VERIFICATION OF A PROBABILISTIC MODEL FOR A DISTRIBUTION SYSTEM WITH INTEGRATION OF DISPERSED GENERATION

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Abstract – In order to assess the present and predict the future distribution system performance using a probabilistic model, verification of the model is crucial. This paper illustrates the error caused by using traditional Monte Carlo (MC) based probabilistic load flow (PLF) when involving tap-changing transformers. This is due to the time-dependent feature of tap position, which is not taken into account in the traditional MC method. A modified MC method using block sampling is proposed to take into account the time-dependent feature of the tap position. Block sampling estimates the tap position in the load flow calculation at the present hour by using results obtained from the previous hours. Simulation results show big improvement when block sampling is used as compared to the traditional MC method. Finally, 100 simulation runs of the PLF are performed to further ascertain the accuracy of the results obtained from the developed probabilistic model.

Keywords: dispersed generation, model verification, Monte Carlo, probabilistic load flow, tap position

1 INTRODUCTION

The Danish distribution systems are highly integrated with dispersed generation (DG), both renewable and non-renewable such as wind turbines (WTs) and combined heat and power (CHP) plants [1]. The power generation from WTs is fluctuating throughout the year due to the uncontrollable prime source, wind speed. CHP units are switched on and off by regulating the power, which reflects the heat demand and electricity tariff. The load demand is also varying with a daily behavior. Therefore, the power fluctuation from DG units and load demand introduces additional uncertain factors (besides network component outage rate) to the normal operation and control of the distribution systems. In order to take into account these uncertain factors when analyzing the system behavior under normal operating conditions, such as voltage variations and system losses, a deterministic approach is no longer sufficient. Instead, a probabilistic approach, e.g. probabilistic load flow (PLF), is needed to obtain a realistic impression of the system performance. In addition, the European standard EN50160 for voltage quality is also specified using probability [2]. A probabilistic analysis provides more information when evaluating the distribution system against the requirement in the standard.

The probabilistic model of a distribution system needs to be validated so that it can be used, e.g. to predict the future behavior of the system with a relatively small number of calculations. The validation procedure is twofold. The first one is to validate the simulation results against the measurement results. For load flow (LF) analysis, voltages obtained from a LF calculation with input of statistical measurement data (SMD) of power in chronological order should be validated against measured voltages. The purpose of this kind of state estimation, or referred to as multi-load flow (MLF) analysis here, is to validate the system model against the physical distribution system. The second one is to verify the PLF, e.g. using a Monte Carlo (MC) method, with respect to the results from the MLF. This verification can be done by judging certain criteria, e.g. the match of cumulative distribution function (CDF) curves and whether or not the 95% confidence intervals of mean voltage values obtained from the PLF are a good estimation of the mean values obtained from the MLF. The verification of PLF needs to be carried out especially when the first validation procedure is not possible, e.g. due to unavailable SMD of voltages.

The verification is ignored in most of the literature on PLF [3][4][5]. Usually, the PLF using MC is used as a reference to validate other simplified algorithms of PLF. However, the validity of the results from the MC simulation is seldom addressed. The results from PLF using MC with simple random sampling (SRS) may not be reliable if time-dependent parameters are not modeled correctly. One such time-dependent parameter is the tap position of tap-changing transformers. Some papers have discussed the implementation of voltage control settings to the PLF [6] [7], and some have also indicated the time-dependent character of the tap position [8], i.e. the operation of the transformer tap at the current hour is based on that at the previous hour. However, results from PLF using MC are taken for granted and not verified. Using SRS will lose the information of tap position at the previous hour of the chosen individual. The problem induced, due to the incorrect tap positions, is that the 95% confidence intervals of mean voltages obtained from the PLF do not provide proper interval estimates of the actual mean values of the voltages.

This paper focuses on the verification of the PLF using MC with respect to the results from MLF. First of all, the basic concept of the MC method is discussed. Secondly, a general PLF equation including tap position is presented. Then, one of the examining criteria, confidence interval, is presented and the limitation of the PLF using SRS is explained. The principle of block sampling and its capability of dealing with time-dependent parameters are introduced and demonstrated in detail. Finally, the MC based PLF using both SRS and block sampling is performed based on a local distribution network in Nordjylland in Denmark and the corresponding measured data from this site.

2 METHODOLOGY

2.1 MC Simulation

The IEEE standard defines MC simulation as a deterministic simulation in which random statistical sampling techniques are employed such that the result determines estimates for unknown values [9]. The two main features of MC simulation are random number generation and random sampling.

There are different algorithms to generate pseudo-random numbers, such as multiplicative congruential algorithms, Marsaglia's subtract with borrow algorithm and the Mersenne Twister algorithm. In this paper, the Mersenne Twister algorithm implemented in Matlab is used. The algorithm generates uniformly distributed random numbers U within the interval $[2^{-53}, 1\text{-}2^{-53}]$. Therefore, a random sampling of a population with size N can be performed by using:

$$PR = \operatorname{ceil}\left(U \times \mathrm{N}\right) \tag{1}$$

where PR is the position of the random chosen individual in the population; function ceil( ) is to round the elements to the nearest integers greater than or equal to the elements. Another way to perform random sampling of a population can be done by inverse-cumulative distribution function (inverse-CDF) sampling:

$$x = \mathrm{F}_X^{-1}\left(U\right) \tag{2}$$

where x is the chosen individual from the stochastic variable X and F_X( ) is the CDF of the stochastic variable X. Equation (2) can be used to generate a random sample when the probabilistic distribution of a stochastic variable is known; whereas equation (1) can only be used for sampling a set of statistical data.

2.2 PLF formulation with time-dependent parameters

There are several time-dependent parameters in power systems, e.g. tap position of tap-changing transformers, connection status of capacitor banks, energy storage devices, etc. These time-dependent parameters are considered differently within different time frames of analysis, such as transient analysis and steady-state analysis. This paper focuses on the tap position of a tap-changing transformer in a steady-state analysis (or hourly analysis). The tap-changing transformer with admittance of y_t and off-nominal tap ratio of p and the corresponding π–equivalent circuit is shown in Figure 1, where the tap is at the same side as bus m. It is noted that the change of the tap position leads to the change of both the diagonal and off-diagonal elements of the admittance matrix at the corresponding buses in the LF algorithm.

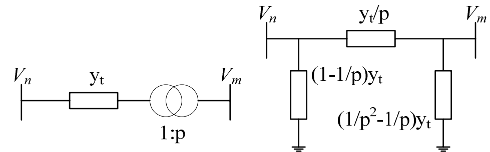

Figure 1 Tap-changing transformer (left) and its π–equivalent circuit (right)

For an electrical system with tap-changing transformers, the conventional PLF equation at i^{th} observation (i^{th} observation in the sample is corresponding to j^{th} hour in the statistical data) is expressed as:

$$\mathbf{Y}_i = \mathrm{g}(\mathbf{X}_i, \mathbf{p}_i) \tag{3}$$

where g( ) is the non-linear LF equations; $\mathbf{Y}_i$ is the input vector of active and reactive power at i^{th} observation; $\mathbf{X}_i$ is the system state vector at the i^{th} observation; $\mathbf{p}_i$ is the corresponding tap ratios of transformers. For the k^{th} tap-changing transformer, the tap ratio is adjusted according to:

$$\mathrm{p}_i^k = \begin{cases} \mathrm{p}_0^k + \Delta\mathrm{p}^k & V_i^k < \mathrm{V}_{\min}^k \\ \mathrm{p}_0^k, & \mathrm{V}_{\min}^k \le V_i^k \le \mathrm{V}_{\max}^k \\ \mathrm{p}_0^k - \Delta\mathrm{p}^k & V_i^k > \mathrm{V}_{\max}^k \end{cases} \tag{4}$$

where k is the transformer number; Δp is the corresponding tap adjustment step; V_i is the corresponding voltage at the tap-regulated bus and $[\mathrm{V}_{\min}, \mathrm{V}_{\max}]$ is the corresponding regulated voltage range. p_0 is the predefined initial tap ratio of the transformer, e.g. corresponding to the reference position, or the tap ratio of the $(i\text{-}1)^{\text{th}}$ observation. Note that $(i\text{-}1)^{\text{th}}$ observation in the sample does not correspond to $(j\text{-}1)^{\text{th}}$ hour in the statistical data. The tap position is readjusted according to equation (4) until the voltage V_i is within the interval $[\mathrm{V}_{\min}, \mathrm{V}_{\max}]$ or the tap reaches its limit positions. The time-dependent feature of the tap position, i.e. the relation between the tap positions at two adjacent hours, however, is not observed in equation (4). Although this is normally the case as the information of tap position is not available in general, the PLF using equation (3) provides biased results of the system state. (The biased results will be shown later.) In other words, the standard MC based PLF needs to be modified so as to provide unbiased results.

The proposed PLF equation considers the time-dependent feature of tap-changing transformers and is expressed as:

$$\left[\mathbf{Y}_i, \hat{\mathbf{p}}_{j-1}\right] = \mathrm{g}(\mathbf{X}_i, \mathbf{p}_i) \tag{5}$$

where $\hat{\mathbf{p}}_{j-1}$ is the vector of the estimated transformer tap ratios at $(j\text{-}1)^{\text{th}}$ hour and the tap ratio of k^{th} tap-changing transformer is instead adjusted according to:

$$\mathrm{p}_i^k = \begin{cases} \hat{\mathrm{p}}_{j-1}^k + \Delta\mathrm{p}^k & V_i^k < \mathrm{V}_{\min}^k \\ \hat{\mathrm{p}}_{j-1}^k, & \mathrm{V}_{\min}^k \le V_i^k \le \mathrm{V}_{\max}^k \\ \hat{\mathrm{p}}_{j-1}^k - \Delta\mathrm{p}^k & V_i^k > \mathrm{V}_{\max}^k \end{cases} \tag{6}$$

The estimated tap ratios $\hat{\mathbf{p}}_{j-1}$ are obtained by using the block sampling method, which is discussed in section 2.5. The PLF using equation (5) provides an unbiased estimation of the results of the system states for an electrical system with time-dependent devices.

2.3 Confidence Interval

Due to the sampling variability of random sampling, a statistic, e.g. of mean value of a voltage (i.e. the estimated mean voltage value), changes from one sample to another. In order to infer the parameter of the mean voltage value (i.e. the real mean voltage value) from one single statistic, confidence interval of the statistic is calculated. A confidence interval of confidence level C can be calculated by:

$$\bar{x} \pm z^* \frac{s}{\sqrt{n}} \tag{7}$$

where $\bar{x}$ is the statistic of the mean value of a stochastic variable; s is the statistic of the standard deviation of the stochastic variable; n is the sample size; z^* is the critical value corresponding to the confidence level C for a normal distribution curve, e.g. z^* is 1.96 for the 95% confidence level [10]. It can be stated that the real mean value of a stochastic variable falls into the calculated confidence interval $\left[\bar{x} - 1.96\frac{s}{\sqrt{n}}, \bar{x} + 1.96\frac{s}{\sqrt{n}}\right]$ for 95% of all the cases.

2.4 SRS

A SRS is a method of selecting n individuals out of N such that every one of the distinct samples has an equal chance of being drawn [10]. The random selection of SRS does not take into account the time dependence among successive hours of system inputs, e.g. load demands and CHP generation. Performing a MC based PLF with SRS on a system with time-dependent devices such as tap-changing transformers may inevitably lead to incorrect results, e.g. the real mean voltage value does not fall into the corresponding 95% voltage confidence interval obtained from the PLF (biased voltage results). Therefore, SRS needs to be modified so that the tap position can be modelled properly in the PLF.

2.5 Block Sampling

The main purpose of the block sampling is to provide an estimation of the tap position, in addition to the random sampling. According to equation (5), the information of the tap position at (j-1)th hour is needed in order to perform the PLF calculation at i^{th} observation. Apparently, $\mathbf{p}_{j-1}$ can be obtained from the PLF equation at (j-1)th hour, i.e.

$$\left[\mathbf{Y}_{j-1}, \mathbf{p}_{j-2}\right] = g(\mathbf{X}_{j-1}, \mathbf{p}_{j-1}) \tag{8}$$

The values of $\mathbf{p}_{j-2}$ can be obtained from the PLF equation at (j-2)th hour. The procedure may be carried out further back to several hours before. If it is assumed that $\mathbf{p}_{j-2}$ are all at the reference position, the estimated tap position at (j-1)th hour $\hat{\mathbf{p}}_{j-1}$ can be obtained through equation (8). The method samples several observations at successive hours with a random number. Therefore, it is referred to as the block sampling. The principle of the block sampling is also illustrated with the diagram shown in Figure 2, where B is the size of the block. Block sampling can also be understood as sampling B individuals at successive hours at a time and performing PLF calculations with the first (B-1) individuals so as to estimate the tap position for the PLF calculation with the last individual.

Block sampling is the same as SRS when the block size B equals 1.The larger B is, the better the estimation of $\mathbf{p}_{j-1}$ can be. For one generated random number, a total number of B PLFs are carried out to obtain one output value of the system state. Therefore, the computation time increases as the block size B increases.

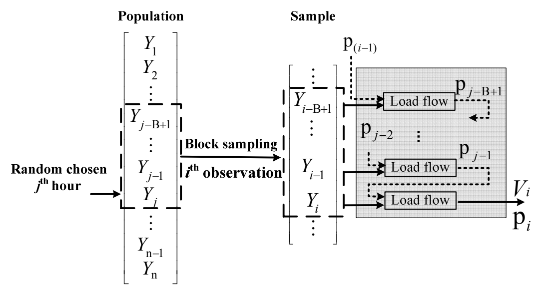

Figure 2: Principle of block sampling.

3 RESULTS AND DISCUSSION

3.1 Network Description

The PLF using MC is performed on a network based on a local 20 kV distribution system at Støvring in Nordjylland in Denmark. The configuration of the distribution network is shown in Figure 3.

There are in total 18 buses and 17 branches of the network. Bus 1 is chosen as the slack bus. There are three main feeders connected to the Støvring substation. Feeder 'STKV' is where CHP units are connected. There are 3 CHP units, each of which has a rating of 3 MW. The CHP units are controlled in such a way that the power factor of the units should be higher than or equal to 0.94, both inductive and capacitive. The power factor of the CHP units for 7 months (5160 hours) is shown in Figure 4, calculated according to the corresponding measured active and reactive power. CHP units are treated as PQ nodes in the PLF algorithm.

There are 3 fixed-speed WTs connected to the end of the 'SØRP' feeder. The rating of each WT is 600 kW. Due to the unavailability of the reactive power measurement data of wind power, the WTs are treated as PQ nodes in the PLF algorithm with a unity power factor. This introduces a certain degree of error to the final results. There are 3 loads connected at the same feeder. The rest of the load and feeders connected to the Støvring substation are lumped as the 'REST' load.

Both 150/60 kV and 60/20 kV transformers are tap-changing transformers with the tap at the secondary side. The set points of the two transformers are 60.2 kV

(1.0033 p.u.) and 20.5 kV (1.025 p.u.), respectively. The 150/60 kV transformer has 13 tap steps, with maximum 4 steps above and 8 steps below its set point. The 60/20 kV transformer has 21 tap steps, with maximum 11 steps above and 9 steps below its set point. Each tap step of both transformers corresponds to 0.0166 p.u. in voltage magnitude. For example, if the voltage at bus 5 exceeds the limit [1.0084, 1.0416] p.u., the tap adjusts one step down or up at a time to bring the voltage within the limit. According to the actual operating situation of the transformers, the tap adjusts around 1 or 2 times a day for most of the cases.

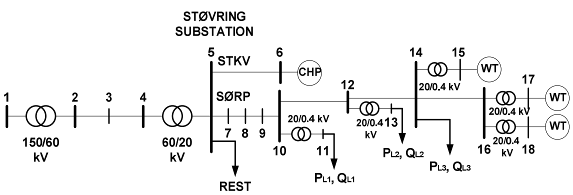

Figure 3: Configuration of the simulated distribution network.

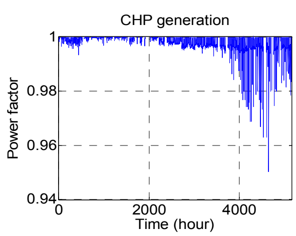

Figure 4: Power factor of CHP units.

3.2 PLF and Data Input

The flow chart of a MC based PLF is shown in Figure 5. Both ways of random sampling by using equation (1) and (2) are implemented. Simulation results are similar to each other. For the following sections, only the results using equation (1) are shown. In fact, there is no adjustment of the tap at the 150/60 kV transformer due to the reason that it is directly connected to the slack bus. Therefore, although implemented in the PLF algorithm, the tap adjustment of the 150/60 kV transformer is not included in the flow chart shown in Figure 5 for simplification.

The tap position of the first LF calculation starts from the reference position (at position 0). This assumption does not affect the simulation results so long as the sample size is not extremely small, e.g. less than 10, for the reason that the tap will anyway readjust according to the actual loading situation. Tap positions of following LF calculations are always based on the previous LF calculation. The difference between a MC using SRS and block sampling is how to obtain the tap positions of the transformers in the present LF as shown in Figure 5. For each LF calculation, the Newton-Raphson method is used. If the LF calculation does not converge at a certain individual of a sample, a new random number is generated to sample a new individual from the power inputs. However, as the data are from the measurement, the divergence of the load flow calculation does not occur during the simulation as expected.

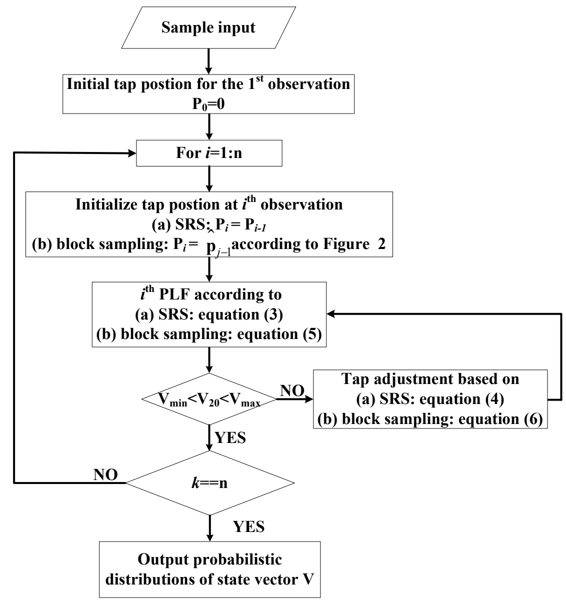

Figure 5: Flow chart of a MC based PLF.

In total, there are 7 months' data for each input stochastic variable. The statistical data are measured every 15 min and averaged to hourly data (5160 hours in total). A SRS with sample size of 2400 is performed on the input power data. Individuals at the same hour of different input variables are sampled, as they are actually a set of observations from the joint probabilistic distribution of all the correlated input power variables. The CDF curves of the SMD (solid curve) and samples from SRS (dotted curve) of total wind power generation, CHP generation, and total 'SØRP' load are shown in Figure 6, Figure 7 and Figure 8, respectively.

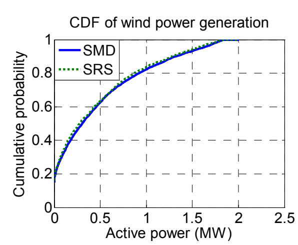

Figure 6: CDF of total wind power generation.

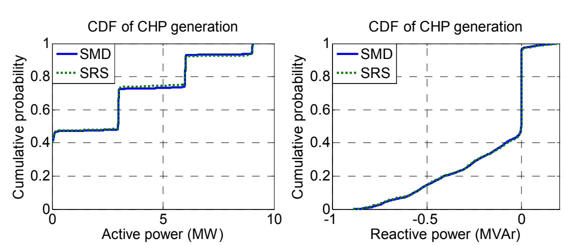

Figure 7: CDF of CHP generation.

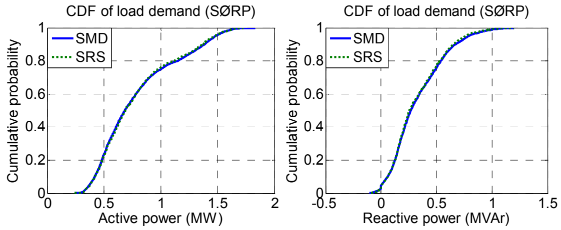

Figure 8: CDF of total load at SØRP feeder.

As shown in the power input figures, the resemblance of CDF curves between SMD and SRS is very good due to the unbiased random sampling. Although not shown here, the real mean value of each stochastic variable falls into its corresponding 95% confidence interval from the SRS.

3.3 Results

First of all, a MLF is performed with the SMD of power input in the time series order. The system states, such as voltage magnitude and voltage angle, obtained from the MLF are taken as reference for the results from the PLF. The results obtained from the MLF are also referred to as SMD in the following figures.

A MC based PLF calculations using SRS and block sampling are performed in Matlab, respectively. The effective tap position estimation using block sampling with a block size of 14 is demonstrated with the one at the 1394th observation as shown in Figure 9. The CDF curves of the voltages from MLF (corresponds to SMD) and PLF (corresponds to SRS) at the Støvring substation (node 5), CHP connection point (node 6), one WT connection point (node 15) and one load terminal (node 11) are shown in Figure 10. The corresponding mean values ($\bar{x}$), and 95% confidence intervals of the voltages as well as the mean voltage values from the SMD (μ) are summarized in Table 1. The corresponding simulation results from PLF using block sampling with sample size of 2400 and block size of 14 are shown in Figure 11 and Table 2.

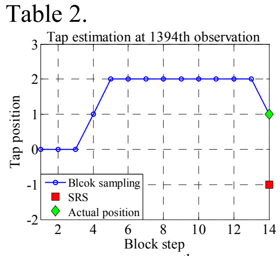

Figure 9 Tap estimation at 1394th observation.

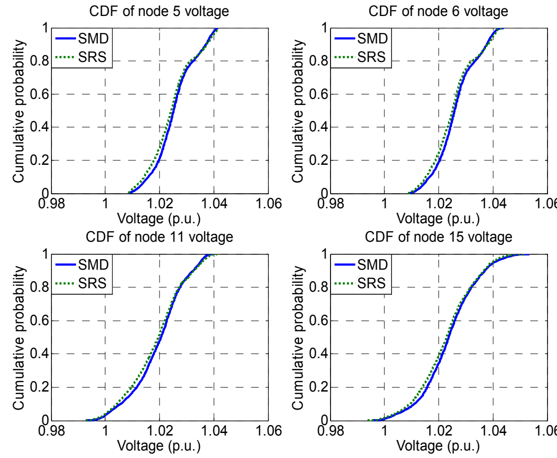

Figure 10: CDF curves of voltage magnitude at 4 critical nodes: SMD of MLF (solid), SRS (n = 2400) of PLF (dotted).

NODE	$\bar{x}$	95% CI	μ
5	1.0248	*[1.0245, 1.0251]*	**1.0257**
6	1.0255	*[1.0252, 1.0258]*	**1.0264**
11	1.0187	*[1.0183, 1.0191]*	**1.0196**
15	1.0227	*[1.0223, 1.0231]*	**1.0238**

Table 1: Mean value ($\bar{x}$) and 95% confidence interval (CI) of the voltage magnitude from SRS (n = 2400) and mean value from SMD (μ) at 4 critical nodes.

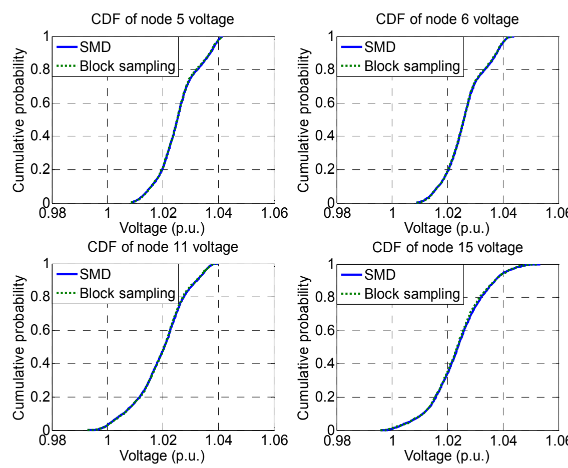

Figure 11: CDF curves of voltage magnitude at 4 critical nodes: SMD of MLF (solid), Block sampling (n = 2400, B = 14) of PLF (dotted).

NODE	$\bar{x}$	95% CI	μ
5	1.0256	*[1.0253, 1.0259]*	**1.0257**
6	1.0263	*[1.0260, 1.0266]*	**1.0264**
11	1.0195	*[1.0192, 1.0199]*	**1.0196**
15	1.0235	*[1.0231, 1.0239]*	**1.0238**

Table 2: Mean value ($\bar{x}$) and 95% confidence interval of the voltage magnitude from block sampling (n = 2400, B = 14) and mean value from SMD (μ) at 4 critical nodes.

3.4 Discussion

As shown in Figure 9, the actual tap position is at position 1. But the tap position used by the SRS is at position -1. The block sampling method with the block size of 14 correctly estimates the tap position at the 1394th observation. The match of the voltage CDF curves between SMD and SRS shown in Figure 10 are good in the tail region due to a large sample and the use of complete non-linear LF equation in the MC simulation. However, the voltage results from SRS are generally smaller than those from SMD.

As also shown in Table 1, the mean voltage magnitudes from SMD at all 4 nodes (mean values from the SMD) do not fall into the 95% confidence intervals from the SRS. Although the same combination of values of input stochastic variables is used, the output voltage results may still differ. The reason is due to the difference of tap positions in the PLF from those in the MLF as shown in Figure 12. The difference of tap positions is more than 30% of all the cases. As illustrated in the left of Figure 13, if the voltage at node 5 is at the position to which the dashed arrow points, with the tap adjusted 1 step up, it may change to the position to which the solid arrow points. However, both voltages are within the regulated limit. Therefore, in this case, the tap position solely depends on what it is at the previous hour. PLF using SRS loses the information of tap position at the previous hour, and thereby gives an incorrect estimation of the actual situation. This explanation can also be further justified by performing PLF using the same tap position as in MLF. The CDF curve of node 5 voltage is shown in the right of Figure 13. The match of the voltage between SMD and SRS are much better as compared to those shown in Figure 10. Although not shown here, the real mean values at all the 4 nodes fall into the 95% confidence interval.

However, the information of tap position is usually not available. Therefore, the PLF using block sampling is developed to deal with the time-dependent feature of tap position in MC simulation. As shown in Figure 11 and Table 2, the PLF using block sampling with sample size of 2400 and block size of 14 gives much better results than those obtained using SRS. All the 4 real mean voltage values fall into the 95% confidence interval obtained from PLF. The reason is due to the improvement of the accuracy of the tap position estimation by using block sampling, as shown in Figure 14. With the block sampling, the difference of the tap positions is less than 5% of all the cases.

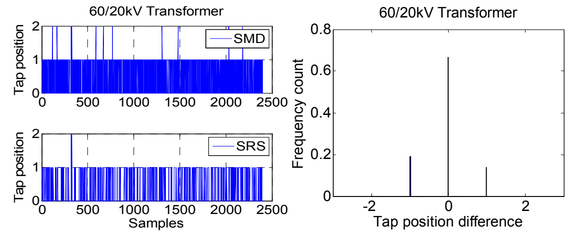

Figure 12: Tap position from SMD and SRS (left), histogram of tap position difference (right).

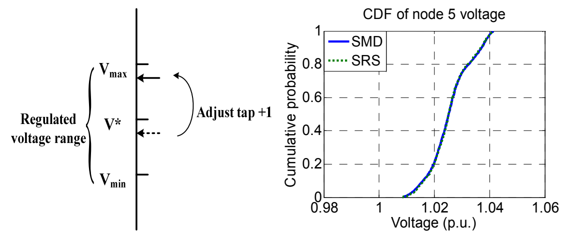

Figure 13: Tap adjustment diagram (left), CDF curves of voltage magnitude at node 5 (right).

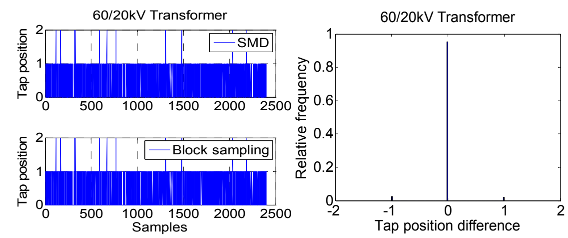

Figure 14: Tap position from SMD and block sampling (left), histogram of tap position difference (right).

In order to demonstrate that the improvement of the results by using block sampling is not due to chance, 100 simulation runs of the PLF using block sampling are performed for different sample sizes and different block sizes. The relative frequencies that the real mean voltage values at all the 4 nodes fall into the corresponding estimated 95% confidence intervals are summarized in Table 3. The tendency is evident that the larger the block size is, the higher the relative frequency is. It is also noted that, especially with small block size such as 1, the larger the sample size the worse is the result. This is mainly due to the reasons that the range of the confidence interval is shorter with larger sample sizes, and that the discrepancy of tap position between MLF and PLF is also larger. Although the relative frequencies with block size of 12 and above is not exactly 95%, the improvement of the results by using blocking sampling is clear as compared to the traditional SRS, i.e. with block size of 1. The relative frequency approaches 95% while the block size increases. More simulation runs, e.g. 1000, will give more stable values of the relative frequency.

SAMPLE SIZE		BLOCK SIZE				
		1	**6**	**12**	**18**	**24**
	100	62%	76%	87%	93%	86%
	200	53%	75%	86%	97%	93%
	400	55%	53%	90%	87%	91%
	600	24%	59%	90%	92%	93%

Table 3: Relative frequency during 100 simulation runs of real mean voltage falling into the 95% confidence interval.

4 CONCLUSION

This paper has first of all shown the inaccurate voltage results from PLF using MC with SRS as compared to the corresponding SMD. One consequence is that the calculated 95% confidence intervals from PLF do not provide a good estimation of the real mean voltage values. The reason has been demonstrated thoroughly, which is due to the ignorance of the time-dependent feature of the transformer tap position. PLF using MC with block sampling has been proposed to deal with the time-dependent feature of tap position, so that more accurate results from PLF can be obtained. As a result, the verified probabilistic model of the distribution system integrated with DG units can be used to assess and predict the behavior of the system.

Block sampling, however, can only be used to improve the results of MC simulation when the statistical data are available in chronological order. In other words, block sampling is not applicable to those analyses with only theoretical probabilistic distributions of input stochastic variables. This is due to the reason that the corresponding hour number in the statistical data of the selected individual of the sample is needed in order to perform the block sampling. How to deal with the same issue provided with only theoretical probabilistic distributions can be part of the future work. In addition, it is illustrated in [11] that there is a time-dependent feature with energy storage devices, i.e. the dependence structure between neighboring values in the sequence. Block sampling can be used to deal with the time-dependent feature of energy storage devices. Detailed analysis of distribution systems with energy storage devices will be carried out in the future.

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