

# MARKOV MODELS FOR RELIABILITY-CENTERED MAINTENANCE PLANNING

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**Abstract** - For reliability-centered planning of preventive maintenance a three-stage combined outage and maintenance model of Markov-type is proposed in which each stage of system degradation is characterized by an individual outage and maintenance rate. By this model the restriction of Markov-theory to constant transition rates is overcome. Performing some simplifications the model can be used in reliability calculation software for large power systems without major modifications. Application results will give detailed insight into the interaction between aging processes, outages, repair- and maintenance-activities.

**Keywords:** *Asset management, Fault management, Network reliability computation, Risk assessment, Supply reliability*

## 1 INTRODUCTION

Present economic conditions force network companies to reduce cost by simplification of facility and network structure, by reduction of maintenance frequency and by higher exploitation of network components which, as a consequence, leads to lower supply reliability. To prevent a degradation of the network, methods for reliability-centered maintenance planning become important tools for a future assessment of the continuity of supply.

In literature different approaches to deal with this problem can be found. In [1] the computation of a weighted list of facility importance is proposed which forms the basis for maintenance activity planning. By the inclusion of information on component health-state this concept allows for a combination of importance- and condition-based maintenance [2]. These approaches do not necessarily require reliability calculations, since importance-screening can be based on network structure and consumer data.

However, for a forecast of the influence of suspended maintenance on future system reliability, the influence of maintenance intensity on outage frequency has to be modeled.

Consequently, constant (time independent) outage rates can no more be used in the outage models.

An approach concentrating on the evaluation of the influence of maintenance intensity and renewals on facility outage rates is presented in [3]. Lifetime-distributions of typical medium voltage network components are modeled by normal- or double-exponential-distributions using information from maintenance protocols and personnel experience. The effect of outage frequencies on consumer supply continuity is assessed by a failure-tree model. In [4] for component lifetime-distributions a combination of exponential- and Weibull-distributions is used. The concept is applied to series- and parallel- structures which form the basic structures in reliability calculations based on the method of minimal cuts. The authors of [5] propose a Markov-model which includes states with different degrees of wear-out, one single outage state and maintenance states with transition rates adapted to the degree of wear-out. An advantage of this model is its high degree of flexibility, a draw-back the usage of only a single outage rate independent on system condition.

In this paper an extension of the Markov-model of [5] is presented. To include exploitation-time dependent outage rates, the time-behavior is approached by a step-by-step trend function. In that way, to each wear-out state a special outage rate can be assigned. Because of its complexity the direct implementation of the proposed model into reliability calculation software for large electrical networks is not applicable in practice. However, by neglecting state transitions which are not relevant for systems with typical component reliability levels, the complex model can be reduced and thus be implemented into conventional reliability calculation software without major modifications. The performance of this approach is demonstrated by the reliability assessment for a real 100 node high-voltage network.

Finally, an approach to extract the information necessary for modeling of the time-dependence of outage rates from outage statistics is presented.

## 2 COMBINED OUTAGE AND MAINTENANCE MODELS

The inclusion of maintenance states into Markov outage-models leads to an increase of the computed value of system unavailability and thus to an increase of computed costs of not delivered energy. The comparison of a simple two-state outage model with a model in which one additional maintenance state is taken into account leads to the cost relation of equation (1), [7].

$$(K_f.P_f + K_r) > (\mu_f/\lambda_f + 1).(K_m.P_m + K_r) \quad (1)$$

$K_f, K_m$  Cost of not delivered energy for failure- and maintenance-outages, €/kWh

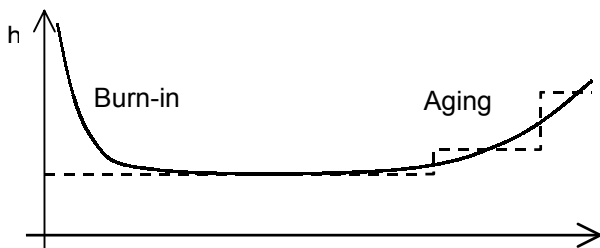
$K_r$  Repair cost for outage and maintenance (personnel, material), €/h

$P_f, P_m$  Interrupted power during outage and maintenance, kW

$\lambda_f, \mu_f$  Failure- outage and repair rate, 1/h

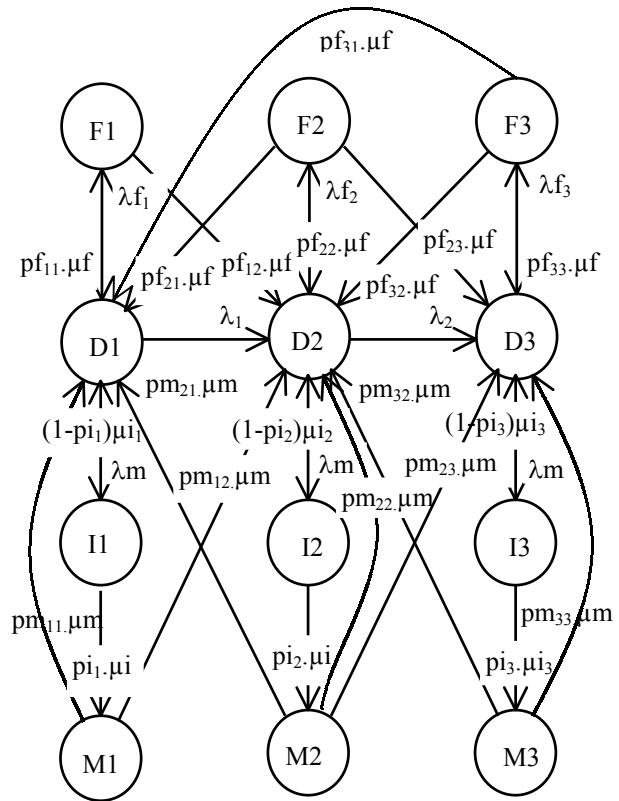
For typical network-component transition rates of  $\lambda_f = 0.1/8760$  and  $\mu_f = 1/100$ , application of (1) results in  $(K_f.P_f + K_r)/(K_m.P_m + K_r) > 876$ . Thus, maintenance would be justified only for rather unrealistic high ratios of outage- to maintenance-cost.

With this simple evaluation it becomes obvious that models with time-independent outage rates will not be realistic approaches for maintenance planning. Because of the restriction of Markov theory to time-independent transition rates, application of general renewal theory seems to be the only way of problem solution. On the other hand, a decisive advantage of Markov theory is its flexibility in modeling complex failure events together with post fault switching. To overcome these restrictions the continuous time dependent function of outage rate is substituted by the discrete step-by-step function shown in figure 1. Burn-in failures are not taken into account in this model.



**Figure 1:** Substitution of the time dependent function of outage rate by a step-by-step function

A suitable base for the implementation of this failure model is the maintenance model presented in [5]. The extended model is shown in figure 2.

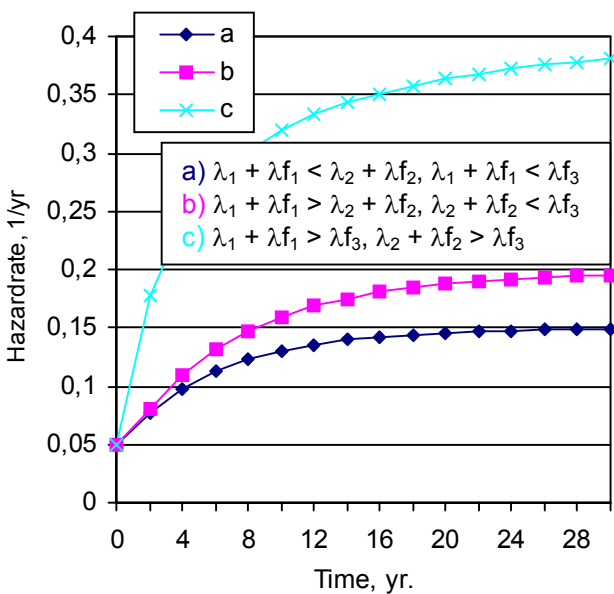


**Figure 2:** Markov model for failure F, maintenance M, inspection I and three states of degradation D

In figure 2, state D1 represents the new system without degradation. Concerning failure states F, perfect repair which transfers the system back to a working state with a lower degradation level (e.g.  $F_2 \rightarrow D_1$ ) and imperfect repair which leads back to a state with increased level of wear-out (e.g.  $F_2 \rightarrow D_3$ ) is modeled. The same principle is applied to transitions from maintenance to operation ( $\lambda_m, \mu_m$ : maintenance outage and repair rates). These transitions are controlled by special probabilities for state improvement ( $pf_{21}, pm_{21}$ ) or state degradation ( $pf_{23}, pm_{23}$ ). For the simulation of condition-based maintenance, inspection is taken into account. The inspection probability  $pi$  represents the percentage of inspections which result in detection of defects, the inspection rates  $\mu_i$  are given by the reciprocal of inspection duration. Substitution (renewal) of a device at the end of its lifetime is not explicitly included in this model, although by interpreting D3 as end-of-lifetime state,  $\mu_3$  as replacement-delay rate and M3 as replacement-state, with some additional modifications substitution could be modeled in a quite straight forward way, too.

In the model of figure 2 no distinction is made between maintenance activities performed to keep the system in working state and to maintain an as large as possible lifetime. Thus, in this approach maintenance comprises all planned activities aimed at maintaining satisfactory system performance.

To investigate the time behavior of the model, at first solely operation states D and failure states F are taken into consideration. The failure states are merged into one single absorbing state without possibility of return to the operation states by repair rates. The time-dependent hazard rate of this outage model is computed by solution of the Markov- equations in the time domain [7]. Results are shown in figure 3.



**Figure 3:** Hazard rate for the outage model of figure 2. (No maintenance states, single absorbing failure state).

The hazard rate  $hz(t)$  starts with the value of  $\lambda f_1$  at  $t=0$  and for  $t \rightarrow \infty$  approaches to a limiting value for which the three solutions a) to c) exist, see figure 3:  $hz(\infty) = \lambda_1 + \lambda f_1$  for solution a),  $hz(\infty) = \lambda_2 + \lambda f_2$  for solution b),  $hz(\infty) = \lambda f_3$  for solution c).

The solution with the largest hazard rate is met only if the outage rate of the last state is smaller than the sums of degradation and outage rates of the forgoing states. Thus, the performance of the solution is similar to that of a pipeline: The outflow cannot be larger than the inflow. In the model of [5] there exists only one single outage state which can be reached by a transition from state D3. For this model the hazard rate starts with 0 for  $t=0$  and approaches to the outage rate for  $t \rightarrow \infty$ .

Since Markov processes deliver stationary state probabilities for large observation times, a Markov

model is never able to produce a hazard rate continuously increasing with time as shown in figure 1, but a hazard rate decreasing with time and tending to a limiting value as shown in figure 3. However, the Markov model of figure 2 is flexible enough to produce after suitable parameter adaptation an outage probability distribution function similar to that given by a hazard rate of figure 1. (See also chapter 5).

If repair is taken into account the probabilities  $pf$  which are a measure for the quality of repair play an important role for the output of the model. Their influence is demonstrated with the stationary solutions for the state probabilities of the model of figure 2 excluding maintenance states. The outage rates are reduced to a single rate by application of equation (2).

$$\lambda_g = \frac{p_{D1} \cdot \lambda f_1 + p_{D2} \cdot \lambda f_2 + p_{D3} \cdot \lambda f_3}{p_{D1} + p_{D2} + p_{D3}} \quad (2)$$

There are following typical cases to be distinguished:

1.) Imperfect repair,  $pf_{33} = 1, pf_{12}, pf_{23} > 0$ :

After having reached state D3, the process keeps oscillating between state D3 and F3 which results in:  $\lambda_g = \lambda_3$ .

2.) Perfect repair,  $pf_{11} = pf_{21} = pf_{31} = 1$ :

$$\lambda_g = \frac{\lambda f_3 \cdot [(\lambda_2 + \lambda f_2) \cdot \lambda f_1 + \lambda_1 \cdot \lambda f_2 + \lambda_1 \cdot \lambda_2]}{\lambda f_3 \cdot (\lambda_2 + \lambda f_2 + \lambda_1) + \lambda_1 \cdot \lambda_2} \quad (3)$$

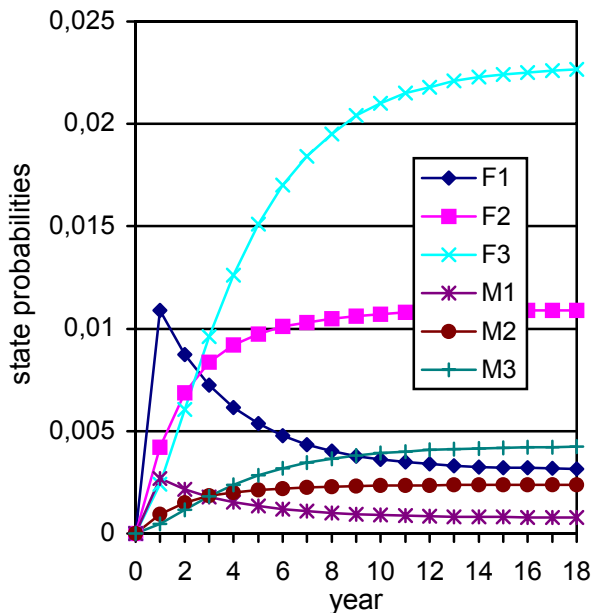
If the outage rates are of similar value ( $\lambda f_1 \approx \lambda f_2 \approx \lambda f_3 \approx \lambda f$ ),  $\lambda_g$  approaches to  $\lambda f$ . If the ratio of  $\lambda f_3 / (\lambda_1 \cdot \lambda_2)$  is large,  $\lambda_g$ -values in the range between  $\lambda f_1$  and  $\lambda f_2$  are met.

We can conclude that for perfect repair the resulting outage rate  $\lambda_g$  can never reach the rate of maximum wear-out  $\lambda f_3$ . Thus, assuming "imperfect repair" seems to be a realistic approach.

Theoretically, the probabilities  $pf$  could be evaluated on the basis of statistical material. However, utilities are rather cautious concerning information about repair quality. Thus, in practice values for  $pf$  have to be assumed by considerations as presented above.

For the calculation of the time behavior of the full model (figure 2) the recursive approach by Markov chains is used [6]. Following input parameter values (transition rates in  $yr^{-1}$ ) are used:  $\lambda_1 = 0.06, \lambda_2 = 0.05, \lambda f_1 = 0.2, \lambda f_2 = 0.4, \lambda f_3 = 1.0, \lambda m_1 = \lambda m_2 = 0.3, \lambda m_3 = 0.5, \mu f_1 = \mu f_2 = \mu f_3 =$

0.00166,  $\mu_{i1} = 0,25$ ,  $\mu_{i2} = 0,20$ ,  $\mu_{i3} = 0.125$ ,  $\mu_{m1} = \mu_{m2} = \mu_{m3} = 0.004$ ,  $pf_{11} = 0.05$ ,  $pf_{12} = 0.80$ ,  $pf_{22} = 0.4$ ,  $pf_{23} = 0.6$  (imperfect repair),  $pf_{33} = 1$ ,  $pm_{11} = 0.9$ ,  $pm_{21} = 0.7$ ,  $pm_{22} = 0.3$ ,  $pm_{32} = 0.8$ ,  $pm_{33} = 0.2$  ("effective" maintenance),  $pi$  between 0.4 and 0.9.



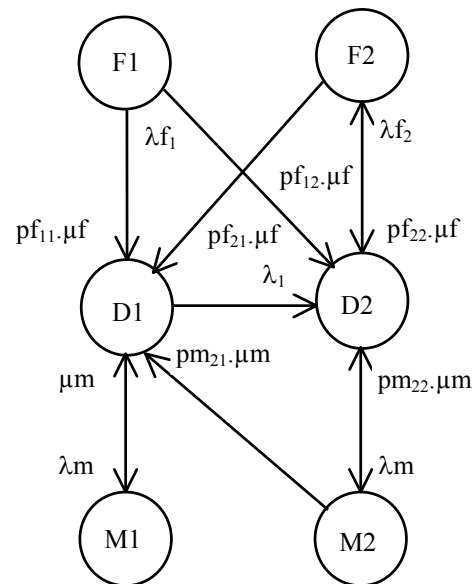
**Figure 4:** State probabilities for the full model (figure 2). F: failure states, M: maintenance states

The system starts its life period in state D1 with state probability  $Pr_{D1}(t) = 1$  at  $t=0$ , see figure 4. The failure state probability for the "new" system state F1 reaches a maximum after the first year and then converges to a far lower value, whereas the failure probabilities for the degradation states F2 and F3 increase continuously. The large stationary value for F3 indicates that the maintenance rate evidently is too low for this degradation level which is confirmed by the comparison of  $\lambda_{f3} = 1.0$  and  $\lambda_{m3} = 0.5$ . The peaks of the curves F1 and M1 are a result of limited resolution; actually, the functions are smooth. The reduced outage rate  $\lambda_g$  reaches a stationary value of 0.56. The stationary values are valid for a time horizon larger than 15 years. Thus, forecast of system performance for shorter periods should be based on model formulations in time domain.

### 3 INFLUENCE OF TIME-BASED AND CONDITION-BASED MAINTENANCE ON OUTAGE COSTS

For simplicity, the model for time-based maintenance shown in figure 5 consists of two stages only. For the computations three-stage models were used. In this model the maintenance

rate  $\lambda_m$  is assumed to be independent of the degradation level of the system. Inspection states are of no relevance.



**Figure 5:** Two-stage model for time-based maintenance

The model for condition-based maintenance is identical to the model of figure 2 with the exception that no maintenance is performed in state D1. Thus, it is assumed that the operator trusts in the system's perfect performance during its first period of lifetime and in his own knowledge of the length of this period.

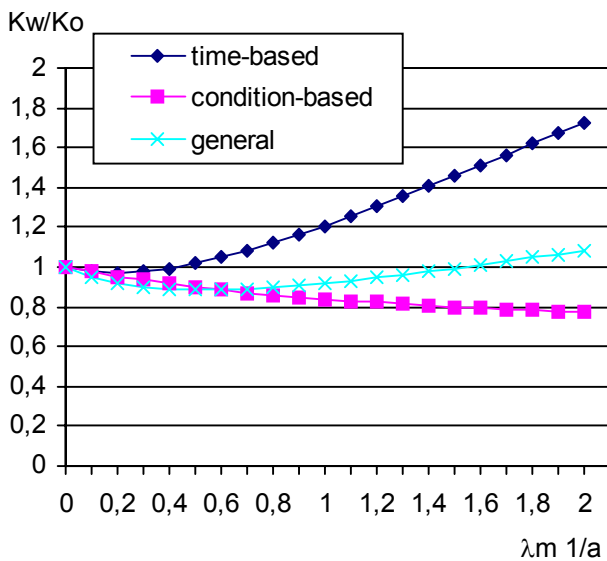
For comparison of the two models an efficiency factor is computed which is given by the ratio of "outage and maintenance cost ( $K_w$ )" to "outage cost solely ( $K_o$ )". For the calculation of cost  $K_o$  the models consist of states D and F only. In the cost models, compare equation (1), repair costs ( $K_r$ ) are neglected. It was assumed that outages as well as maintenance and inspection lead to interruptions of power ( $P_m = P_i = P_f$ ). For the cost factors relative values are taken as:  $K_i = 2$  (inspection),  $K_m = 4$  (maintenance),  $K_f = 8$  (outage). The other parameter values are similar to those given in chapter 2.

It can be concluded from figure 6 that time-based maintenance is less efficient than condition-based maintenance. The region where the time-based maintenance strategy becomes efficient ( $K_w/K_o < 1$ ) is very narrow and limited to rather low maintenance rates. Of course, for a higher ratio of  $K_f/K_m$  efficiency of time-based maintenance becomes larger [7], but this is also true for the other strategies. For (ideal) condition-based maintenance, efficiency continuously increases

with increasing maintenance rates and reaches a limit at a value of  $K_w/K_o \approx 0.75$ . The practical meaning of this result is: After having reached state D2 or higher order degradation states, maintenance activities should be started as soon as possible ("with infinitely high maintenance rate") whenever inspection indicates their need. Thus, the system is transferred as quickly as possible (with probability  $p_{m21}$ ) back to state D1 where maintenance can be suspended till condition indicators signalize system degradation. However, residence in the domain of  $K_w/K_o < 1$  (figure 6) is bound to following conditions:

- Inspection- and maintenance costs are sufficiently low with respect to outage costs.
- Inspection- and maintenance durations are lower than outage durations.
- The probability for system condition improvement by maintenance is sufficiently high.

For quantification of "sufficient" special formulae were derived and presented in [7].

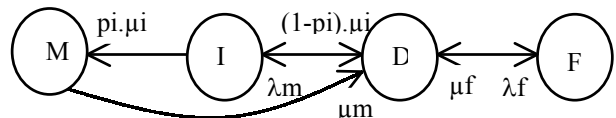


**Figure 6:** Comparison of the efficiency of different maintenance strategies

In practice, this ideal model is feasible only if network operators are disposing of very efficient methods for online state condition monitoring. Since such devices do not yet exist, presently the "general model" of figure 2 constitutes the most realistic approach to condition-based maintenance. As shown in figure 6 it produces efficiency indexes which form a superposition of the indices of the time-based and ideal condition-based model.

#### 4 APPLICATION OF THE COMBINED OUTAGE AND MAINTENANCE MODEL TO LARGE SYSTEMS

Representation of each system component by the Markov model of figure 2 would lead to an inadmissibly large number of system states even for moderately large systems. Thus, for application in reliability evaluations of real systems the model has to be simplified. The presented approach is based on the fact that for electrical energy network components  $\lambda$ -type transition rates (outage) are very small with respect to  $\mu$ -type transition rates (repair). Under these circumstances  $\lambda$ -type transitions between different system components can be neglected and the combined outage and maintenance models of the components become independent of each other. By this approach the 12-state component models can be reduced to 4-state models, see figure 7, which can be easily implemented into conventional reliability calculation software.



**Figure 7:** Reduced four-state Markov model for outage and maintenance

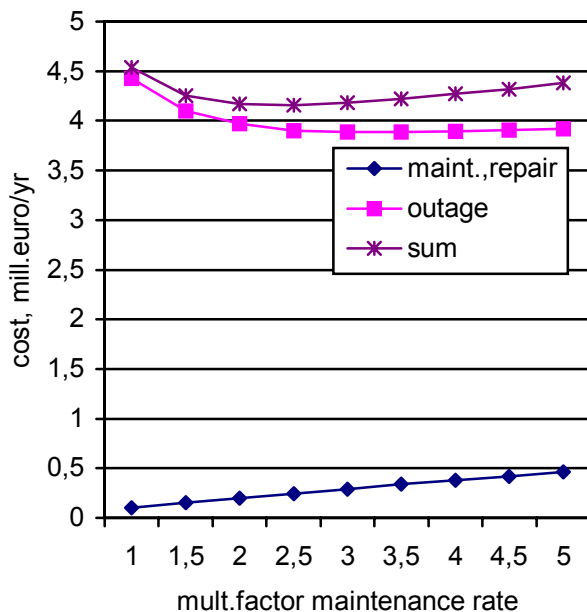
Reduction of the transition rates is performed by equations similar to (2). Mathematical proofs can be found in [8] for a two-stage model and a system of two components. Test results permit to believe that the approach is sufficiently accurate for larger systems, too.

Real systems are composed of components with different age. Thus, a classification of system components with respect to their age has to be done. For components which have already reached a state of progressed degradation, state D1 represents the present-time state with the present-time outage rate  $\lambda f_1$  and not the "new"-state. In that case the "new"-state can never be reached by the proposed model. This restriction constitutes a principal feature of the proposed approach. Since it is based on stationary Markov theory, it cannot represent the effect of component renewals with different time shift. Thus, for modeling of renewals in system reliability analysis a simulation in time domain would be necessary.

For demonstration purposes reliability evaluations for a real 100 node urban high voltage network with voltage levels of 380-kV and 110-kV

are performed. A basic description of the reliability software can be found in [9]. The three-stage maintenance models of figure 2 and the reduction procedure for the model of figure 7 are implemented in a subroutine which is included into the data input procedure. Model extension thus does not affect the central part of the algorithm.

Values for maintenance costs were provided by the utility. For outage costs a value of 6 €/kWh was taken. The basic component reliability data was evaluated from the outage and maintenance history of the network. It is assumed that the devices contained in the component "bus" are of the same age. Thus, identical model parameters can be used for all components. For other component types than busses effects of maintenance are not taken into consideration. For modeling of aging, assumptions had to be made. Again, similar values as in chapter 2 were used. Therefore, computation results presented in figure 8 represent examples for demonstration purposes, but not an image of real system reliability.



**Figure 8:** Influence of maintenance rate on maintenance, repair and outage energy cost for busses. Actual values for maintenance rates (mult.factor = 1): outdoor: 3 /yr, indoor: 1,2 /yr, gas isolated: 0,2 /yr

In figure 8 the influence of maintenance rates of the system busses (plus breakers) on operation costs is demonstrated. The curves suggest that the choice of maintenance intervals is based on too optimistic assumptions concerning the effect of maintenance activities on component reliabilities. Thus, maintenance intervals should be reduced.

## 5 PRACTICAL EVALUATION OF MULTI-STAGE OUTAGE MODEL PARAMETERS

It was shown in chapter 2 that realistic values for the probabilities of system improvement  $pf$  and  $pm$  can be estimated by logical considerations. Defect detection probabilities and repair rates can be extracted from maintenance reports. However, for the assessment of degradation rates ( $\lambda_1, \lambda_2$ ) and allocation of outage rates to degradation states either experience must exist concerning the outage behavior of components when maintenance is suspended, or the physical processes leading to system degradation have to be known in detail. Presently, physical degradation models exist for cables only. Since for electrical network components maintenance is performed regularly, there is no practicable way to evaluate the outage rates of the degradation states from outage statistics. Thus, the evaluation has to be based on assumptions. The principles of this approach are described below.

At first, on the basis of information gained from failure statistics and network operator experience, for each component type (transformer, bus, breaker etc.) an outage probability distribution  $F_w$  is constructed, which would result if no preventive maintenance would have been performed.

$$F_w(T) = \text{Prob.}(TTF < T) \quad (4)$$

TTF time to failure

Generally,  $F_w$  will be a distribution with its maximum time to failure (TTF) being smaller than the TTF of the real outage probability distribution where maintenance is taken into account. In the second step the parameters of the outage model of figure 2 are adapted in such a way that the assumed outage distribution is reproduced by the distribution, computed by a time-domain analysis for the model. (For this evaluation the same outage model as for hazard rate calculations in chapter 2 is used). Efficient parameter estimation techniques will be the topic of future research.

## 6 CONCLUSIONS

Markov-theory is restricted to constant transition rates, whereas in reality outage rates are increasing with operation time. Thus, using conventional Markov- models for maintenance planning, maintenance can be justified only for unrealistic high ratios of outage costs to maintenance costs. To overcome these restrictions a three-stage outage model is proposed in which each stage of system

degradation is characterized by an individual outage and maintenance rate. Due to its flexibility this model can be used for simulation of different maintenance strategies like time-based or condition-based maintenance. The model can be included into existing software for stationary network reliability calculations without major modifications. Some simplifications necessary for its implementation are admissible for power systems. Evaluation of the influence of component renewals on system reliability is not possible by this model. For such applications time-domain-simulations have to be performed.

Calculations with the proposed model for synthetic and real systems delivered some principal results: Maintenance activities should be planned carefully to avoid unnecessary delay of start and to guarantee short maintenance durations. Quality of maintenance should be sufficiently high to transfer the system back to states with lower degradation levels. If the system is returned to an "as new" state by repair after forced outages (e.g. due to replacement of faulted components by new ones), preventive maintenance will rather not improve system performance. Into maintenance planning considerations societal costs of not supplied energy should be included. Maintenance cost can be reduced by application of condition-based instead of time-based maintenance. However, distinct cost savings can be reached only if operators dispose of efficient devices for state condition monitoring.

Parameter estimation for combined maintenance and repair models is a problem still to be solved.

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