OPTIMAL FAULT ANALYSIS FORMULATION FOR UNBALANCED DISTRIBUTION SYSTEMS CONSIDERING FAULT RESISTANCE ESTIMATION

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Abstract – Fault resistance (R_F) is a critical parameter of faulted power systems due to its stochastic nature. If not considered, R_F may interfere in fault analysis and protection schemes efficiency. To overcome these limitations, a novel fault analysis formulation for unbalanced three-phase distribution systems is proposed. The formulation is based on short-circuit analysis through bus impedance matrix (Z_{bus}) and a novel iterative algorithm for fault resistance estimation. The method's robustness and potential for online applications is demonstrated through numeric simulations.

Keywords: Bus impedance matrix, Distribution systems, Fault analysis, Fault resistance, Power system protection

1 INTRODUCTION

Power systems are daily exposed to faults. A fault is defined as any failure which interferes with the normal current flow [1]. This phenomenon may be considered stochastic, and can be classified as temporary or permanent. Temporary faults are normally caused by lightning. In this case, system service is restored after approximately 20 cycles with the circuit breakers (CB) opened. Permanent faults are associated with bigger system damages. On these events, system restoration becomes maintenance crew dependant, which must search and repair the faulted line. The use of correct fault location techniques may restrict the search area, improving the restoration process.

Nevertheless, one-terminal data fault location algorithms may present poor performance for non-negligible fault resistances. The errors added according to R_F magnitude may interfere in the fault location process [2]-[3]. To analyze the accuracy and reliability of fault location algorithms, power systems protection engineers must simulate the faulted system using common Energy Management System (EMS) tools. Fault analysis and power flow techniques are commonly used, comparing the simulated results with the fault records.

Fault analysis is an important tool used for electric power system planning and operation. Three approaches are basically applied for such analysis: classical symmetrical components, phase variable and complete time-domain simulation [4]. Classical short circuit analyses of electrical power systems (EPS) use symmetrical component-based approaches [5]-[6]. However, for typical unbalanced power distribution systems (PDS) this method does not yield accurate results. This limitation is consequence to the inaccurately representation of specific distribution systems characteristics, as the presence of single-phase or double-phase laterals [7].

Thus, the symmetrical components approach has been substituted by phase variable based formulations for PDS fault analysis [8]. In the phase variable approach, system voltages and currents are related through the system impedance and admittance matrices, described on phase frame. In such case, the short-circuit analysis is evaluated using the bus impedance matrix. Several techniques have been proposed for the bus impedance matrix building, improving the time, memory and computational requirements [9]-[10].

However, fault analysis is still fault resistance dependant. Due to fault resistance stochastic nature, fault analysis studies normally supposes the fault path as an ideal short-circuit, with negligible fault resistance. To overcome such limitation, recent studies have proposed the usage of fault resistance estimation techniques [11]-[14]. In such cases a fault resistance estimate is provided, using symmetrical components or modal analysis techniques. Hence, the application of these methods is restricted to the application in balanced systems with equally transposed lines.

In order to improve the fault analysis studies for application in power distribution systems, a novel formulation is presented. The proposed technique develops an iterative method based on phase frame representation to estimate the fault resistance from one-terminal fault records. Using the obtained fault resistance estimate, the fault analysis is realized through the bus impedance matrix in phase frame representation.

This paper is organized as follows. Section II presents the fault resistance estimation technique. Section III presents the fault analysis formulation. The PDS test case-system is presented in section IV. Finally, sections

V and VI discuss the results and conclusions obtained from this work.

2 FAULT RESISTANCE ESTIMATION

Fault resistance is associated with the fault impedance path [15]. The proposed fault resistance estimation formulation uses an iterative process for such calculation. The method is developed on phase frame and uses as input data the sending-end voltages and currents, as well as system parameters, such as system topology, loads, and lines series impedances.

2.1 Mathematical Development

Referring to the single line-to-ground (SLG) fault illustrated in Fig.1, the sending-end voltages are given by (1), which describes the steady-state fault conditions:

$$V_{Sfa} = V_{Fa} + x \cdot \left(z_{aa} \cdot I_{Sfa} + z_{ab} \cdot I_{Sfb} + z_{ac} \cdot I_{Sfc} \right) \quad (1)$$

where

- V_{Sfm} relay point phase m voltage
- x fault distance
- z_{mm} phase m self impedance per unit length
- z_{mn} phases m and n mutual impedance per unit length
- I_{Sfm} relay point phase m currents
- V_{Fm} fault point phase m voltage
- m, n phases a, b, and c

$$V_{Fa} = Z_F \cdot I_{F_a} \quad (2)$$

- Z_F fault impedance
- I_{F_a} phase a fault current

Supposing the fault impedance strictly resistive and constant, (1) may be expanded into its real and imaginary parts, as given by (3):

$$\begin{bmatrix} V_{Sf_{a(r)}} \\ V_{Sf_{a(i)}} \end{bmatrix} = \begin{bmatrix} M_1 & I_{F_{a(r)}} \\ M_2 & I_{F_{a(i)}} \end{bmatrix} \cdot \begin{bmatrix} x \\ R_F \end{bmatrix} \quad (3)$$

where the subscripts r and i represent the real and imaginary components, R_F the fault resistance, and:

$$M_1 = \sum_{k=\{a,b,c\}} \left\lfloor z_{ak(r)} \cdot I_{Sf_{k(r)}} - z_{ak(i)} \cdot I_{Sf_{k(i)}} \right\rfloor \quad (4)$$

$$M_2 = \sum_{k=\{a,b,c\}} \left\lfloor z_{ak(r)} \cdot I_{Sf_{k(i)}} + z_{ak(i)} \cdot I_{Sf_{k(r)}} \right\rfloor \quad (5)$$

From (3), fault distance and resistance estimates may be obtained as function of the sending-end voltages and currents, as well as the line parameters, as given by:

$$\begin{bmatrix} x \\ R_F \end{bmatrix} = \frac{1}{M_1 I_{F_{a(i)}} - M_2 I_{F_{a(r)}}} \begin{bmatrix} I_{F_{a(i)}} & -I_{F_{a(r)}} \\ -M_2 & M_1 \end{bmatrix} \cdot \begin{bmatrix} V_{Sf_{a(r)}} \\ V_{Sf_{a(i)}} \end{bmatrix} \quad (6)$$

Fault distance and resistance independent equations are obtained from (6), as given by (7) and (8), respectively.

$$x = \frac{V_{Sf_{a(r)}} \cdot I_{F_{a(i)}} - V_{Sf_{a(i)}} \cdot I_{F_{a(r)}}}{M_1 \cdot I_{F_{a(i)}} - M_2 \cdot I_{F_{a(r)}}} \quad (7)$$

$$R_F = \frac{M_1 \cdot V_{Sf_{a(i)}} - M_2 \cdot V_{Sf_{a(r)}}}{M_1 \cdot I_{F_{a(i)}} - M_2 \cdot I_{F_{a(r)}}} \quad (8)$$

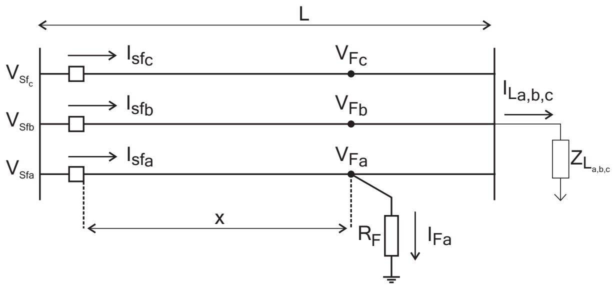

Figure 1: Single-line-to-ground fault.

Hence, from (7) and (8) the fault distance and fault resistance may be estimated. To yield such estimates, the system parameters, sending-end voltages and currents should be known. However, the fault current, which is an unknown variable, is present in both expressions. To estimate the fault current, an iterative procedure is developed and presented in the following.

2.2 Fault Current Estimate

Since the fault current (I_F) is the only unknown variable in the fault resistance expression (8), its estimate is calculated through measured data and system topology. Referring to Fig. 1, the fault current estimate may be calculated by the difference between the load current and the sending-end current, as given by (9):

$$I_{Fa} = I_{Sf\,a} - I_{La} \quad (9)$$

where

- I_{Sf_a} phase a sending-end current
- I_{L_a} phase a load current

Assuming the fault period load current different from the pre-fault load current, due to system dynamics, an iterative algorithm is developed to estimate the first [16] as follows:

I) Fault period load current is initially considered equal to pre-fault load current:

$$I_{La} = I_{Sa} \quad (10)$$

where I_{Sa} is phase a pre-fault sending-end current;

II) Fault current (I_{Fa}) is calculated through (9);

III) Fault location and resistance are estimated by (7) and (8), respectively;

IV) Fault point voltages are obtained by (11):

$$\begin{bmatrix} V_{F_a} \\ V_{F_b} \\ V_{F_c} \end{bmatrix} = \begin{bmatrix} V_{Sf_a} \\ V_{Sf_b} \\ V_{Sf_c} \end{bmatrix} - x \cdot \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_{Sf_a} \\ I_{Sf_b} \\ I_{Sf_c} \end{bmatrix} \quad (11)$$

V) An equivalent admittance matrix between the load and line section after the fault point is provided (12):

$$[Y_L] = \left((L - x) \cdot [Z] + [Z_L] \right)^{-1} \quad (12)$$

where Z is the line impedance matrix, Z_L is the load impedance matrix, and L is the total line length.

VI) New load current is calculated using the obtained equivalent admittance matrix and the fault point voltages:

$$[I_L]_{abc} = [Y_L] \cdot [V_F]_{abc} \quad (13)$$

VII) Return to step II with the updated load current and continue the iterative procedure until the fault resistance estimate converges to a certain value, as (14):

$$|R_F(n) - R_F(n-1)| < \delta \quad (14)$$

where δ is a convergence parameter, set according to the accuracy and computational time desired.

Fault resistance and distance are the outputs from the iterative procedure presented in this section. In this case, the fault distance may be used as a fault location estimate. However, for high fault resistance values, this estimate may be erroneous. An additive error is introduced to the fault distance estimate due to the fault resistance effects in the fault current estimate [2]. This additive error provides a much higher effect in fault distance estimate than the fault resistance one. Since classical power distribution systems are radial feeders with intermediate loads and laterals, the fault distance is used for convergence analysis, as will be presented in the following.

2.3 *Typical Power Distribution Systems*

Power distribution systems are typically composed by a main radial feeder with laterals and intermediate loads. The application of the proposed fault resistance estimate for PDS with laterals is obtained through equivalent radial systems. In this case, the laterals are substituted by Thèvenin equivalents and considered as intermediate loads. This procedure is executed to each lateral. In case of a PDS with several laterals, several fault resistance estimates will be calculated. For example, supposing a PDS with *n* laterals, *n* different equivalent systems are obtained, and consequently, *n* fault resistances are estimated. The identification of the correct faulted lateral, if unknown, and consequently the correct fault resistance estimate selection, may be obtained using a current pattern matching diagnosis [16]. Another possible approach for the faulted lateral identification is the usage of artificial intelligence, like artificial neural networks (ANN), as proposed by the authors in [17].

The equivalent radial systems are calculated through the transformation of lines and loads not part of the path being analyzed, into constant impedances along the system. Based on the pre-fault bus voltages and currents, estimated from three-phase power flow for radial power distribution feeders [18], the equivalent impedances may be calculated by (15):

$$Z_{eq\,km} = V_{S_k} \cdot I_{S_{km}}^{-1} \quad (15)$$

where

V_{Sk}	pre-fault voltage at bus *k*
I_{Skm}	pre-fault current flowing from bus *k* to bus *m*
k	bus index
m	lateral to be modeled as equivalent constant impedance

The fault resistance estimation algorithm considers initially the first line section. In case the fault is not diagnosed as internal, the analyzed voltages and currents are updated to the next section (*k+1*), using (16) and (17), respectively:

$$[V_{k+1}]_{abc} = [V_k]_{abc} - L \cdot [Z] \cdot [I_k]_{abc} \quad (16)$$

$$[I_k]_{abc} = [I_{k-1}]_{abc} - [I_{L_k}]_{abc} \quad (17)$$

where *L* is the section *k* line length and I_{L_k} is the bus *k* load current, given by (18).

$$[I_{L_k}]_{abc} = [V_k]_{abc} \cdot [Y_{L_k}] \quad (18)$$

With the updated voltages and currents, the proposed technique is executed again to the next line section. The process is repeated until the fault is diagnosed as internal.

3 FAULT ANALYSIS FORMULATION

Fault analysis is a fundamental tool used in power systems planning and operation. Due to different techniques available, the methodology choice is determined by the specific system characteristics. Power distribution systems are typically unbalanced, with untransposed feeders and single-phase loads. Accordingly to these specific conditions, the bus impedance matrix method [6] should be chosen. The application of this fault analysis technique on short-circuits studies allow the analysis of any fault type through modifications in the phase coordinate base-case impedance matrix. In this case, the typical PDS unbalanced nature is considered by the technique.

3.1 *Bus Impedance Matrix*

The base-case three-phase admittance matrix Y_{bus} is calculated from the three-phase sub-matrices of the system elements, as lines and transformers. The diagonal sub-matrix of a hypothetical bus *p* is obtained by (19), which represents the sum of all sub-matrices of elements connected to bus *p*.

$$[Y_{abc}]_{p,p} = \sum_{i=1}^{M} \left[\tilde{Y}_{abc}\right]_{i,p} \quad (19)$$

The off-diagonal sub-matrices are obtained from (20), where $[\tilde{Y}_{abc}]_{p,j}$ is the admittance matrix of the element between buses *p* and *j*.

$$[Y_{abc}]_{p,j} = -\left[\tilde{Y}_{abc}\right]_{p,j} \quad (20)$$

From (19) and (20), a *3n* x *3n* three-phase admittance matrix (Y_{bus}) is obtained, where *n* is the number of the power distribution system buses.

3.2 *Pre-Fault Calculations*

The fault analysis formulation uses a three-phase load flow specifically designed for radial distribution systems, based on the iterative ladder technique [18] to estimate the pre-fault system voltages. The applied technique considers the nonlinear characteristics of the feeder and performs a forward and a backward sweep to solve the load flow problem.

3.3 Fault Calculations

The fault voltages and currents from the three-phase impedance matrix are obtained through the superposition technique [7]. The method is based on different modifications in the bus impedance matrix (Z_{bus}), accordingly to the fault type. In this case, the base-case impedance matrix is modified to include the fault resistance estimate previously obtained.

Supposing the single-line-to-ground fault, as illustrated in Fig. 1, the base-case impedance matrix (Z_{bus}) must be augmented to include the R_F estimate. Hence, the fault resistance estimate is included in Z_{bus} as a new node K, yielding a $(3n+3)\times(3n+3)$ new impedance matrix Z_{new}. In this fault condition, the phase a fault current (I_{Fa}) may be calculated by (21):

$$I_{Fa} = \frac{-V_{Fa}}{Z_{new}(K,K)} \quad (21)$$

where V_{Fa} is the node K faulted phase a pre-fault voltage, and $Z_{new}(K,K)$ is given by:

$$Z_{new}(K,K) = Z(K,K) + R_{Fa} \quad (22)$$

whereas R_{Fa} is the fault resistance estimated previously by (8).

From the injected fault current (I_{Fa}) and the impedance matrix Z_{new}, the superimposed voltages at each bus can be calculated using (23):

$$\Delta V_i = Z_{new}(i,K)\cdot I_{Fa} \quad (23)$$

where $i = 1 \rightarrow (3n+3)$ and represent the bus index for each phase.

The sum from the superimposed voltages to each pre-fault bus voltage (V_{PF}), results on the bus voltages (V_F) during the fault period:

$$V_{F_i} = V_{PF_i} + \Delta V_i \quad (24)$$

Finally, from the fault period bus voltages and the admittance matrix is possible to calculate the fault period currents at each feeder (I_{ij}):

$$\lfloor I_{ij} \rfloor = \lfloor Y_{ij} \rfloor \cdot \left(\lfloor V_i \rfloor - \lfloor V_j \rfloor\right) \quad (25)$$

where

Y_{ij} admittance matrix between nodes i and j

V_i node i phase voltages vector

V_j node j phase voltages vector

where $i = 1 \rightarrow (n+1)$, which represents the bus index for each phase.

4 STUDY-CASE

In order to analyze the proposed technique performance, a real distribution feeder of the Electric Energy Distribution State Company (CEEE-D), southern Brazil was simulated with BPA's ATP/EMTP [19]. Also, a modified Fourier filter [20] was implemented to remove the decaying DC component and estimate the voltages and currents fundamental components.

The Y-connected with neutral grounding feeder, denominated Particular West 1 (*PW1*) is illustrated in Fig. 2. The system is composed by two load buses connected to a main feeder by two independent three-phase laterals. The line sections were modeled as RL series circuits, obtained from modified Carson's equations [18]. The three-phase impedance matrix per unit length is given by (26):

$$Z = 1\times10^{-3}\cdot\begin{bmatrix} 0.260+j0.4516 & 0.2998+j0.1550 & 0.2998+j0.1550 \\ 0.2998+j0.1550 & 0.4260+j0.4516 & 0.2998+j0.1550 \\ 0.2998+j0.1550 & 0.2998+j0.1550 & 0.4260+j0.4516 \end{bmatrix} \quad (26)$$

where the per unit length line impedance matrix (Z) is given in Ω/meter.

The *PW1* system unbalanced loads were modeled as constant impedances Y-connected with neutral grounding and are given in Table I.

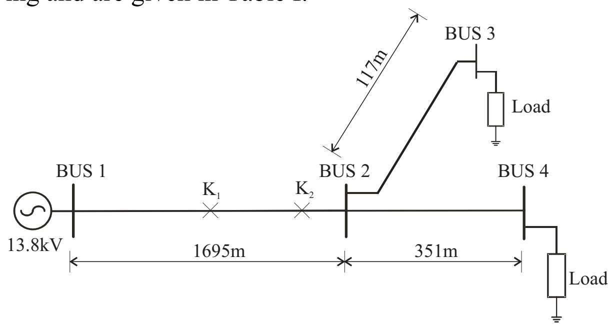

Figure 2: Distribution feeder *PW1*.

Bus	Phase A $R_a + jX_a$ (Ω)	Phase B $R_b + jX_b$ (Ω)	Phase C $R_c + jX_c$ (Ω)
3	538.8 + j109.4	484.92 + j98.46	592.68 + j120.34
4	538.8 + j109.4	484.92 + j98.46	592.68 + j120.34

Table 1: Distribution feeder *PW1* load data.

5 TEST RESULTS

The study system *PW1* had 33 different fault locations and different fault resistances simulated using ATP/EMTP. This test set yielded 167 different simulated faults. In this section, the proposed technique performance is analyzed in two different approaches: fault resistance estimates and fault analysis. The results represent exclusively the proposed formulation performance, without any additional errors, as due to measurement devices.

5.1 Fault Resistance Estimation

The obtained results for the single-line-to-ground faults with 0.001, 10, 20, 50, and 100 Ω are presented in Table 2. The errors from fault resistance estimates were calculated considering the absolute difference between the estimated and simulated fault resistances values, as given by (27):

$$Error = \left| R_{F_{calc}} - R_{F_{real}} \right| \quad (27)$$

where R_{Fcalc} and R_{Freal} are the estimated and the simulated fault resistances, respectively.

5.1.1 Fault Resistance Effect

Analyzing the results, it can be seen the proposed fault resistance estimate technique is not affected by the fault resistance value. The formulation yielded similar errors for 0.001 Ω and 100 Ω. In this case, an average error between 0.0014 Ω and 0.0071 Ω is introduced by the proposed technique in the fault resistance estimate. As well as, a maximum error equal to 0.024 Ω is introduced by the technique in the most severe simulated test-case (R_F = *100 Ω*). For lower simulated fault resistances, the maximum associated errors are also lower. For fault resistances equal do 10, 20, and 50 Ω, the highest fault resistance estimates were 10.0026 Ω, 20.0033 Ω, and 50.0077 Ω, respectively. In the solid fault test-case (R_F = *0.001 Ω*), the maximum error produced by the technique was 0.002 Ω.

5.1.2 Fault Distance Effect

The fault resistance estimation procedure was also analyzed in the fault distance aspect. In this case, the obtained results for the fault resistance estimate are analyzed over the 33 simulated fault points. Figure 3 illustrates the fault distance effects on the fault resistance estimates. The obtained results over the 33 fault points had similar fault resistance estimates, with negligible differences between each estimate. In this case, the fault distance has an insignificant effect on the proposed fault resistance estimates.

Simulated R_F (Ω)	Average Estimate R_F (Ω)	Maximum Absolute Error (Ω)	Average Error (Ω)
0.001	0.0014	0.002	0.0014
10	10.002	0.0026	0.002
20	20.003	0.0033	0.003
50	50.007	0.0077	0.0071
100	100.02	0.024	0.0022

Table 2: Fault resistance estimates for single line-to-ground faults.

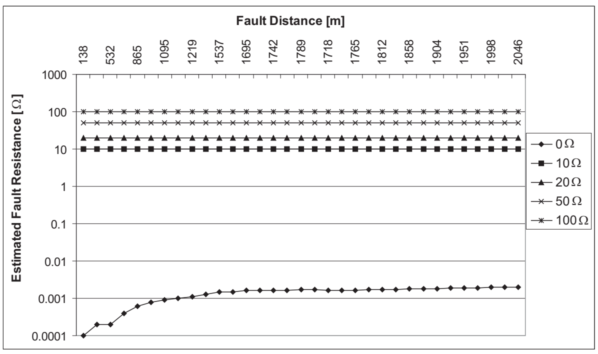

Figure 3: Fault distance effect on the fault resistance estimate.

5.2 Fault Analysis

The proposed fault analysis method was tested considering the *PW1* feeder simulations and estimated fault resistances. Two different fault locations are here analyzed. The investigated fault points K_1 and K_2, as illustrated in Fig. 2, are located at 865 meters and 1653 meters from the substation, respectively.

In this section, the estimated fault current and bus voltages are analyzed. The obtained results are presented in Tables 4 to 8 and compared with the simulated cases. The percentage errors were calculated considering the absolute difference between the estimated and simulated values, as given by (28):

$$Error(\%) = \left| \frac{f_{calc} - f_{simul}}{f_{simul}} \right| \times 100 \qquad (28)$$

where f_{calc} and f_{simul} are the calculated and simulated fundamental components of the voltages or currents.

The fault analysis performance was also analyzed considering the fault distance estimates, obtained from (7). In this case, the fault location technique performance is analyzed. Table 3 presents the fault distance estimates for the fault point K_2, considering different fault resistance values.

Simulated R_F (Ω)	Fault Distance	Error (%)
0.001	1649.5	0.21
1	1649.5	0.21
5	1649.4	0.22
10	1649.1	0.24
20	1647.8	0.31
50	1640.4	0.76
100	1619.3	2.04

Table 3: Fault distance estimates for single line-to-ground faults at K_2 fault point.

The fault distances estimates presented in Table 3 show negligible errors for lower fault resistances values, resulting in errors smaller than 1% for faults until 50 Ω fault resistances. However, as the fault resistance increases, the associate errors to one-terminal fault distances formulations also increase [2]. Hence, considering the yielded results from the fault location technique considering a fault with R_F = *100 Ω* at K_2, the associated error was close to 2%.

Table 4 presents the estimated fault currents at K_2 for a single line-to-ground fault considering the estimated fault distance. The maximum error yielded was 3.33% for a fault with negligible R_F. However, for faults with considerable fault resistances, the associated errors were close to zero. For fault resistances between 5 Ω and 100 Ω, the errors associated to the fault current estimates were between 0.28% and 0.17%.

The yielded results for fault current at K_2 considering the correct fault distance (1653 meters) are presented in Table 5. In such case, the highest error associated to the fault current estimate was 4.86%. The obtained results for solid faults (0.001 Ω) from the proposed fault analysis considering the estimated and the correct fault distance are presented in Tables 4 and 5. In this fault condition, the error associated to the difference between both fault current estimates was 1.53%, which represents a difference equal to 100 A. For higher fault resistances values, the proposed technique yielded the same fault currents in both conditions.

Table 6 and 7 presents the estimated voltages at each bus during a single-line-to-ground fault at K_2. The obtained results for a 1 Ω fault resistance, presented in

Table 6, illustrate a highest error equal to 7.71%, which represents an error of 322.19 Volts of the faulted phase bus voltage's The existence of transient phenomenon, which is considered by BPA's ATP/EMTP, but neglected by the bus impedance technique, contributes to these errors.

The comparison between the simulated and the calculated voltages for a single-line-to-ground fault at K_2 with fault resistance equal to 10 Ω is given by Table 7. In this test case, it can be seen a decrease to the errors obtained for a 1 Ω single-line-to-ground fault. On this fault condition, the highest error yielded was 1.62%, which represents a difference equal to 134.42 Volts between the simulated and the estimates voltages.

Table 8 presents the estimated bus voltages for a 20 Ω single line-to-ground fault at K_1. In this fault condition, the highest error obtained was 5.16%, which represents an error of approximately 0.4 kV.

Simulated R_F (Ω)	Estimated R_F (Ω)	Simulated Fault Current (A)	Estimated Fault Current (A)	Error (%)
0.001	0.0025	7198.4∟-48.85°	7438.1∟-45.58°	3.33
1	1.0017	4181.7∟-24.98°	4218.1∟-23.83	0.87
5	5.0019	1381.5∟-6.87°	1377.6∟-7.50	0.28
10	10.0022	741.96∟-2.91°	739.97 ∟-3.97°	0.27
20	20.0029	384.47∟-0.71°	383.62∟-2.00°	0.22
50	50.0076	157.10∟0.69°	156.81∟-0.75°	0.18
100	100.0236	79.10∟1.17°	78.97∟-0.32°	0.17

Table 4: Fault current estimates for single line-to-ground faults at K_2 fault point using the fault distance estimates.

Simulated R_F (Ω)	Estimated R_F (Ω)	Simulated Fault Current (A)	Estimated Fault Current (A)	Error (%)
0.001	0.0025	7198.4∟-48.85°	7548.1∟-46.46°	4.86
1	1.0017	4181.7∟-24.98°	4218.1∟-23.83	0.87
5	5.0019	1381.5∟-6.87°	1377.6∟-7.50	0.28
10	10.0022	741.96∟-2.91°	739.97 ∟-3.97°	0.27
20	20.0029	384.47∟-0.71°	383.62∟-2.00°	0.22
50	50.0076	157.10∟0.69°	156.81∟-0.75°	0.18
100	100.0236	79.10∟1.17°	78.97∟-0.32°	0.17

Table 5: Fault current estimates for single line-to-ground faults at K_2 fault point using the real fault distance (1653 m).

Bus	Phase	Simulated Voltage (V)	Estimated Voltage (V)	Error (%)
1	A	7769.99 ∟ -1.43°	7803.27∟-4.79°	0.43
1	B	7989.06 ∟-118.32°	7988.45∟-121.55°	0.01
1	C	7988.43 ∟ 121.79°	7989.16∟118.56°	0.01
2	A	4181.82∟ -20.77°	4504.29∟-20.77°	1.04
2	B	9361.50∟-130.79°	8787.99∟-130.79°	1.68
2	C	9343.74∟129.15°	9163.08∟129.15°	0.19
3	A	4181.98 ∟-24.97°	4504.17∟-20.76°	7.71
3	B	9360.42∟-130.78°	8786.84∟-130.79°	6.13
3	C	9343.34∟134.14°	9162.81∟129.15°	1.93
4	A	4182.28 ∟-24.96°	4503.02∟-20.72°	7.67
4	B	9358.26 ∟-130.79°	8783.51∟-130.77°	6.14
4	C	9342.52 ∟134.13°	9161.94∟129.15°	1.93

Table 6: Bus voltages estimates for single line-to-ground faults with R_F=1 Ω at K_2 using the real fault distance (1653 m).

Bus	Phase	Simulated Voltage (V)	Estimated Voltage (V)	Error (%)
1	A	7969.02 ∟1.11°	7972.21∟-2.11°	0.04
1	B	7989.94∟-118.32°	7989.14∟-121.55°	0.01
1	C	7987.92 ∟121.79°	7988.80∟118.57°	0.01
2	A	7419.40∟-2.91°	7443.42∟-3.72°	0.25
2	B	8307.51∟-120.17°	8174.08∟-121.56°	0.10
2	C	8045.72∟124.61°	8056.14∟122.29°	0.30
3	A	7419.15 ∟-2.91°	7443.16∟-3.72°	0.32
3	B	8306.97∟-120.17°	8173.51∟-121.56°	1.61
3	C	8045.57∟124.61°	8055.97∟122.30°	0.13
4	A	7418.65 ∟-2.92°	7442.02∟-3.70°	0.31
4	B	8305.87∟-120.18°	8171.45∟-121.54°	1.62
4	C	8045.26∟124.60°	8055.23∟122.30°	0.12

Table 7: Bus voltages estimates for single line-to-ground faults with R_F=10 Ω at K_2 using the estimated fault distance (1649.1 m).

Bus	Phase	Simulated Voltage (V)	Estimated Voltage (V)	Error (%)
1	A	7974.65∟2.2°	7975.05∟2.2°	0.01
1	B	7990.24∟-117.51°	7989.12∟-117.51°	0.01
1	C	7987.57∟122.6°	7988.84∟122.6°	0.02
2	A	7819.87∟1.04°	7416.54∟1.04°	5.16
2	B	8071.95∟-118.09°	8291.99∟-118.09°	2.73
2	C	7991.3∟123.32°	8012.51∟123.32°	0.27
3	A	7819.56∟1.04°	7416.27∟1.04°	5.16
3	B	8071.5∟-118.09°	8291.39∟-118.09°	2.72
3	C	7991.13∟123.32°	8012.36∟123.32°	0.27
4	A	7818.94∟1.03°	7415.09∟1.03°	5.16
4	B	8070.58∟-118.1°	8289.28∟-118.1°	2.71
4	C	7990.78∟123.31°	8011.68∟123.31°	0.21

Table 8: Bus voltages estimates for single line-to-ground faults with R_F=20 Ω at K_1 using the estimated fault distance (861.7 m).

6 CONCLUSIONS

In this paper, a novel fault analysis formulation using the bus impedance matrix and a fault resistance estimate is proposed. The technique is suitable for generic unbalanced power distribution systems with laterals and intermediate loads. The usage of fault records, provided by protection relays and digital fault recorders, allow the online application.

The proposed fault analysis consists of a fault resistance estimation algorithm and the bus impedance matrix used to calculate currents and bus voltages. Also, a fault distance estimate is yielded by the formulation. Test results show the robustness of the fault analysis method for contingency analysis used in electric power system planning and operation. The availability of a correct fault resistance estimate also provides a different set of possible EPS protection and operational studies.

The fault resistance estimation procedure is nowadays used as a software tool by a southern Brazilian electrical energy company.

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