use tools that allows us to accurately simulate the behavior of the system inverter-module and select the technology maximizing the future PV plant power generation, according to the ranges of environmental conditions— $G&T$ —. In the specific literature, most studies about the characterization of $I - V$ curves describe the matching of mathematical models for crystalline silicon modules. However, there is a lack of information about models applied to other technologies. In this way, the main contribution of this paper is to propose an alternative and practical non-linear model to estimate the electrical behavior of CdTe thin-film modules. The proposed model is derived from the data-sheet provided by the manufacturers, and the number of fitting parameters is considerably lower in comparison with previous contributions. From data collected in field test, the actual performance of the modules can be checked by the proposed model. This information will be used in future works, such as preventive maintenance analysis or comparison of efficiency levels.

The organization of this paper is as follows. Section 2 roughly describes PV module model contributions previously published in the specific literature. The proposed non-linear regression technique is discussed in Section 3. Finally, we give the conclusions of the research in Section 5.

2 PHOTOVOLTAIC MODULE MODELING

Mainly, two different topologies have been proposed along the last years to model the $I - V$ curve of PV solar cells as a function of $G&T$ environmental parameters: Single Diode Model and Double Diode Model.

2.1 Single Diode Model (SDM)

It is based on the modified Shockley–diode equation, [8], and includes a diode quality factor to consider the effect of recombination in the space-charge region —see Fig. 1—. The photo current generated by the illuminated cell is modeled by an ideal current source, considering both shunt and series effects by means of two resistances, [8–10]. Mathematically, the assumption of a single exponential for the dark saturation current is described as follows.

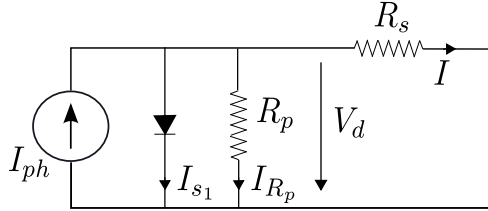


Figure 1: Single Diode Model Scheme

$$I = I_{ph} - I_{s1} \cdot \left(e^{\frac{q \cdot V_d}{k \cdot T \cdot A}} - 1 \right) - \frac{V_d}{R_p} \quad (1)$$

$$I_{ph} = K_0 \cdot G \cdot (1 + K_1 (T - 298)) \quad (2)$$

$$V_d = V + I \cdot R_s, \quad (3)$$

where:

- I_{ph} Photo-current
- I_{s1} Dark Saturation Current
- V_d Diode voltage
- R_s Series resistance
- R_p Parallel resistance
- A Diode ideality factor
- q Boltzman constant
- e Electron charge
- T Absolute Temperature

This expression involves several parameters according to the specific solar cell modeled. Unfortunately, full information to fit these parameters is not often available, and they have to be adjusted by iterative process whose convergence strongly depends on the starting points. In [10], a SDM–Seven parameters suitable for on-line $G&T$ model is proposed by Sera et al. Other authors propose to reduce the model complexity with the aim of improving its convergence and stability. In [9], several effects are neglected, proposing implicit expressions for I_{ph} in a SDM–Five parameters.

2.2 Double Diode Model (DDM)

This is another approach commonly referenced, [11] [12] [13], where the space-charge recombination effect is simulated by incorporating an additional current component with its own exponential voltage dependence, see Fig. 2.

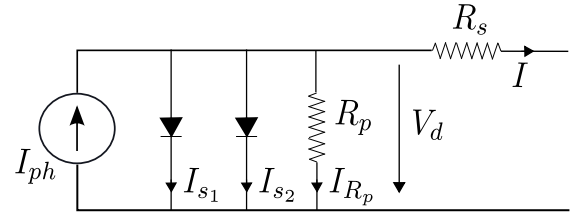


Figure 2: Double Diode Model Scheme

$$I = I_{ph} - I_{s1} \cdot \left(e^{\frac{q \cdot V_d}{k \cdot T}} - 1 \right) - I_{s2} \cdot \left(e^{\frac{q \cdot V_d}{k \cdot A \cdot T}} - 1 \right) - \frac{V_d}{R_p} \quad (4)$$

This approach is derived from the physics of the $p-n$ junction and is usually accepted to simulate the behavior of such cells, specially for polycrystalline silicon [14]. Manning and Gow proposed in [13] a cell DDM–Twelve parameters considering current diodes and resistances dependent on $G&T$. Other authors propose a simplified DDM by reducing the parameter dependences on $G&T$ environmental conditions. In [12] analytical expressions are found, where an iterative process is only required to assess the model parameters, reducing considerably non-convergence problems.

DDM is usually considered a more accurate estimation of solar cell performance, [11, 14]; particularly under low solar irradiance conditions, where the SDM series resistance values can not be appropriately fitted [15]. However, for crystalline silicon cells, various authors suggest that SDM equation represents more accurately the $I-V$ relation and its variations with usual operating $G&T$ parameters [10] [9].

3 PROPOSED NON-LINEAR REGRESSION MODEL

Both SDM and DDM approaches are based on assumptions that may not always be justified [12]. As an example, the current flowing through the solar cells is considered as a linear superposition of two currents: a current due to the junction bias and another one due to illumination. Additionally, and taking into account that thin-film solar cells are topologically different from crystalline cells, assumptions carried out about these model approaches should be reviewed before applying on thin-film solar cells, [16]. In this paper, a completely different route is taken, since it does not rely on a physical model for the quantities of interest, but is based on a fit of the curves provided by the manufacturer. As will be seen, the quality of the fit is good and a major advantage of the proposed model is the small number of unknown parameters to be estimated. The building block of our modeling process are the Gompertz curves, which are popular in growth modeling.

3.1 A symmetrized Gompertz curve

The Gompertz curve is a three-parameters curve described by the following equation:

$$y = k e^{-e^{-bx}}, \quad (5)$$

where k , a and b are essentially positive quantities. The associated function $y(x)$ is increasing and approaches k as x becomes infinite. There is a point of inflexion when $x = a/b$ and the corresponding ordinate is k/e . An example of Gompertz curve is the solid line curve in Figure 3.

The profile of dependence of I with respect to V is decreasing and close to the symmetric of a Gompertz curve with respect to the vertical line $x = a/b$, which is a point of inflexion. The current I hits the Ox axis obliquely at $V = V_{oc}$. We therefore decide to use as a model for I versus V a symmetrized Gompertz curve for V between 0 and a/b which is moreover shifted so that for $V = a/b$, $I = 0$. This is illustrated in Figure 3 where a Gompertz curve is plotted with $a = 12$, $b = 6$ and $k = 2$ (solid line) together with the vertical line at $x = a/b$, the symmetric curve and the new Ox axis.

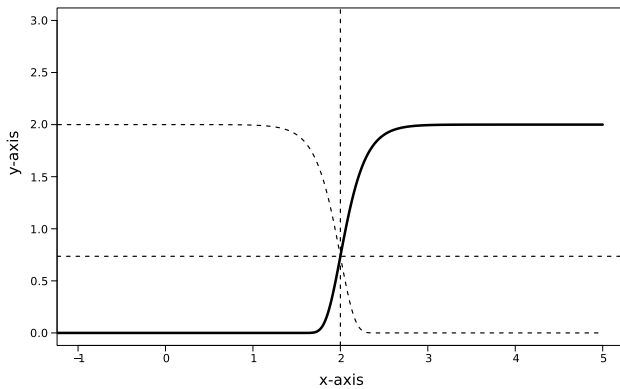


Figure 3: A Gompertz curve (solid line) together its symmetric and the change of Ox axis. The parameters were set to $a = 12$, $b = 6$ and $k = 2$.

The suggested three-parameters model for I versus V is therefore:

$$I = ke^{-e^{-a+bV}} - k/e. \quad (6)$$

It is of course relevant to relate the three parameters a , b and k to the natural physical quantities for the phenomenon under study. On one hand, k is clearly a scaling parameter. Moreover

$$I_{sc} = ke^{-e^{-a}} \quad (7)$$

$$V_{oc} = a/b. \quad (8)$$

Furthermore the slope of the curve when it hits $I = 0$, i.e. at V_{oc} is $-bk/e$. The parameter b mainly determines the rate of decay of I when V approaches V_{oc} . Let us introduce the parameter $\gamma = a/b$ for a even more natural parametrization of our model and finally consider

$$I = ke^{-e^{b(-\gamma+V)}} - k/e, \quad 0 \leq V \leq \gamma. \quad (9)$$

It then holds

$$I_{sc} = ke^{-e^{-\gamma}} \quad (10)$$

$$V_{oc} = \gamma. \quad (11)$$

Relation (9) constitutes a parsimonious model for the dependence of I on V and its parameters relate naturally to relevant physical quantities.

Moreover the estimation of γ , b and k is easily performed using for example the non-linear least squares fit function `nls` in R, [17], from a set of points in the curve. Typically the manufacturer provides I_{sc} , V_{oc} and the pair of maximum power (V_{MPP}, I_{MPP}) , which therefore correspond to three points on the curve. These three points moreover span the whole interval for V and therefore allow a precise estimation of the involved parameters. To avoid numerical instabilities in `nls`, we need at least an extra point. In practice, it was checked that it is enough to add an artificial point linearly interpolated between (V_{MPP}, I_{MPP}) and $(V_{oc}, 0)$, but much closer to the latter.

4 EXPERIMENTAL RESULTS

The proposed non-linear model has been fitted from data-sheets provided by the manufactures. In our case, FS-270 module series from FirstSolar [18] has been selected to show the validity of the model. Using the points of open circuit voltage, short circuit current and maximum power data provided by the manufacture, the parameters of our non-linear model has been fitted.

Fig. 4 shows the adjustment results for different $I - V$ performance estimations considering the module temperature as constant —current and voltage values are relative to standard test conditions (STC): $1000W/m^2$ and $25^\circ C$ —. In addition, Fig. 5 shows the results when solar irradiation (G) is assumed as constant. As can be seen from these figures, the proposed non-linear model based on Gompertz curve approach offer a proper adjustment of the CdTe thin film module electric behavior, using significantly less parameters than previous models: both SDM and DDM proposals.

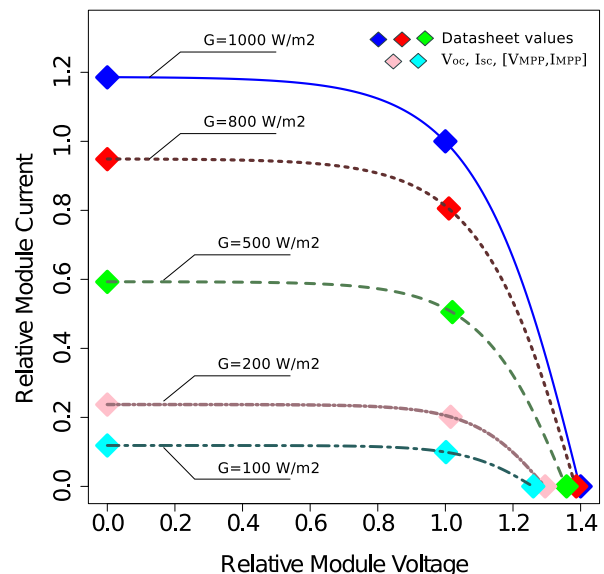


Figure 4: Non-linear proposed model: results ($T = 25^\circ C$)

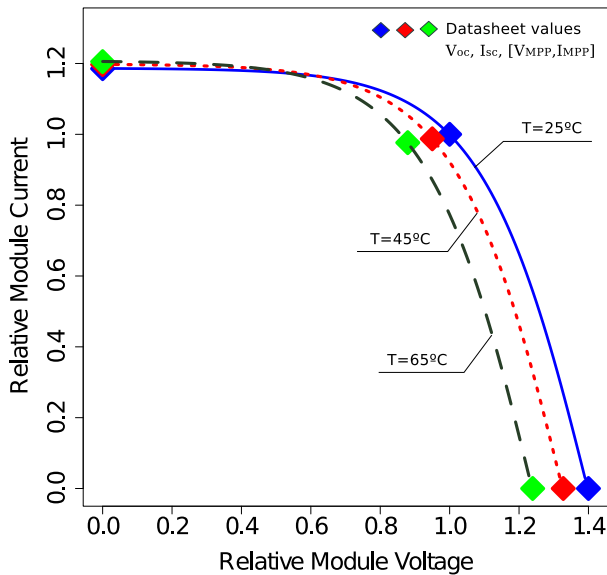


Figure 5: Non-linear proposed model: results ($G = 1000\text{W}/\text{m}^2$)

Table 1 summarizes the values of the parameters for the corresponding $I - V$ examples as well as the estimated error (%) with respect to the MPP value.

G (W/m^2)	T ($^{\circ}\text{C}$)	k	γ	b	MPP Error (%)
100	25	0.19	1.26	8.63	-1.18
200	25	0.38	1.29	8.28	-0.57
500	25	0.94	1.36	6.88	0.03
800	25	1.50	1.38	6.11	0.11
1000	25	1.88	1.40	5.66	0.12
100	45	0.19	1.18	8.59	-1.42
200	45	0.38	1.22	8.26	-0.71
500	45	0.95	1.27	6.98	-0.24
800	45	1.52	1.30	6.23	-0.02
1000	45	1.90	1.33	5.68	0.04
100	65	0.19	1.09	8.43	-1.36
200	65	0.38	1.13	8.06	-1.14
500	65	0.95	1.18	6.88	-0.56
800	65	1.53	1.22	6.16	-0.15
1000	65	1.91	1.24	5.71	-0.01

Table 1: Non-linear model: Results

G (W/m^2)	T ($^{\circ}\text{C}$)	MPP value Error (%)		
		Proposed Model	SDM7	DDM9
800	25	0.11	-3.4	-0.34
1000	25	0.12	5.6	-6.84
800	45	-0.02	-2.3	0.79
1000	45	0.04	4.3	-8.64
800	65	-0.15	-3.1	0.93
1000	65	-0.01	4.2	-9.23

Table 2: Non-linear model: comparison with SDM and DDM approaches

A comparison with SDM and DDM approaches is also included in the present paper taking into account a previous contribution of the authors, [19], where both SDM and DDM examples were applied on CdTe thin film modules —see Appendix I for further information—. As an example of these results, Fig. 6 shows the obtained

$I - V$ curves when a seven-parameters Single Diode Model (SDM7) is implemented.

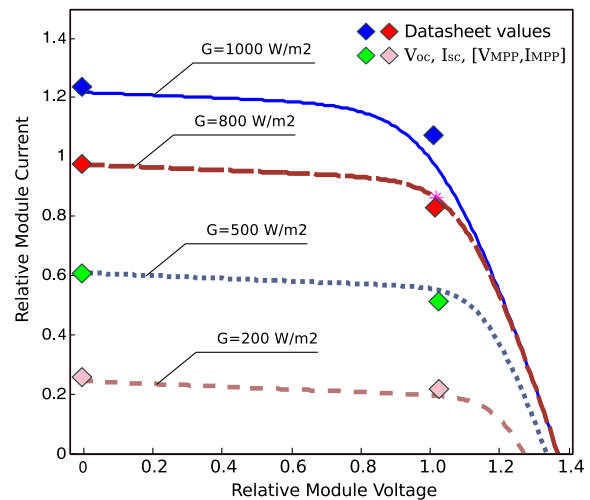


Figure 6: Seven-parameters Single Diode Model (SDM7): results ($T = 25^{\circ}\text{C}$)

The errors (%) at the MPP value is then compared considering seven-parameters SDM (SDM7) and nine-parameters DDM (DDM9) proposals —see Table 2—. In all cases, the estimated error is considerably lower by using the proposed non-linear model, with a remarkable less number of fitting parameters. The current solution is thus suitable to estimate $I - V$ the electrical behavior of CdTe thin film modules under different G & T environmental conditions.

5 CONCLUSIONS

This paper presents and assesses the application of a Non-Linear model for the electrical behavior of CdTe Thin-Film solar modules. The proposed approach is based on a symmetrized and shifted Gompertz curve, it only depends on three parameters which relate naturally to the relevant physical quantities like I_{sc} or V_{oc} .

The model estimates $I - V$ relation according to the G & T environmental conditions, minimizing the number of fitting parameters in comparison with previous approaches. It is checked that the estimation procedure is easy and the agreement of the fitted model with data provided by the manufacturer is very good. Several examples are included in the paper from real data collected in Spanish PV plants. From the results, the model validity to determine the expected electrical behavior of CdTe Thin-Film solar modules is assessed. As an additional property, the proposed non-linear model can be also applied to check the range of electrical power generation, according to the expected datasheet values provided by the manufacturers. This fact avoids the necessity of removing and analyzing the solar modules in a laboratory environment. This efficiency information can be used in future works, such as preventive maintenance analysis or comparison of efficiency levels, which are currently topics of interest of the authors.

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APPENDIX I

I.1. Seven-parameters Diode Single Model (SDM7) Equations

The datasheet for SDM7 should include the following information in STC: I_{sc} , V_{oc} , V_{MPP} , I_{MPP} , tkI_{sc} , I_{sc} dependence on T coefficient; and tkV_{oc} , V_{oc} dependence on T coefficient. Considering then eq. (1),

$$\left(e^{\frac{q \cdot V_d}{k \cdot T \cdot A}} - 1 \right) \approx e^{\frac{q \cdot V_d}{k \cdot T \cdot A}} \quad (12)$$

$$\frac{dP}{dV} = I_{MPP} + \frac{V_{MPP} \frac{\partial I(V,I)}{\partial V}}{1 - \frac{\partial I(V,I)}{\partial I}} \quad (13)$$

From these assumptions, the non-linear system can be expressed as:

$$\begin{aligned} IMPP = & I_{sc} - \frac{V_{MPP} + I_{MPP} R_s - I_{sc} R_s}{R_p} - \\ & - \left(I_{sc} - \frac{V_{oc} - I_{sc} R_s}{R_p} \right) e^{\frac{V_{MPP} + I_{MPP} R_s - V_{oc}}{ns \cdot vt}} \\ \frac{dP}{dV} I = I_{MPP} = & I_{MPP} + \\ + V_{MPP} \frac{-\frac{I_{sc} R_p - V_{oc} + I_{sc} R_s}{ns \cdot vt} \cdot e^{\frac{V_{MPP} + I_{MPP} R_s - V_{oc}}{ns \cdot vt}} - 1}{R_p + \frac{I_{sc} R_p - V_{oc} + I_{sc} R_s}{ns \cdot vt} \cdot e^{\frac{V_{MPP} + I_{MPP} R_s - V_{oc}}{ns \cdot vt}} + \frac{1}{R_s}} \\ -1 = & \frac{-\frac{I_{sc} R_p - V_{oc} + I_{sc} R_s}{ns \cdot vt} \cdot e^{\frac{I_{sc} R_s - V_{oc}}{ns \cdot vt}} - 1}{R_p + \frac{I_{sc} R_p - V_{oc} + I_{sc} R_s}{ns \cdot vt} \cdot e^{\frac{I_{sc} R_s - V_{oc}}{ns \cdot vt}} + \frac{1}{R_s}} \end{aligned}$$

Since these expressions can not be solved analytically, numerical methods have to be used. As is described in [10], vt is assessed from (14) and R_p from (14). Considering R_s , R_p and A as constant and their dependence with $G&T$ is neglected, the following step is focused on determining the I_{ph} and I_{s1} dependence with the $G&T$ values.

$$I_{s1} = \left(I_{sc} - \frac{V_{oc} - I_{sc} R_s}{R_p} \right) e^{-\frac{V_{oc}}{ns \cdot vt}} \quad (14)$$

$$I_{ph} = I_{sc} e^{\frac{V_{oc}}{ns \cdot vt}} + \frac{V_{oc}}{R_p} \quad (15)$$

$$I_{scG} = K_4 G \quad (16)$$

$$I_{phG} = K_5 G \quad (17)$$

$$V_{ocG} = \ln \left(\frac{K_4 G R_p - V_{ocG}}{I_s R_p} \right) ns \cdot vt \quad (18)$$

The V_{oc} dependence with G is not analytically defined, and it must be calculated by applying the Newton-Raphson method from the expression (18).

$$V_{ocT} = V_{ocG} (1 + tk V_{oc} (T - T_{stc})) \quad (19)$$

$$I_{scT} = I_{scG} (1 + tk I_{sc} (T - T_{stc})) \quad (20)$$

$$I_{s1T} = \left(I_{scT} - \frac{V_{ocT} - I_{scT} R_s}{R_p} \right) e^{-\frac{V_{ocT}}{ns \cdot vt}} \quad (21)$$

$$I_{phT} = I_{s1T} e^{\frac{V_{ocT}}{ns \cdot vt} + \frac{V_{ocT}}{R_p}} \quad (22)$$

$$\begin{aligned} I_{s1T} = & [I_{scG} (1 + tk I_{sc} (T - T_{stc})) - \\ & - \frac{V_{ocG} (1 + tk V_{oc} (T - T_{stc}))}{R_p} + \\ & + \frac{I_{scG} (1 + tk I_{sc} (T - T_{stc})) R_s}{R_p}] \cdot \\ & \cdot e^{-\frac{V_{ocG} (1 + tk V_{oc} (T - T_{stc}))}{ns \cdot vt}} \end{aligned} \quad (23)$$

$$I_{phT} = I_{scG} (1 + tk I_{sc} (T - T_{stc})) \cdot e^{\frac{V_{ocG} (1 + tk V_{oc} (T - T_{stc}))}{ns \cdot vt} + \frac{V_{ocG} (1 + tk V_{oc} (T - T_{stc}))}{R_p}} \quad (24)$$

In our case, Table 3 shows the parameters obtained for the SDM7 approach.

Table 3: SDM7: parameter values

Parameter	Value	
R_s	3.4948	Ω
R_p	494.64	Ω
A	1.7781	-
K_5	0.0012	m^2/V
$I_{s1} (stc)$	$6.46 \cdot 10^{-8}$	A
$tk V_{oc}$	-0.0025	%
$tk I_{sc}$	0.0004	%

I.II. Nine-parameters Double Diode Model (DDM9) Equations

$$\frac{-1}{R_{s0}} = \frac{dI}{dV} \Big|_{V=V_{oc}} = 0.03 \frac{I_{MPP}}{V_{oc} - V_{MPP}} \quad (25)$$

$$\frac{-1}{R_{p0}} = \frac{dI}{dV} \Big|_{V=0} = 400 \frac{I_{sc} - I_{MPP}}{V_{MPP}} \quad (26)$$

From these expressions, the DDM9 parameters can be analytically evaluated:

$$\begin{aligned} I = & -\frac{V}{R_{p0}} - \frac{V_{oc}}{R_{p0}} + 2 vt e^{\frac{V-V_{oc}}{vt}} e^{\frac{I R_s}{vt}} (R_{s0} - R_s)^{-1} - \\ & - 2 \left(I_{sc} - \frac{V_{oc}}{R_{p0}} - \frac{vt}{R_{s0} - R_s} \right) e^{\frac{V-V_{oc}}{vt}} e^{1/2 \frac{I R_s}{vt}} \end{aligned} \quad (27)$$

$$e^{x R_s} \approx 1 + x R_s + 1/2 x^2 R_s^2 \quad (28)$$

$$I_{s1} = \left(\frac{V_{oc}}{R_{p0}} - I_{sc} + 2 \frac{vt}{R_{s0} - R_s} \right) e^{-\frac{V_{oc}}{vt}} \quad (29)$$

$$I_{s2} = 2 \left(I_{sc} - \frac{V_{oc}}{R_{p0}} - \frac{vt}{R_{s0} - R_s} \right) e^{-1/2 \frac{V_{oc}}{vt}} \quad (30)$$

$$\begin{aligned} R_p = & ((R_{p0} - R_s)^{-1} - I_{s1} e^{\frac{I_{sc} R_s}{vt}} vt^{-1} - \\ & - \frac{1}{2} I_{s2} e^{1/2 \frac{I_{sc} R_s}{vt}} vt^{-1})^{-1} \end{aligned} \quad (31)$$

$$I_{ph} = I_{s1} \left(e^{\frac{V_{oc}}{vt}} - 1 \right) - I_{s2} \left(e^{1/2 \frac{V_{oc}}{vt}} - 1 \right) + \frac{V_{oc}}{R_p} \quad (32)$$

Estimated parameters for this DDM9 model are shown in Table 4

Table 4: DDM9: parameter values

Parameter	Value	
R_s	14.285	Ω
R_p	1.3739e+003	Ω
A	2	-
$I_{s1} (stc)$	$7.0928 \cdot 10^{-14}$	A
$I_{s2} (stc)$	$2.6437 \cdot 10^{-7}$	A
R_{s0}	17.19	Ω
R_{p0}	1386	Ω
$tk I_{sc}$	0.0004	%
$tk V_{oc}$	-0.0025	%