AN ADAPTIVE MEDIAN POST-FILTER FOR IMPEDANCE ESTIMATION BASED ON DIFFERENTIAL EQUATION ALGORITHM

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Abstract – This paper presents an application of the adaptive median post-filter to improve impedance estimation based on the differential equation algorithm. The main feature of the post-filter is the automatic selection of the optimal window size that balances between fast tracking and filtering performance. For sudden impedance change (fault inception), small window size is selected to make fault detection fast. Large window size is selected if the impedance is slow varying or steady-state (pre-fault and fault conditions) to increase filtering efficiency. The post-filter is also very efficient in eliminating impulsive noise created by numerical instability of the differential equation algorithm under certain conditions. The simulation results presented in the paper show the adaptive median post-filter enhances filtering performance of the differential equation algorithm while preserving fast tracking of the impedance during fault inception.

Keywords: Impedance estimation, Distance protection, Median filters, Adaptive technique

1 INTRODUCTION

Fast and accurate estimation of the fault impedance is an essential component of the transmission line relaying algorithms. Two types of the impedance estimation algorithms, Fourier transform based algorithms and the differential equation based algorithms, are widely accepted by the protective relying industry [1]. The Fourier transform based algorithms possess good filtering characteristic and computation time can be improved using recursive calculation [1]. However, the presence of the exponentially decaying DC component affects the estimation accuracy. The differential equation based algorithms are computationally fast and not sensitive to DC component but exhibit poor filtering characteristic. Linear post-filtering has been proposed to solve the problem [1]. Unfortunately in some cases the computation exhibits numerical instability which results in large outliers in the estimated values [1, 2]. These outliers disrupt linear filtering producing large errors. To solve the problem a median post-filter with fixed window size has been suggested [2]. This paper focuses on further improvement of the differential equation algorithm by using the median post-filter with adaptive window size.

The window size of the median post-filter is varying to adapt to an unknown impedance (resistance and reactance) change. For sudden impedance change (when a fault occurs), the window size reduces to track the change quickly. If the impedance is constant or slow changing, the window size automatically increases to filter higher frequencies, Gaussian and impulsive (outliers) noise more efficiently. The Intersection of Confidence Interval (ICI) rule [3, 4] has been implemented here for a data-driven window size selection. The ICI rule was successfully implemented in the past for phasor and frequency estimation based on adaptive linear filters [5, 6]. The adaptive median filter is built as a number of parallel filters, which are different only in their window size, and the selector, which determines the best size and the corresponding estimate.

The second section of the paper presents an overview of the basic differential equation algorithm and identifies potential sources of errors. The third section introduces a novel approach to enhance the performance of the differential equation algorithm. This approach is based on the adaptive median post-filter. The paper concludes with the presentation of the representative simulation results. The results are showing that the differential equation algorithm with adaptive median post-filtering is very accurate and fast in tracking impedance change during various faults and disturbances.

2 DIFFERENTIAL EQUATION ALGORITHM

This algorithm assumes that the current and voltage waveforms contain a DC component but are otherwise free from high-frequency oscillations. In the algorithm derivation, a single-phase line differential equation model with resistance and inductance as parameters has been assumed [1]:

$$v = Ri + L\frac{di}{dt} . \tag{1}$$

Applying numerical integration of the differential equation (1) over the intervals (t_k, t_{k+1}) and (t_{k+1}, t_{k+2}),

$$\int_{t_k}^{t_{k+1}} v dt = R\int_{t_k}^{t_{k+1}} i dt + L(i_{k+1} - i_k),$$

$$\int_{t_{k+1}}^{t_{k+2}} v dt = R\int_{t_{k+1}}^{t_{k+2}} i dt + L(i_{k+2} - i_{k+1}),$$

using the trapezoidal rule, two discrete time equations has been formulated [1]:

$$\frac{T_s}{2}(v_{k+1}+v_k)=R\frac{T_s}{2}(i_{k+1}+i_k)+L(i_{k+1}-i_k) \text{ and}$$

$$\frac{T_s}{2}(v_{k+2}+v_{k+1})=R\frac{T_s}{2}(i_{k+2}+i_{k+1})+L(i_{k+2}-i_{k+1}).$$

In matrix form, the above two equations can be combined to give

$$\begin{bmatrix} \frac{T_s}{2}(i_{k+1}+i_k) & (i_{k+1}-i_k) \\ \frac{T_s}{2}(i_{k+2}+i_{k+1}) & (i_{k+2}-i_{k+1}) \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix} = \frac{T_s}{2} \begin{bmatrix} (v_{k+1}+v_k) \\ (v_{k+2}+v_{k+1}) \end{bmatrix} \quad (2)$$

Resistance and inductance estimates are calculated by solving the matrix equation (2)

$$R_{k+1}=\frac{(v_{k+1}+v_k)(i_{k+2}-i_{k+1})-(v_{k+2}+v_{k+1})(i_{k+1}-i_k)}{2\left(i_k i_{k+2}-i_{k+1}^2\right)} \quad (3)$$

$$L_{k+1}=\frac{T_s}{2}\frac{(v_{k+2}+v_{k+1})(i_{k+1}+i_k)-(v_{k+1}+v_k)(i_{k+2}+i_{k+1})}{2\left(i_k i_{k+2}-i_{k+1}^2\right)} \quad (4)$$

where T_s is the sampling interval, and (v_k, i_k) are voltage and current samples respectively.

The algorithm needs only three samples of voltage and current to calculate impedance. This is considered a great improvement in response time compared to the algorithms based on phasor calculation using Discrete Fourier Transform (DFT) [1]. However, two major problems prevent the widespread practical application of this algorithm:

- Numerical calculation of derivative or integral based on only two samples is very sensitive to measurement noise and high frequencies.

- The denominator in (3) and (4) is not constant but varies in time and under some conditions it is approaching zero. In such cases, impulsive changes in estimates (outliers) with large errors are detected [1, 2].

3 ADAPTIVE MEDIAN POST-FILTER

Post-filtering has been proposed as a solution to the above-mentioned numerical problems with differential equation algorithm [1, 2]. The optimal post-filter should have fast response to track sudden changes of impedance (fast fault detection), and at the same time to filter high-frequencies, Gaussian and impulsive noise after fault inception to improve accuracy in the impedance estimation. Previously proposed filters based on fixed window size [1, 2] are not able to find such optimal solution. We introduce here the novel optimal solution based on the median post-filter proposed in [2] but with the adaptive window size.

Suppose that the resistance and inductance samples calculated using (3) and (4), for a specified window size, are ranked in ascending order. Then the median estimate for this window size is the middle sample if there are an odd number of samples in the window. It is well known that the median filter with respect to the Gaussian noise is nearly as good as the linear filter, while the median estimate demonstrates a good resistance to the random impulse noise (outliers). In this application, the window size of the median filter is varying to adapt to an unknown impedance change. For sudden impedance change, the window size is small to track the change quickly (fault inception). If the impedance is constant or slow changing (after fault inception), the window size automatically increases till the optimal size which realizes efficient filtering of higher frequencies, Gaussian and impulsive noise.

To find the optimal window size one needs to calculate median estimates for several windows. Then each candidate estimate should be assessed to select the best one. In the implementation, assessment of the optimality follows immediately after increase of window size and the procedure stops when the optimal size is discovered without examining larger window sizes. The procedure is the same for resistance and inductance post-filtering. We will use the resistance post-filtering to explain the method. The assessment is based on the Intersection of Confidence Intervals method (ICI) [3, 4]. In this method, after calculating the resistance median estimate $\hat{R}(N)$ for a central window of length *N*, the confidence interval is estimated as follows:

$$\hat{D}_R(N)=\left[\hat{R}(N)-\kappa\hat{\sigma}_R(N),\ \hat{R}(N)+\kappa\hat{\sigma}_R(N)\right], \quad (5)$$

where the standard deviation is estimated using the following expression [4, 7]:

$$\hat{\sigma}_R(N)=\hat{\sigma}_{\varepsilon R}\sqrt{\frac{\pi}{2N}}, \quad (6)$$

and κ is a threshold of the confidence interval [4]. The standard deviation $\hat{\sigma}_{\varepsilon R}$ in (6) of the resistance (3) estimation error ε is estimated in the robust way using Median Absolute Deviation (MAD) estimator [8]:

$$\hat{\sigma}_{\varepsilon R}=\frac{median(|\Delta R|)}{0.6745\sqrt{2}}, \quad (7)$$

where each entry of the vector ΔR is calculated as the difference between two consecutive samples of R estimated using (3).

For each specified window size N, the median estimates $\hat{R}(N)$ and $\hat{L}(N)$, and the corresponding confidence intervals $\hat{D}_R(N)$ and $\hat{D}_L(N)$ are calculated. The Intersection of Confidence Intervals (ICI) method that determines the optimal window size can be summarized as follows [3, 4]:

The optimal window lengths $N_R{*}$ and $N_L{*}$ for a time instant kT_s are those for which the sets $\bigcap_{N \le N_R{*}} \hat{D}_R(N)$ and $\bigcap_{N \le N_L{*}} \hat{D}_L(N)$ respectively are nonempty.

The asymptotic properties of the ICI method are provided in [9]. The adaptive median filter is built as a number of parallel filters, which are different only in their window size, and the selector, which determines the best size and the corresponding estimate.

4 SIMULATION EXAMPLE

This example has been selected to illustrate performance features of the adaptive median filter as a post-filter for the resistance and inductance estimation based on equations (3) and (4). The data used in the simulation is typical for a 100km long 220kV line. The sampling frequency is 1kHz and the fault simulated is at 20% along the line length. To simulate measurement noise, voltage and current signals are distorted with normally distributed noise (zero mean and 1% standard deviation). All window sizes between 5 and 35 are used by the adaptive median filter.

Figure 1 shows voltage and current (pre-fault and fault) entering the differential equation algorithm with adaptive median post-filtering. It should be noted that the fault current contains large exponentially decaying DC offset. This component as well as some high frequencies could cause in some cases numerical instability of the differential equation algorithm. In this example there are two characteristic points that influence numerical stability of the algorithm. Those points could be identified in Figure 2 where samples of denominator in (3) and (4) are plotted. In Figure 2, point A is the fault inception and at point B the denominator approaches zero. At these points estimation equations (3) and (4) produced large impulsive errors (outliers). Those errors can be observed on the dashed-dot lines in upper parts of Figures 3 and 4 (points A and B) for resistance and reactance estimation respectively.

The adaptive median post-filter is able to eliminate the impulsive errors (points A and B in Figures 3 and 4). Output of the filter is shown using solid line in upper parts of Figures 3 and 4 for resistance and reactance estimation respectively. Another positive feature of the filter is the fast adaptation to a sudden impedance change (fault inception). As can be seen in lower parts of Figures 3 and 4, the optimal window size during sudden impedance change reduces from 35 to 5. Small window size allows fast impedance tracking but at the same time filtering efficiency has been degraded. In practical implementation this feature is very important for fast fault detection and activation of the protection algorithm. In the pre-fault and fault stages, the window size automatically increases as shown in lower parts of Figures 3 and 4 (up to 35 samples). This feature makes the filter very efficient in filtering higher frequencies, Gaussian and impulsive noise. This can be examined in Figures 3 and 4; the solid line (filter output) is much smoother than erratic dash-dot line (estimates calculated using (3) and (4)). In upper parts of Figures 3 and 4 we also show true values of the fault resistance (0.002 pu) and reactance (0.0628 pu) respectively.

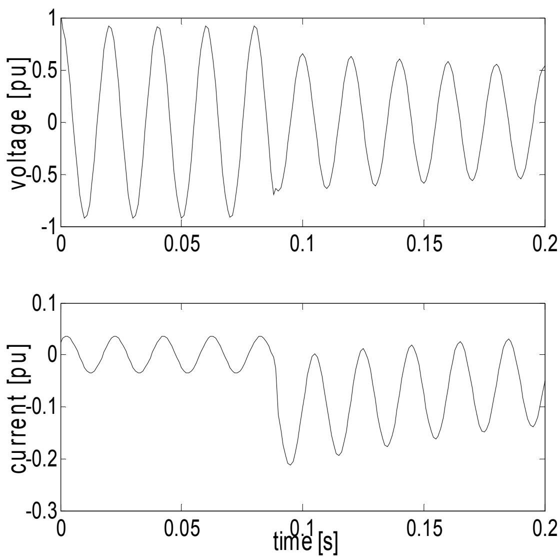

Figure 1: Voltage and current entering the differential equation algorithm

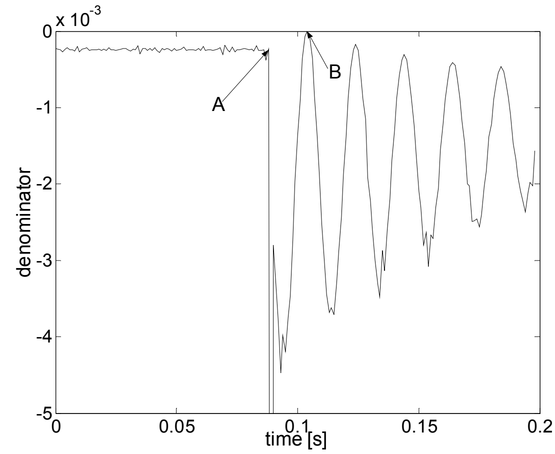

Figure 2: Denominator in resistance and inductance estimation equations (3) and (4)

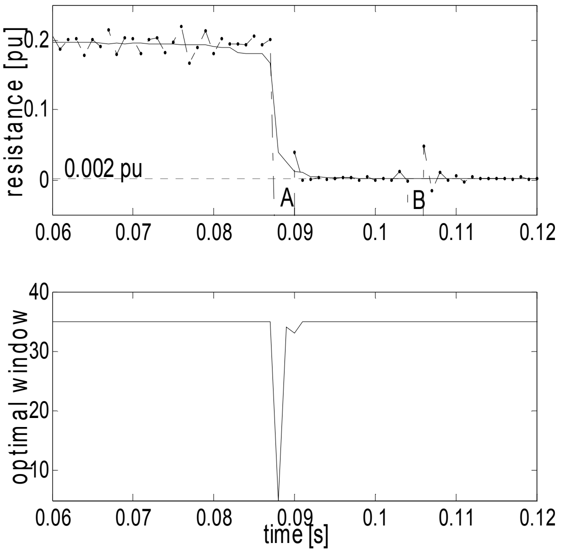

Figure 3: Upper figure is the resistance estimation (dash-dot line is the estimate using (3), solid line is the result of adaptive median post-filtering and doted straight line is the true fault resistance); lower figure is the optimal window size for each sample selected for median post filtering

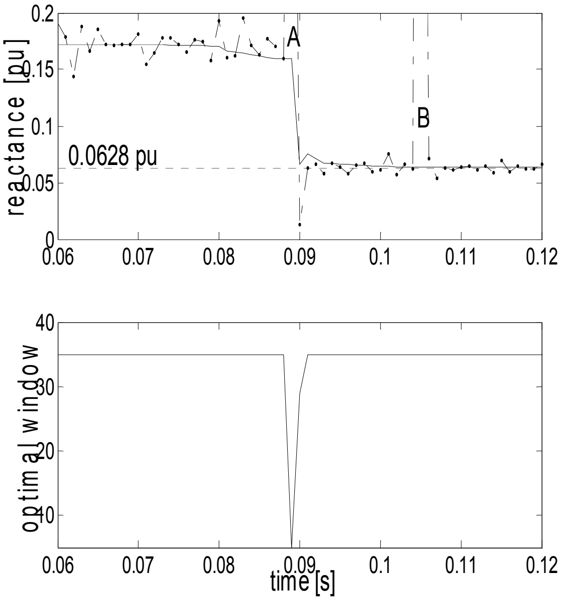

Figure 4: Upper figure is the reactance estimation (dash-dot line is the estimate using (4), solid line is the result of adaptive median post-filtering and doted straight line is the true fault reactance); lower figure is the optimal window size for each sample selected for median post filtering

5 CONCLUSIONS

The paper describes an application of the adaptive median post-filtering that enhances performance of the differential equation algorithm for impedance estimation. Compared to the previously proposed post-filtering techniques (linear or median filters with fixed window size), the novel adaptive approach (median filter with varying window size) is able to find the optimal solution under various conditions. One such condition is a sudden change of the measured impedance seen during fault inception. This condition is efficiently handled by reducing the window size there by eliminating the biased estimates and tracking the change quickly. With this feature fast fault detection of the differential equation algorithm has been preserved. The adaptive median post-filter improves accuracy in the impedance estimation by automatically selecting larger window size during the pre-fault and fault conditions. With larger window size higher frequencies, Gaussian and impulsive noise are efficiently eliminated. The simulation results presented in this paper show that the novel adaptive post-filtering technique is able to improve performance of the differential equation algorithm under various practical conditions.

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