CAPACITOR PLACEMENT IN RADIAL DISTRIBUTION NETWORKS THROUGH A LINEAR DETERMINISTIC OPTIMIZATION MODEL

Roberto S. Aguiar, Pablo Cuervo
Electric Engineering Department, University of Brasilia
Brasilia, Brazil
robertosfa@bol.com.br, pablo@ene.unb.br

Abstract – The classical optimization problem for reducing investment costs and energy loss in radial distribution systems is addressed through a strategy of linear approximations and new variable representation. The linear approximation of the problem is performed in two main aspects: one is by using Support Hipper-planes (SH) allowing the approximation of the loss function in each branch of the distribution system. The other is related to the treatment given to the product of integer variables and continuous variables. The classical problem is then transformed into a new mixed linear integer optimization problem, easy to solve by traditional large scale robust solvers. Rigid upper and lower limits to the voltage variables, instead of using penalty functions to prevent violation of limits, allow evaluating voltage profile improvement against corresponding cost savings. Examples on several distribution networks and planning periods show the robustness and performance indexes which indicate this method as an appealing alternative to utilities interested in planning radial distribution networks.

Keywords: Capacitor placement, Support Hipper-planes, Linear Approximation Process, Mixed Linear Optimization.

1 NOMENCLATURE

Parameters:

nb	number of system buses
k_e^l	energy costs for load level l [\$/MWh]
k_c	Fixed Cost of a capacitor bank unit [\$/unit]
T^l	Number of hours of load level l in a planning horizon period of T hours
r_i	Series resistance in branch i [Ω]
x_i	Series reactance in branch i [Ω]
$p_{L_i}^l$	Active power load at bus i at load level l
$q_{L_i}^l$	Reactive power load at bus i at load level l
$b_{c_i}^{sh}$	Nominal Capacitor Susceptance installed at bus i [S]
nh	Number of Support Hypper-planes
$V_i^{\min}$	Minimum voltage level at bus i [kV]
$V_i^{\max}$	Maximum voltage level at bus i [kV]

Variables:

V_i	Bus voltage at bus i [kV]
P_i	Active power flow supplied from bus i
Q_i	Reactive power flow supplied from bus i
Q_{ci}	Reactive power injected by Capacitors installed at bus i [MVar]
u_i	Integer variable representing the number of capacitor units installed at bus i

2 INTRODUCTION

The main purpose of radial distribution systems is to attend customers in a reliable and good quality manner reducing investment costs and energy losses. These objectives can be achieved in some degree by correctly installing capacitor banks, and lately by allowing installing Distributed Generation (DG) owned by demand supplier companies and/or independent producers [3]. The first initiative is addressed in this paper by establishing a new procedure that further will be the basis for considering the DG sizing and placement planning problem.

The purpose of capacitor placement planning is to define capacitor types, sizes, locations and control schemes for a period that can vary from one to ten years looking for minimizing investment and energy loss costs. Unfortunately, the solution of this problem is difficult to find because of the combinatorial and multimodal characteristics coming from the classic formulation of the problem. This mathematical formulation characterizes a mixed integer nonlinear program [1,3,5,7]. In general, this kind of optimization problems is hard to solve and even does not have a guaranteed solution by conventional optimization tools. Historically, this problem was first treated in an approximately manner through the use of mathematical programming tools such as Dynamic Programming [3] and Benders Decomposition [7]. Some authors have worked the problem using Heuristic [3] or Meta Heuristic approaches like Tabu Search [1,5], Genetic Algorithms [4,6,10], and Simulated Annealing [3]. More recently, it has been suggested the use of hybrid methods for obtaining sub-optimal solutions, like in [1,3]. The difficulty in these methods is the calibration of complex and arbitrary parameters that could depend on operation conditions and distribution systems characteristics.

The suggested methodology here is based on the classical formulation of the problem, but creates variables in a different space of possible solutions with the advantage of not necessarily creating the usual multi-modal functions difficult to be handled by optimization solvers. This is possible thanks to the use of important results obtained from a loss linearization theorem [8] and also due to the introduction of equivalent additional linear constraints to reproduce the non-linear behavior introduced by the existence of products of integer and continuous variables in the original classical formulation [9]. These two main linear approximations allow formulating a mixed linear integer optimization model for the capacitor placement planning problem, which can be solved by robust available solvers for large scale optimization problems. The main contribution of the proposed approach is the possibility of solving the capacitor placement planning problem by a complete deterministic procedure, without the need to employ random optimization methods, using very few load flow simulations and avoiding calibration of complex and arbitrary parameters. Other advantages are: the ease use because it is based on load flow simulation cases, the possibility of taking advantage of historical post-operative data, the possibility of considering rigid upper and lower limits to the voltage variables, instead of using penalty functions to prevent the violation of limits, the possibility of considering different topologies, load levels, capacitor types and sizes and locations.

The paper is organized as follows: in section 3 is presented the formulation of the classical model; in section 4 is described the linear approximation approach background; in section 5 are presented models involving fixed size and switched capacitor types; in section 6, the binary linear optimization problem formulation is presented. Sections 7 and 8 show numerical examples and performance indexes analysis. Finally in section 9, several conclusions are presented.

3 CLASSIC MIXED NON-LINEAR FORMULATION

The general formulation of the capacitor placement problem aims the reduction of energy loss and investment costs, over a period of time that can vary from one to ten years, involves several aspects like capacitor types, sizes, locations and control schemes. From the operational point of view, the main concern is to keep voltage levels in an adequate range, even when loads change. We re-write here the classical non-linear formulation problem presented in [1,2,3,5,7]. Without lost of generality and for the sake of simplicity, the formulation presented in equations (1) to (7) corresponds to a distribution network composed of just one main feeder and just one load level. Buses and corresponding down stream branches have the same index i (i=1,…,nb). They are numerated in an increasing order starting from the first bus connected to a substation.

$$Min \quad f = \sum_{i=1}^{nb} C_{inv\ i}(Q_{c\,i}) + \sum_{i=1}^{nb} k_e . T . P_{loss\,i} \tag{1}$$

s.a.

$$P_{i+1} = P_i - r_i . \frac{P_i^2 + Q_i^2}{V_i^2} - P_{L\,i+1} \tag{2}$$

$$Q_{i+1} = Q_i - x_i . \frac{P_i^2 + Q_i^2}{V_i^2} - Q_{L\,i+1} + Q_{c\,i+1} \tag{3}$$

$$V_{i+1}^2 = V_i^2 - 2(\, r_i \ P_i + x_i \ Q_i \,) + \frac{(r_i^2 + x_i^2)(P_i^2 + Q_i^2)}{V_i^2} \tag{4}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{5}$$

$$Q_{ci} = u_i . b_{ci}^{sh} . V_i^2 \tag{6}$$

$$u_i \in \{0,1,2,3,4\} \tag{7}$$

In this model, active and reactive powers, voltage levels, location and sizes of capacitor units are variables. Load levels, voltage limits, and line parameters are considered known. The objective function in (1) has two components: The first term is related with total investment cost for installing capacitors of any type. This cost function is discrete and increases with the number of units installed [1,3,5,7]; the second term is related with the total energy loss cost during the planning period, T. This function is non-linear due to the behavior of transmission losses. Constraints (2), (3) and (4) are the load flow equations specialized for radial distribution networks [1,5,7]. Inequalities in constraints (5) represent adequate voltage ranges in each bus. The square current module in constraints (2) to (4) is defined as shown in equation (8).

$$\mathrm{I}^2{}_i = \frac{(P_i^2 + Q_i^2)}{V_i^2} \tag{8}$$

This equation represents a non-linear relationship among variables P_i, Q_i e V_i and is used to calculate corresponding active and reactive power losses in branch i. Constraint (6), that is part of (3), represents the reactive injection if capacitor units are installed. The level of reactive injection depends on the number of banks installed (integer variable u) in each candidate bus. The reactive injection, Q_{ci} , is a non-linear term since it is the product of continuous variable V_i^2 and integer variable u_i, both of them unknown during the planning period. The capacitive susceptance parameter, b_{ci}^{sh} , can be considered fixed and obtained from standard nominal voltage and nominal reactive power. The number of equations and variables increases proportionally when several load levels are considered. Also, if more complex topologies are considered, several extra indexes are needed as well as boundary conditions related with power balances in bifurcations of feeders. The optimization problem described before characterizes a mixed non-linear program extremely difficult to solve by standard optimization algorithms.

4 BACKGROUND OF LINEAR APPROXIMATIONS

In this section are presented the fundamentals of linear approximations adopted.

4.1.1 *Loss linear approximation*

A graphical interpretation of the loss linear approximation theorem is given in figure 1 (demonstration is available in [8] but not presented here because of space limitations).

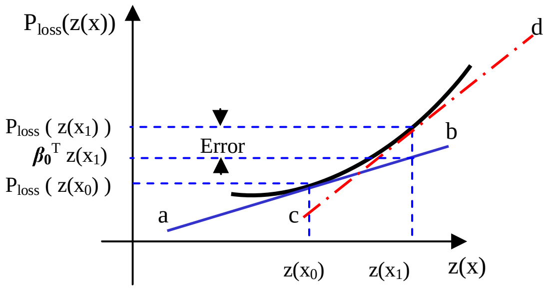

Figure 1: Linear approximation of losses in a distribution branch

In this figure, the dark nonlinear curve represents actual active power losses in a particular branch. Let $\mathbf{z_0} = \mathbf{z(x_0)}$ be the unique vector of injections corresponding to the operating point $\mathbf{x_0}$ in branch i. In other words, $\mathbf{z}$ can be considered as $\mathbf{z} = [\, \mathbf{V_i^2}\ \mathbf{V_{i+1}^2}\ \mathbf{P_i} \,]^T$. Associated with this operating point, the actual active power loss in branch i is $P_{Loss}(\mathbf{z}(\mathbf{x}_0))$. One of the results of the theorem indicates that there exists a Support Hipperplane (SH), $\beta_0^T \mathbf{z}(\mathbf{x}_0)$, at $\mathbf{x_0}$ such that $P_{Loss}(\mathbf{z}(\mathbf{x}_0)) = \beta_0^T \mathbf{z}(\mathbf{x}_0)$ (where β_0^T is a vector of constant coefficients characterizing the SH slope, and it is symbolized by segment a-b in figure 1.). This point corresponds to the tangency point ($P_{loss}(\mathbf{z}(\mathbf{x_0}))$, $\mathbf{z}(\mathbf{x_0})$). Other result of the theorem is that in any other operating point like, $\mathbf{z}(\mathbf{x_1})$, the relationship between the loss function and the SH is $P_{loss}(\mathbf{z}(\mathbf{x}_1)) > \beta_0^T \mathbf{z}(\mathbf{x}_1)$. As it can be observed in figure 1, the first SH will introduce inaccuracy to represent actual losses $P_{loss}(\mathbf{z}(\mathbf{x_1}))$ at $\mathbf{z}(\mathbf{x_1})$. Then, for a better linear approximation of losses at the operating point $\mathbf{z}(\mathbf{x_1})$, it is necessary to use one extra SH symbolized in figure 1 as segment c-d. According with the theorem, several SH´s can be generated to represent more accurately the loss function in branch i for different operating points.

Hence, the active losses in branch i can be approximated linearly by a set of SHs (k = 1, …, nh) as follows :

$$P_{losses\ i} \geq \boldsymbol{\beta}_{ki}{}^T \mathbf{Z}_i \tag{9}$$

This linear approximation based on SH allows introducing the following definitions of variables in the original optimization problem:

$$P_{losses\ i} \overset{\Delta}{=} \frac{r_i(P_i^2 + Q_i^2)}{V_i^2} \quad ; \quad Q_{losses\ i} = P_{losses\ i} \cdot \frac{x_i}{r_i} \tag{10}$$

4.1.2 *Expressing the Product of a Binary and a Continuous Variable as a Finite Number of Linear Expressions*

First, the square voltage module is considered as a variable rather than the voltage module in the optimization problem; that is, by definition, $VS_i = V_i^2$. Second, based on the theory of expressing the product of a binary and continuous variable as a finite number of linear expressions [9], we consider the simple case where only one fixed size capacitor bank can be installed in one candidate bus (a more general case is presented later). The non-linear constraint (6) can be treated as linear by introducing the following definition and inequalities:

a) The introduction of variable $\sigma_i = VS_i \cdot u_i$, where $u_i \in \{0,1\}$, results in the following linear expression in terms of σ_i,

$$Q_{ci} = \sigma_i . b_{ci}^{sh}. \tag{11}$$

b) Simultaneously, the following set of inequalities is introduced:

$$VS_i - VS_i^{\max}(1 - u_i) \leq \sigma_i \leq VS_i - VS_i^{\min}(1 - u_i) \tag{12}$$

$$u_i \cdot VS_i^{\min} \leq \quad \sigma_i \quad \leq u_i \cdot VS_i^{\max} \tag{13}$$

Equations (11) – (13) work as follows: a) when u_i =0, there is no capacitor installed at bus i, then Q_{ci}=0 in eq.(11), because equation (13) forces $\sigma_i = 0$. Also, the corresponding bus voltage is forced to be inside its adequate range because of equation (12). b) When u_i =1, there is a capacitor bank installed at bus i, then $Q_{ci} = \sigma_i . b_{ci}^{sh}$. (MVar) because $\sigma_i = VS_i$ in constraint (12) and the corresponding voltage is again forced to be inside its adequate range, this time because of constraint (13).

5 MODEL INCLUDING CAPACITOR TYPES

The previous formulation can be extended to consider two types of capacitors: fixed size and switched capacitors. We focus attention on the objective function and linear constraints (11) – (13), for the purpose of modeling capacitor types in the linear approximation approach, other constraints as power flow equations and linear approximation of losses remain the same.

5.1 *Fixed Size Capacitors*

In this case, the decision of installing fixed size capacitor banks considers different load levels, l, and respect voltage limits. To illustrate the model, the linear constraint (11), with corresponding additional inequalities, (12) and (13), are handled for considering the possibility of installing up to four fixed size capacitors banks in the same candidate location.

In this case, reactive power injected by capacitor banks at bus i is $Qf_{\ i} = u_i \cdot VS_i^l \cdot b_c^{sh}$, where $u_i \in \{0,1,2,3,4\}$ and b_c^{sh} is the nominal susceptance of the standard module size of a capacitor bank. Since corresponding additional inequalities (12) and (13) only work with binary variables, the following extension of the definition of variable σ_i is necessary.

$$\sigma_i \overset{\Delta}{=} \sum_{j=1}^{4} \sigma_{i\,j} \;,\quad \sigma_{i\,j} = u_{i_j} \cdot VS_i^l.,\quad u_{i_j} \in \{0,1\} \qquad (14)$$

In order to avoid combination of binary variables providing the same integer number, the following additional set of linear binary constraints is necessary.

$$\begin{aligned} &.u_{i\,j=1} \le u_{i\,j=2} \;,\quad u_{i\,j=3} \le u_{i\,j=4} \\ &u_{i\,j=3} + u_{i\,j=4} \le u_{i\,j=1} + u_{i\,j=2} \qquad (15) \\ &(u_{i\,j=1} + u_{i\,j=2}\;) - (u_{i\,j=3} + u_{i\,j=4}) \le 1 \end{aligned}$$

These binary constraints can be represented in a more compact way as shown in equation (16).

$$bc(u_{i_j}) \le 0 \qquad (16)$$

In addition, each variable $\sigma_{i\,j}$ should work together with a set of constraints of the type (12) and (13). Therefore, with σ_i defined by equation (14), the reactive power injection at bus *i*, due to the installation of fixed size capacitors banks, has the following linear expression in terms of σ_i,

$$Qf_{\;i} \;=\; \sigma_i \cdot b_c^{sh} \;. \qquad (17)$$

Equation (14) can be rewritten as

$$\sigma_i \overset{\Delta}{=} \sum_{j=1}^{4} \sigma_{i\,j} = \sum_{j=1}^{4} u_{i\,j} VS_i^l. = VS_i^l.\sum_{j=1}^{4} u_{i\,j} = VS_i^l.u_i \;. \qquad (18)$$

Equation (17) and (18) should work with additional constraints of type (12) and (13) as shown in the following constraints:

$$VS_i^l - VS_i^{l\max}(1-u_{i_j}) \le \sigma_{i_j} \le VS_i^l - VS_i^{l\min}(1-u_{i_j}) \qquad (19)$$

$$u_{i_j} VS_i^{l\min} \le \sigma_{i_j} \le u_{i_j} VS_i^{l\max} \qquad (20)$$

Constraints (17) – (20) represent the linear approximation of constraint (6) for fixed size capacitors.

Defining k_f as the cost of investment per unit of capacity (\$/module of 300 kVar) and considering *l* loads levels, as well as the previous definitions of u_i in (18), the linear binary objective function in the optimization problem is

$$Min \quad k_f \sum_{i=1}^{nb} u_i + \sum_{l} \;\sum_{i=1}^{nb} k_e^l T^l P^l_{loss\;\;i} \;. \qquad (21)$$

Note that this formulation can be extended for the case of installing more than four fixed capacitor banks at the same location. However, in real distribution networks installing more than four banks in the same bus is most of the times unpractical.

5.2 Switched Capacitors

In this case, the reactive power injection of capacitor banks installed can vary according with the need of the system to attend variations of load. The model in this case adopts the same structure as in the previous case, but the definition of variables change in the following way. For each load level *l*, and bus i, a equation similar to (17) is considered, as shown in equation (22),

$$Qs_i^l \;=\; \delta_i^l \cdot b_c^{sh} \;. \qquad (22)$$

Where,

$$\delta_i^l \overset{\Delta}{=} \sum_{j=1}^{4} \delta_{i\,j}^l \;,\quad \delta_{i\,j}^l = v_{i\,j}^l \cdot VS_i^l.,\quad v_{i\,j} \in \{0,1\} \qquad (23)$$

As in the previous case, additional constraints of type (12) and (13) are necessary, as well as binary linear constraints for each set of binary bus variables δ_i^l as follows:

$$VS_i^l - VS_i^{\max}(1-v_{i\,j}^l) \le \delta_{i\,j}^l \le VS_i^l - VS_i^{\min}(1-v_{i\,j}^l) \qquad (24)$$

$$v_{i\,j}^l VS_i^{\min} \le \quad \delta_{i\,j}^l \quad \le v_{i\,j}^l VS_i^{\max} \qquad (25)$$

$$bc(v_{i_j}^l) \le 0 \qquad (26)$$

It is considered here that switched capacitor size changes in equal discrete steps (the size of these steps depends on the fabricant and in this model it is possible to consider the installation of capacitors from different fabricants). The cost of these capacitors depends on the total available capacity of each installed unit, as in the case of fixed size capacitors. The difference when a switched capacitor bank is installed is that the corresponding reactive injection can vary with the load variation in order to improve voltage profile and losses reduction. Defining investment cost per capacity of a bank module as k_v (\$/module of 300 kVar). The objective function in this case is as follows,

$$Min \quad k_v \sum_{i=1}^{nb} \rho_i + \sum_{i=1}^{nb} k_e^l \cdot T^l \cdot P^l_{loss\;i} \;. \qquad (27)$$

Where the total available capacitor capacity in each bus i is represented by ρ_i, which is calculated by the following additional constraints:

$$\rho_i \ge \sum_{j=1}^{4} v_{i\,j}^l \;. \qquad (28)$$

As well as in the fixed size case, the extension for allowing the installation of more than four switched capacitors is straightforward, but we consider unnecessary for practical reasons.

6 MIXED BINARY LINEAR MODEL

The optimization mixed binary linear optimization program (MBLOP), obtained after the introduction of linear approximations, takes the following form when both fixed and switched capacitors types are allowed to be installed in candidate bus. For simplicity, but without loss of generality, the maximum number of capacitor banks that can be installed at the same bus is four. The rule for branch index notation is that a branch connected to bus *i* adopts the same index of bus *i*, if bus *i* is the first bus in the direction from substation to loads (additional sub-indexes can be introduced to consider bifurcations, but they are not included here for the purpose of simplicity in describing equations). In this formulation l load levels are considered and i varies from 1 to nb.

$$Min \quad k_f \sum_{i=1}^{nb} \sigma_i + \sum_{l} \{ k_v \sum_{i=1}^{nb} \rho_i + \sum_{i=1}^{n} ke^l . T^l . P^l_{loss\ i} \} \quad (29)$$

Subject to :

- *Load flow Constraints:*

$$P^l_{i+1} = P^l_i - P^l_{loss\ i} - P^l_{L\ i+1} \quad (30)$$

$$Q^l_{i+1} = Q^l_i - \frac{x_i}{r_i} P^l_{loss\ i} - Q^l_{L\ i+1} + Qc_i \quad (31)$$

$$VS^l_{i+1} = VS^l_i - 2(r_i P^l_i + x_i Q^l_i) + \frac{(r_i^2 + x_i^2)}{r_i} P^l_{loss\ i} \quad (32)$$

$$P^l_{loss\ i} \geq \boldsymbol{\beta}_{k\ i}{}^{\mathbf{T}} z^l_i \,, \quad k = 1,...,nh \quad (33)$$

$$Qc_i = Qf_i + Qs_i \quad (34)$$

$$VS_i^{l\min} \leq VS^l_i \leq VS_i^{l\max} \quad (35)$$

$$\delta^l_i + \sigma_i \leq 4 \quad (36)$$

- *Fixed Size Capacitor Constraints:* (16) to (20).
- *Switched Capacitor Constraints:* (22)-(25), (28).

In this formulation, fixed parameters are the same as defined in Section I. The objective function (29) considers the possibility of installing both types of capacitors with corresponding investment costs. Constraints (30) to (35) represent a linear radial load flow approximation, where linear constraints (33) are built by a set of predefined SH. This set is previously built by an automatic procedure through the variation of reactive load and load flow simulations. In this formulation, the active power loss in each branch is a single linear variable as defined by equation (9). Constraints (34) represent reactive power injected at each bus by capacitor banks, constraints (35) are voltage limits and constraints (36) limits the number of capacitor banks to four at each candidate bus. The number of variables and constraints of MBLOP increases in relation to the non linear version; nevertheless, powerful optimization tools are available to solve large scale MBLOP for realistic distribution networks.

7 NUMERICAL EXAMPLES

7.1 Fixed and Switched Capacitor Placement for the radial IEEE 9 Bus system

This System has been widely used for testing various implementations of capacitor placement methods. It consists of one single feeder without bifurcations. Cost parameters, load levels, and peak load level are as follows: Energy cost K_e=0,06 \$/kWh, peak cost K_p=168 \$/kW/year, capacitor investment cost for both fixed and variable type is K_c=4.9 \$/kvar, high level load of 1.1 (time duration of 1000 h), medium level load of 0.6 (time duration of 6760 h), and low level load of 0.3 (time duration of 1000 h), as described in [1] and [8]. In this case, MBLOP uses 36 SH per branch for the linear approximation of the loss function. In other words, it means to run only 36 load flows.

When no capacitor additions are allowed in the MBLOP, the obtained total system cost is \$328,780.00 and represents the cost of energy losses. This value is similar to the same cost presented in [1] and [6] which is \$329,039.00. This means that the linear approximation of the loss function produces a relatively small error. Table 1 illustrates buses where exist capacitor placement and number of capacitor banks allocated by MBLOP in two cases: First, when only fixed size capacitor banks are considered for installation (columns F) and second, when both, fixed size and switched capacitor banks, are considered for installation (columns FS). In the first case, the system operational cost is now \$ 310,752.00, representing a cost reduction of 5.8% (close to the 6.1% of cost reduction reported in [1] which performs 2000 power flow simulations for this small system). The average error in voltage levels is 0,25% and in the total active losses is 0,44% when compared with a load flow solution of the optimal configuration.

Bus	**6**		**5**		**4**		**1**	
Load	**F**	**FS**	**F**	**FS**	**F**	**FS**	**F**	**FS**
0.3	4	3	2	0	1	1	1	1
0.6		4		2		1		1
1.1		4		2		1		1

Table 1: Number and types of Capacitor Placement.

In the second case, the total cost found is \$ 309,190.00, representing a 5,9% of reduction, against a 6,5% of reduction reported in [1] and the percentage of error in total losses is 0.17%. The MBLOP allocates a total of 8 banks (5 of fixed size type and 3 of switched type), against 9 banks presented in literature. The lowest voltage level is 0.8401 p.u at bus 1 (this bus is more distant from substation than the others). When the tolerated range of voltages is forced to be more constrained (between 0.860 and 1.025 for all buses), the MBLOP solution allocates 13 banks. In this case, cost reduction is only 1.70% due to the investment in more capacitor banks. In the second case, it is possible to obtain a better voltage profile and less total losses than in the case with only fixed size capacitor banks. Voltage profile average increases 4.2% in relation to the case without capacitor installation, which is better than the first case (fixed capacitors only) with 3.1% of increase.

7.2 Fixed Size Capacitor Placement for the radial IEEE 70 Bus system considering 12 load levels

This system test has been analyzed for a period of ten years with annual load growth of 9.55% up to the fourth year. The same data and parameters as reported in literature are used [1,6]. Total cost is composed of investment cost and cost of energy losses: C_{inv} + C_{losses}. A maximum of 4 capacitor banks per bus is allowed. In this case, 20 candidate buses and 81 SH are considered. Candidate buses are selected by performing a Sensitivity Analysis (SA) [1]. When there are no capacitor banks installed, total cost of energy losses is $ 1,074,700. Table 2 shows results obtained when only fixed size capacitors are allowed. Total cost in this case is $726,174.65 which represents a reduction of 32.7% in relation to the original operation of this system. Comparing total losses obtained by a load flow solution with the final optimized configuration and total losses obtained by MBLOP, an error of 0.83% is found. This means that SH correctly approximates the loss function. The reported savings in literature is 34.2%, which is a little better but on the other hand MBLOP has a complete control on the voltage ranges and needs only 81 load flow simulations against a lot more reported in other methods.

Bus	**19**	**45**	**46**	**51**	**52**	**54**
No. Banks	1	1	1	1	2	1
C_{inv} ($)	**C_{losses} ($)**		**Savings (%)**		**Error (%)**	
10,948.00	715,226.65		32.7		0.83	

Table 2: Fixed size capacitor placement.

7.3 Fixed and Switched Capacitor Placement for the radial IEEE 70 Bus system considering 12 load levels

Table 3 shows results when both type of capacitors are considered for installation for the same set of candidate buses as in the previous case. As can be seen, only at bus 17 and bus 50 fixed size capacitors are installed. The other listed buses receive switched capacitors whose control variation a long the planning period is showed in this table. Total cost obtained by MBLOP is $715,080.29, which represents a total cost saving of 33.4% (close to the 34.3% reported in literature by other random methods).

Year	**Load**	**BUS**				
		14	**17**	**45**	**50**	**53**
1	**0.50**	0	1	0	4	0
	0.80	0	1	0	4	0
	1.00	1	1	1	4	0
2	**0.55**	0	1	0	4	0
	0.88	0	1	1	4	0
	1.10	1	1	1	4	1
3	**0.60**	0	1	0	4	0
	0.96	0	1	1	4	0
	1.20	0	1	1	4	1
4 to 10	**0.66**	0	1	0	4	0
	1.05	1	1	1	4	0
	1.31	1	1	1	4	1
C_{inv}($)		**C_{losses}($)**		**Savings (%)**	**Error (%)**	
14,063.00		701,017.29		33.4	1.5	

Table 3: Fixed and switched capacitor placement.

Comparing costs of Table 2 and Table 3, it is possible to observe that the installation of switched capacitors reduce total energy loss but increases total investment cost. Moreover, it was also observed that average voltage profile improves with switched capacitors.

7.4 Capacitor Placement for a Real Radial network

Part of the distribution system of the local company was reduced to 47 buses without changing its electrical characteristics for the purpose avoiding electrical redundant buses. Load levels considered are 1.41 MW (66% of the time), 2.19 MW (20.1% of the time) and 3 MW (13.9% of the time). Energy cost is 20.47 $/MWh, cost of a 300 kVar fixed size capacitor bank is $2,037, and cost of a 300 kVar switched capacitor bank is $2,710. In this case, 63 SH are used and 20 candidate buses are selected from a sensitivity analysis ranking. The planning horizon is six years with an increase in load of 5% each two years and energy price increase of 10% each two years. The obtained cost of energy losses is $4,944.32 per year without installing capacitors. Because of the long period of low level load, only two fixed size capacitor banks are installed, one at bus 37 and another one at bus 45. The solution of the MBLOP provides 14.3% of cost savings with 2.9% of error in the total system power loss calculation. The lowest voltage level found is 0.97 p.u.

8 COMPUTATIONAL ISSUES

Taking as a basis the previous case of the IEEE 70 bus system for the same ten year planning horizon that includes 12 load levels, several cases were simulated with different number of candidate buses selected from a SA ranking list.

No. C. B.	**Binary variables**		**Time (s)**		**Savings (%)**		**Error (%)**	
	F	**FS**	**F**	**FS**	**F**	**FS**	**F**	**FS**
10	10	130	48	37	30.07	30.89	1.03	0.88
30	30	390	306	196	32.94	33.49	1.14	0.79
69	69	897	560	316	33.37	34.46	1.07	1.64

Table 4: Performance indexes.

In this example, only one fixed size and/or one switched capacitor bank is allowed to be installed at each candidate bus (C.B.).

Table 4 shows several performance indexes like number of binary variables, processing times, percentage of savings obtained and percentage of error in total active power losses calculation with respect to the number of selected candidate buses (No. C.B.). Columns with F correspond to cases where only fixed capacitors are considered for installation and columns with FS correspond to cases where both types of capacitors are allowed for installation.

As can be seen, the number of binary variables increases with the number of candidate buses and capacitor type. In this case, the flexibility provided by FS allows getting good lower limits in the Branch & Bound optimization program, in spite of the number of binary variables, reducing in this way the processing times. Savings increase when the number of candidates evaluated increases getting even better results than in case of table 3. The total active power loss error is small and independent of the quantity of bus candidates.

When the possibility of installing from zero to four capacitor banks in each candidate bus is considered, processing times are high due to the fact of that the number of binary variables is high (i.e., for 69 candidate buses the number binary variables is 3588 for the case considering fixed and switched capacitors). However, a detailed analysis shows that cost savings obtained by MBLOP as a function of processing times for both: only fixed (F) and fixed and switched capacitors (FS) reveled that the differences in percentages of cost savings obtained for a processing time of 10 minutes and five hours is less than 1% for both cases. A near sub-optimal (feasible) solution can be obtained if the optimization process is stopped by a time limit. In this case, for a ten years planning period, the time limit could be 10 minutes. GAMS/CPLEX solver was used to find solutions for MBLOP in all the cases presented. The equipment used was a PC with 733 MHz and 256 MB.

9 CONCLUSIONS

This paper demonstrates that it is possible to solve the capacitor placement planning problem through a binary mixed linear deterministic model based on load flow simulations. The obtained linear model is easily handled by existent robust solvers specialized in mixed integer linear optimization problems. One characteristic of this approach is avoiding the need to calibrate arbitrary complex parameters normally present in random and evolutionary methods. Other important characteristic is the possibility of taking into account historical post-operative data available in distribution companies for generating some of the required linear approximations, as well as the need for few load flow simulations. Also, the possibility of directly evaluating the improvement in voltage profile against corresponding cost savings is one of the advantages of this approach.

The suggested linear model determines size, location and control scheme of fixed and switched capacitors, as well as total economy obtained in several planning periods. The obtained solutions rigidly respect load flow equations and operational limits for several load levels. In all tested cases, accurate results were found when compared with the corresponding load flow solutions. A cost reduction of 14% was obtained for a horizon plan of six years in a distribution system of the local company.

Further research includes the application of this tool for studying Distributed Generation (DG) sizing and placement planning under a market environment.

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Roberto Stefan Fernandes de Aguiar received his B.Sc. from Universidade Federal da Paraíba – UFPB in Campina Grande, Brazil, and the M.S. from University of Brasilia - UnB, Brazil. His current research interest is distribution system planning.

Pablo Cuervo Franco (SM’03) received his B.Sc. at Andes University in Bogotá, Colombia, and the M.S. and D.S. degrees from University of Campinas – UNICAMP, Brazil. He developed post-doctoral research activities at McGill University, Canada. Currently, he is professor of Electrical Engineering at Brasilia University, Brazil. His current research interests are in analysis of power systems under competition and distribution system planning.