ON-LINE STABLE STATE DETERMINATION IN DECENTRALIZED POWER GRID MANAGEMENT

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Abstract - Both the coordination of international energy transfer and the integration of a rapidly growing number of decentralized energy resources (DER) throughout most countries causes novel problems for avoiding voltage band violations and line overloads. Traditional approaches are typically based on global off-line scheduling under globally available information and rely on iterative procedures that can guarantee neither convergence nor execution time. In this paper we focus on stability problems in power grids based on widely dispersed (renewable) energy sources. In this paper we will introduce an extension of the DEZENT algorithm, a multi-agent based coordination system for DER, that allows for the feasibility verification in constant and predetermined time. We give a numerical example showing the legitimacy of our approach and mention ongoing and future work regarding the implementation and utilization.

Keywords: Voltage band violation, congestion management, distribution network, decentralized energy resources, power flow feasibility boundary

1 INTRODUCTION

The increasing integration of decentralized energy resources (DER) into distribution networks leads to new operational requirements in order to guarantee a secure and reliable energy supply. An increasing number of additional energy sources increases the complexity and amount of power flow distributions. In general this is taken into account by dimensioning the rated power of DER with respect to worst case power flow scenarios. This approach does not exploit the opportunities of increasing line usage rates by a coordinated operation of both, loads and DER on a particular network.

One of the major issues related to this coordination is the recognition of critical operational states from a small set of information; in most cases the complex power balance of a connection point. Standard algorithms cannot be used in this time critical environment, as they cannot guarantee their convergence and execution time. In this paper a new approach is presented that does have these properties and provides a coordinator with even more complex information than the standard algorithms do.

2 PREVIOUS AND RELATED WORK

In our earlier work on DEZENT [1, 2, 3, 4] we introduced a bottom-up principle of power distribution and balancing, as part of a completely decentralized management of renewable electric energy production and consumption on the basis of real-time multi-agent systems. For the sake of higher fault tolerance it exploits the widely distributed renewable source structure as a basis for efficient fault control [4, 7]: Failures would have a limited local or regional impact only, and every energy producer represents a potential back-up/ reserve facility. The completely decentralized approach adapts naturally to unpredictable situations including power failures: In the distributed landscape of production facilities breakdowns have typically a local origin, and DEZENT handles them in the same mode as for normal functioning in short time intervals of mere milliseconds. In [5, 6] we introduced an integrated staged management of distributed electrical power grids on the basis of our DEZENT multi-agent system and imposed the need for a fast and reliable real-time determination of unfeasible supply configurations especially under novel and increasingly complex power flow distributions.

So far a lot of research has been conducted on determining the feasibility boundaries in the domain of nodal power. In [5] the feasibility bounds in the body of active nodal power have been investigated. [6] analyzes the boundaries of feasible power flows and an analytical approach is presented to analyze their convexity properties. In [7] the convexity of sets of feasible power injections is investigated and evaluated in terms of financial transmission rights.. In this context we want to introduce an approach for the determination of the subspace of feasible combinations of nodal power in the domain of complex nodal power. Hereby we focus on distribution networks that have a single central feeder.

3 ORGANIZATION OF THE PAPER

The objective of this paper is to show that the power flow feasibility boundary can be derived from a geometrical interpretation of the basic constraints, like voltage band and rated line current.

Therefore, after some notation conventions and definitions in **section 4**, it will be shown in **section 5** that voltage band constraints and rated line currents lead to a non-convex subspace of feasible combinations in the domain of complex nodal voltage. It will fatherly be shown, that this non-convex subspace can be described using convex sets.

Section 6 will discuss the mapping of the before defined subspaces and sets from the domain of complex nodal voltage to the domain of complex nodal power, also called the parameter space.

After a basic numerical study in **section 7**, we present in **section 8** an example for the implementation of a numerical calculation of the subspace of feasible nodal power combinations in the parameter space.

The paper will be concluded by a discussion about the possible integration into a decentralized power grid management.

4 NOTATIONS & DEFINITIONS

$\underline{A}, \underline{A}_i$	Vector or matrix and i -th row vector
$\overline{A}_i$	i -th complex element of vector $\underline{A}$
$\overline{Y}_{ij}$	Serial admittance between node i and j
$\underline{CR}(\underline{A})$	Orthonormal base vector set spanning the column range of matrix $\underline{A}$
$\underline{RR}(\underline{A})$	Orthonormal base vector set spanning the row range of matrix $\underline{A}$
$\underline{NS}(\underline{A})$	Orthonormal base vector set spanning the nullspace of matrix $\underline{A}$
$\underline{LN}(\underline{A})$	Orthonormal base vector set spanning the left nullspace of matrix $\underline{A}$
$\underline{S}$	Diagonal matrix containing the singular values of a matrix
A	Set **A**
$diag(\underline{A})$	Diagonal matrix with the elements of vector $\underline{A}$ on its main diagonal

Definitions:

- $CR(\underline{A})$ is the image of the domain of $\underline{A}$.
- $RR(\underline{A})$ is the preimage of $CR(\underline{A})$
- $NS(\underline{A})$ is the preimage of $\underline{0}$
- $LN(\underline{A})$ is the image of $\underline{0}$

5 GEOMETRICAL INTERPRETATION OF STANDARD POWER FLOW ALGORITHMS

5.1 Constraints imposed by the voltage band

Traditionally, the operational state of a network can be described by a vector of complex nodal voltages. Every other operational state value of a network, like complex nodal power, line currents, etc. can be calculated from this voltage profile. These complex voltages are linked by the power flow equations and do not vary over time under normal (stable) operating conditions. Mathematically they can be interpreted as being completely independent from each other in the domain of complex nodal voltage. This is also true for the domain of complex nodal power, under the condition that one node is excluded forming the *reference node* that provides the balancing current (in the following the *reference node* is denoted with index 1).

Traditional algorithms deployed for the determination of voltage band violations or line overloads translate a certain point from the domain of complex nodal power into the domain of complex nodal voltage. In order to check for voltage band violations the translated point $\underline{V}$ in the domain of complex nodal voltage is verified against the following inequality for every node i of the network:

$$V_{\min,i} \le \left|\overline{V}_i\right| \le V_{\max,i} ; \forall i \in \{2,\ldots,n\} \tag{1}$$

where $V_{\min,i}$ specifies the lower voltage limit and $V_{\max,i}$ the upper voltage limit for a given node i .

The voltage $\overline{V}_1$ of the reference node is specified according to (2):

$$\overline{V}_1 = V_{ref} \cdot e^{j0°} \tag{2}$$

with $V_{ref} = const$.

Equation (1) can be interpreted geometrically. All complex nodal voltages of the same absolute voltage form a ring in the complex plane corresponding to their node. Thus, minimum and maximum voltages can be interpreted as convex bodies in the complex plane (see **figure 1**). The hull of the outer body consists of all combinations of complex nodal voltages $\overline{V}_i$ for all nodes i that fulfill the following condition:

Maximum Voltage:

$$\left|\overline{V}_i\right| \le V_{\max,i} ; \forall i \in \{2,\ldots,n\} \tag{3}$$

Embedded in this body of maximum voltages is the body of minimum complex nodal voltages:

Minimum Voltage:

$$\left|\overline{V}_i\right| \le V_{\min,i} ; \forall i \in \{2,\ldots,n\} \tag{4}$$

An operational state is feasible under the condition that the corresponding point in the domain of complex nodal voltage lies within the body, that is formed by the maximum voltage, but does not lie within the body formed by the minimal voltage magnitudes for every node of the network.

The two bodies defined for a node i can be represented as convex sets according to (5) and (6):

$$\mathbf{V}_{\max,i} := \left\{\underline{V} : \left|\overline{V}_i\right| \le V_{\max,i}\right\} \tag{5}$$

$$\mathbf{V}_{\min,i} := \left\{\underline{V} : \left|\overline{V}_i\right| \le V_{\min,i}\right\} \tag{6}$$

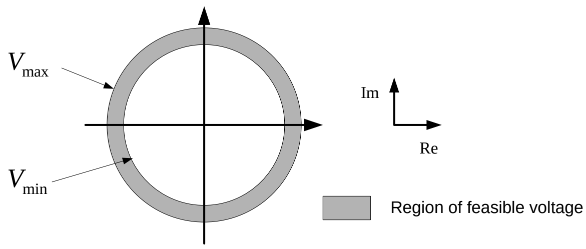

Figure 1: Concept of nested bodies

Additionally, the set including all voltage profiles with the required (constant) reference voltage can be formulated following (7).

$$\mathbf{V}_{ref} := \left\{ \underline{V} : \overline{V}_1 = V_{ref} \cdot e^{j0^\circ} \right\} \tag{7}$$

With (1)-(7), we state that a combination of complex nodal voltages $\underline{V}$ is feasible with respect to the voltage band constraints if:

$$\begin{aligned} &\underline{V} \in \mathbf{V}_{feasible,V} \\ &\Rightarrow \underline{V} \in \mathbf{V}_{ref} \wedge \underline{V} \in \mathbf{V}_{\max,i} \wedge \underline{V} \notin \mathbf{V}_{\min,i}; \forall i \in \{2,\ldots,n\} \end{aligned} \tag{8}$$

The sets $\mathbf{V}_{\max,i}$ and $\mathbf{V}_{\min,i}$ defined for every node i of the network can be combined according to (9).

$$\mathbf{V}_{\max} := \bigcap_{i=2}^{n} \mathbf{V}_{\max,i} \qquad \mathbf{V}_{\min} := \bigcup_{i=2}^{n} \mathbf{V}_{\min,i} \tag{9}$$

$\mathbf{V}_{\max}$ is constructed as an intersection of ($n-1$) convex sets and thus itself is convex. $\mathbf{V}_{\min}$ is also constructed from ($n-1$) convex sets, but as $\mathbf{V}_{\min}$ is a union of them, it is *not necessarily* convex.

Based on (7) and (9) the feasibility condition (8) can be expressed by:

$$\begin{aligned} &\underline{V} \in \mathbf{V}_{feasible,V} \\ &\Rightarrow \underline{V} \in \mathbf{V}_{\max} \wedge \underline{V} \notin \mathbf{V}_{\min} \wedge \underline{V} \in \mathbf{V}_{ref} \end{aligned} \tag{10}$$

The sets $\mathbf{V}_{\max}$ and $\mathbf{V}_{\min}$ are both n-dimensional, but a vector of complex nodal voltages $\underline{V}$ also has to be element of $\mathbf{V}_{ref}$. Thus, (9) and (10) can be combined to (11).

$$\begin{aligned} &\underline{V} \in \mathbf{V}_{feasible,V} \\ &\Rightarrow \underline{V} \in \left(\mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right) \wedge \underline{V} \notin \mathbf{V}_{\min} \end{aligned} \tag{11}$$

It is possible to reduce the extend of $\mathbf{V}_{\min}$ in terms of size and the number of dimensions by constructing the intersection with $\mathbf{V}_{\max}$ and $\mathbf{V}_{ref}$ ($\mathbf{V}_{ref}$ has ($n-1$) dimensions). All elements of $\mathbf{V}_{\min}$ omitted by this intersection would not fulfill (11) and thus are categorized as unfeasible by the first condition. Hence, (12) is equal to (11).

$$\begin{aligned} &\underline{V} \in \mathbf{V}_{feasible,V} \\ &\Rightarrow \underline{V} \in \left(\mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right) \wedge \underline{V} \notin \left(\mathbf{V}_{\min} \cap \mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right) \end{aligned} \tag{12}$$

Hereby, two ($n-1$)-dimensional sets $\mathbf{V}_1$ and $\mathbf{V}_2$ are constructed that allow for the feasibility decision for a complex nodal voltage configuration in the domain of complex nodal voltage.

$$\begin{aligned} &\mathbf{V}_1 := \left(\mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right) \\ &\mathbf{V}_2 := \left(\mathbf{V}_{\min} \cap \mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right) \end{aligned} \tag{13}$$

$\mathbf{V}_1$ is constructed by the intersection of 2 convex sets and thus is convex itself. Because $\mathbf{V}_2$ partly originates in the union of convex sets, this is not necessarily true for $\mathbf{V}_2$. But the separation into ($n-1$) sets defined by (14) gives ($n-1$) convex sets. This property is important for the representation of the embodied subspaces and will be used later on.

$$\mathbf{V}_{2,i} := \left(\mathbf{V}_{\min,i} \cap \mathbf{V}_{\max} \cap \mathbf{V}_{ref} \right); \forall i \in \{2,\ldots,n\} \tag{14}$$

The algorithm presented later in this paper translates these bodies, defined in the domain of complex nodal voltage, to the domain of complex nodal power in order to make the feasibility decision not in the domain of complex nodal voltages but directly in the domain of complex nodal power.

5.2 Constraints imposed by the rated line currents

In standard algorithms the results of a power flow calculation are applied to a complex matrix as given in (15) with $\overline{Y}_{ij}$ being the serial admittance of a line between node i and j in order to detect line overloads.

$$\underline{Y}_{line} = \begin{bmatrix} \overline{Y}_{12} & -\overline{Y}_{12} & & \\ & \overline{Y}_{23} & -\overline{Y}_{23} & \\ & & \overline{Y}_{34} & -\overline{Y}_{34} \\ & \overline{Y}_{24} & & -\overline{Y}_{24} \end{bmatrix} \tag{15}$$

The matrix $\underline{Y}_{line}$ represents a mapping between the domain of complex nodal voltage and complex line currents. For a network topology of n nodes and m lines connecting all nodes, m can vary according to (16).

$$(n-1) \le m \le \frac{n \cdot (n-1)}{2} \tag{16}$$

As a result the dimension of $\underline{Y}_{line}$ is ($m \times n$) varying according to the network topology.

In order to check for line overloads with standard algorithms, a point of complex nodal voltages is translated by $\underline{Y}_{line}$ from the domain of complex nodal voltages to the domain of complex line currents, where it is verified against the inequality given in (17) for every line k in the network.

$$\left|\bar{I}_k\right| \le I_{\max,k} ; \forall k \in \{1,\ldots,m\} \qquad (17)$$

where $\bar{I}_k$ is the complex current over line k and $I_{\max,k}$ being the rated current of line k.

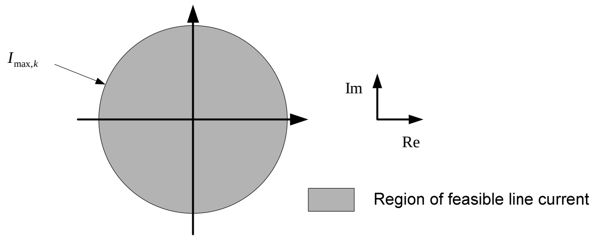

Figure 2: Representation of feasible line currents

Geometrically, this again may be interpreted as the check whether or not a point $\underline{I}_k$ is within an area, limited by a circle with the radius $I_{\max,k}$ for every line k of the network (see **figure 2**). This can be formulated as a set of valid points for every line k. $\underline{I}$ is feasible with respect to the lines' rated currents according to (18):

$$\begin{aligned} &\underline{I} \in \mathbf{I}_{feasible} \\ &\Rightarrow \underline{I} \in \mathbf{I}_{\max,k} := \left\{\left|\bar{I}_{\max,k}\right| \le I_{\max,k}\right\}; \forall k \in \{1,\ldots,m\} \end{aligned} \qquad (18)$$

where $\underline{I}$ is the vector of complex line currents.

A point has to be element of all m sets in order to be feasible. Thus $\underline{I}$ is a feasible current configuration according to (19):

$$\begin{aligned} &\underline{I} \in \mathbf{I}_{feasible} \\ &\Rightarrow \underline{I} \in \mathbf{I}_{\max} := \bigcap_{i=1}^{n} \mathbf{I}_{\max,i} \end{aligned} \qquad (19)$$

The dimension of $\mathbf{I}_{\max}$ is m and can vary according to (16). On any given network only a few of these line current combinations can occur simultaneously. This is due to the following facts:

- Line currents only depend on the voltage difference between the connected nodes, and not on their actual voltages.
- Kirchhoff's mesh law imposes that the sum of voltages differences in a mesh has to be zero.

In order to determine which combination can occur one has to calculate the column range $CR\left(\underline{Y}_{line}\right)$ of the matrix $\underline{Y}_{line}$. The column range of $\underline{Y}_{line}$ has the dimension of ($n-1$) and a vector of line currents has to be an element of the column range of $\underline{Y}_{line}$ in order to be realizable (see 20).

$$\begin{aligned} &\underline{I} \in \mathbf{I}_{realizable} \\ &\Rightarrow \underline{I} \in CR\left(\underline{Y}_{line}\right) \end{aligned} \qquad (20)$$

We say $\underline{I}$ is feasible and realizable if $\underline{I}$ is element of $\mathbf{I}_{feas}$. Expression (19) and (20) can be combined according to:

$$\begin{aligned} &\underline{I} \in \mathbf{I}_{feas} \\ &\Rightarrow \underline{I} \in \mathbf{I}_{feasible} \wedge \underline{I} \in \mathbf{I}_{realizable} \\ &\Rightarrow \underline{I} \in \mathbf{I}_{feasible} \cap CR\left(\underline{Y}_{line}\right) \end{aligned} \qquad (21)$$

As the set $\mathbf{I}_{feas}$ formulated in (21) originates in the intersection of convex sets, $\mathbf{I}_{feas}$ itself is convex. $\mathbf{I}_{feas}$ can then be translated to the domain of $\underline{Y}_{line}$ forming a convex set of possible voltage profiles that would have $\mathbf{I}_{feas}$ as its image over $\underline{Y}_{line}$. The calculated set lies within the row range of $\underline{Y}_{line}$. In order to meet the voltage of the reference node the calculated set has to be moved within the nullspace of $\underline{Y}_{line}$ until the voltage of the reference node is equal to the reference node's $\bar{V}_1$ specified in (2). The resulting set is denoted as $\mathbf{V}_{I\max}$ and is ($n-1$)-dimensional. As the nullspace of $\underline{Y}_{line}$ is a 1-dimensional space and is orthogonal to the row range of $\underline{Y}_{line}$, the required shift in the domain of complex nodal voltage preserves the convexity of the set.

In this section it has been shown that the voltage band and rated line current constraints cause the subspace of feasible voltage combinations to be non-convex. Furthermore it has been demonstrated that this non-convex subspace can be described using convex sets. The translation of these sets into the domain of net nodal power will be described in the following section.

6 CALCULATING THE SUBSPACE OF FEASIBLE COMPLEX NODAL POWER COMBINATIONS

In section 4 three ($n-1$)-dimensional sets in the domain of complex nodal voltage, $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_{I\max}$, were constructed that allow for the decision of the feasibility of a certain point of operation $\bar{V}$. In order to be able to make this decision directly in the domain of complex nodal power both sets have to be translated between both domains using (22).

$$\underline{S} = diag(\underline{V}) \cdot \left(\underline{Y} \cdot \underline{V}\right)^* \qquad (22)$$

Where $\bar{V}$ is a complex nodal voltage configuration of $\underline{S}$ and $\underline{Y}$ is the nodal admittance matrix of the particular network. The reference node is excluded in order to equilibrate the power balance within the network. In order to formulate this, the first row of $\underline{Y}$ and $diag\left(\underline{V}\right)$ is excluded. Both elements will be denoted as $\underline{Y}_{2\ldots n}$ and $diag\left(\underline{V}\right)_{2\ldots n}$ respectively.

$$\underline{S}_{2\ldots n} = diag(\underline{V})_{2\ldots n} \cdot \left(\underline{Y}_{2\ldots n} \cdot \underline{V}\right)^{*} \tag{23}$$

In a first step a vector $\underline{V}$ is translated from the n-dimensional domain of complex nodal voltage to the ($n-1$)-dimensional domain of complex nodal current using $\underline{Y}_{2\ldots n}$. As the rank of $\underline{Y}_{2\ldots n}$ is ($n-1$) there exists a 1-dimensional nullspace of $\underline{Y}_{2\ldots n}$, denoted $NS\left(\underline{Y}_{2\ldots n}\right)$, and a ($n-1$)-dimensional row range of $\underline{Y}_{2\ldots n}$, denoted $RR\left(\underline{Y}_{2\ldots n}\right)$, in the domain of $\underline{Y}_{2\ldots n}$. When multiplying $\underline{Y}_{2\ldots n}$ with a vector $\underline{V}$, the vector $\underline{V}$ is projected onto $RR\left(\underline{Y}_{2\ldots n}\right)$ and then translated to the domain of complex nodal current. The properties of this projection have to be evaluated for the sets $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_{\mathrm{I}\max}$ separately.

6.1 *Translating the sets of feasible voltages*

In case of translating the two ($n-1$) dimensional sets $\mathbf{V}_1$ and $\mathbf{V}_2$, two sets $\mathbf{I}_{V1}$ and $\mathbf{I}_{V2}$ are constructed. These two resulting sets are ($n-1$)-dimensional under the condition that none of the base vectors spanning $RR\left(\underline{Y}_{2\ldots n}\right)$ has a zero scalar product with one of the Euclidean base vectors $\underline{e}_{2\ldots n}$ spanning the vector space of complex nodal voltage.

6.2 *Translating the sets of feasible line currents*

As the set $\mathbf{V}_{I\max}$ containing all voltage combinations has the orientation of $RR\left(\underline{Y}_{line}\right)$, the number of the dimension of its image over $\underline{Y}_{2\ldots n}$ depends on how $\mathbf{V}_{I\max}$ is projected onto $RR\left(\underline{Y}_{2\ldots n}\right)$. The translation of the ($n-1$)-dimensional set $\mathbf{V}_{I\max}$ into the domain of complex nodal current results in the ($n-1$)-dimensional set $\mathbf{I}_{I\max}$ under the condition that none of the base vectors spanning $RR\left(\underline{Y}_{2\ldots n}\right)$ has a zero scalar product with one of the base vectors spanning $RR\left(\underline{Y}_{line}\right)$.

6.3 *Behavior of the power flow equations*

A given vector $\underline{V}$ in the domain of $\underline{Y}_{2\ldots n}$ can be separated into two components. The first one is the part of $\underline{V}$ that lies in the row range of $\underline{Y}_{2\ldots n}$, the second one is the part lying in the null space of $\underline{Y}_{2\ldots n}$.

The row range $RR\left(\underline{Y}_{2\ldots n}\right)$ of $\underline{Y}_{2\ldots n}$ is ($n-1$)-dimensional and will be denoted as $\underline{RR}\left(\underline{Y}_{2\ldots n}\right)$ representing a matrix consisting of ($n-1$) orthonormal column vectors that span the row range of $\underline{Y}_{2\ldots n}$. By this the part of $\underline{V}$ lying in the row range of $\underline{Y}_{2\ldots n}$ can be calculated according to (24).

$$\underline{V}_{RR(\underline{Y})} = \underline{RR}\left(\underline{Y}_{2\ldots n}\right) \cdot \underline{RR}\left(\underline{Y}_{2\ldots n}\right)^{T} \cdot \underline{V} \tag{24}$$

And thus the part of $\underline{V}$ lying in the nullspace of $\underline{Y}_{2\ldots n}$ can be calculated using (25).

$$\underline{V}_{NS(\underline{Y})} = \underline{V} - \underline{V}_{RR(\underline{Y})} \tag{25}$$

As the nullspace of $\underline{Y}_{2\ldots n}$ is one-dimensional, $\underline{V}$ can be described as follows:

$$\underline{V} = \underline{V}_{RR(\underline{Y})} + \overline{\lambda} \cdot \underline{V}_{NS(\underline{Y})} \tag{26}$$

with the complex factor $\overline{\lambda}$ being determined by (29). This guarantees that the first element of the vector $\underline{V}$ determined in (27) is equal to $V_{ref} \cdot e^{j0^{\circ}}$.

$$\overline{\lambda} = \frac{V_{ref} \cdot e^{j0^{\circ}} - \underline{V}_{RR(\underline{Y}),1}}{\underline{V}_{NS(\underline{Y}),1}} \tag{27}$$

As the nullspace $NS\left(\underline{Y}\right)$ is the preimage of the zero element $\underline{0}$. The component of $\underline{V}$ that lies in the nullspace does not affect the image of $\underline{V}$ over $\underline{Y}_{2\ldots n}$. Thus, instead of (23) we say:

$$diag\left(\underline{V}_{RR(\underline{Y}),2\ldots n} + \overline{\lambda} \cdot \underline{V}_{NS(\underline{Y}),2\ldots n}\right) \cdot \left(\underline{Y}_{2\ldots n} \cdot \underline{V}_{RR(\underline{Y})}\right)^{*} \tag{28}$$

In the following we assume that the translation from the domain of complex nodal voltage preserves the convexity of the before defined sets.

Observations in numerical calculation show that the translation (22) also introduces a shift of the images of $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_{I\max}$. We recognized a significant influence on the counterpart of expression (12) in the domain of complex nodal power.

In the following section we present a basic numerical study in order to illustrate the concept of the presented approach and the before mentioned impact of the image shifting on the feasibility decision.

7 NUMERICAL STUDY

In order to study the approach presented in this paper, we evaluate a numerical example using the standard Newton-Raphson power flow algorithm. The network segment consists of two nodes and one line. The first node serves as the reference node (see **figure 3**).

In order to evaluate the approach presented in this paper, a set of combinations of active and reactive power is spread randomly across the complex power plane of the second node. Afterwards the power combinations are verified using the Newton-Raphson power flow algorithm in order to detect voltage band violations and a line overloads. Each point in **figure 4** represents a combination that is found to comply with the stability constraints given in (29), while each point in **figure 5** indicates a combination that violates at least one of these stability constraints.

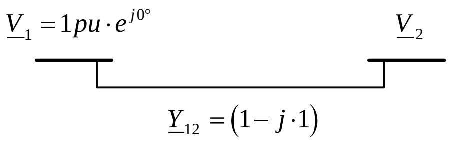

Figure 3: Two-node example network

$$0.9\,pu \leq \left|\underline{V}_2\right| \leq 1.1\,pu \qquad \left|\underline{I}_{12}\right| \leq 1\,pu \tag{29}$$

Figure 4 and **5** also depict the boundaries imposed by the minimum, maximum voltage and line current constraint according to the approach presented in this paper.

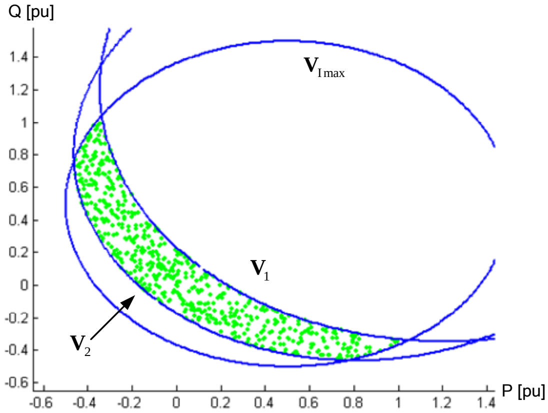

Figure 4: Subspace of feasible power injections

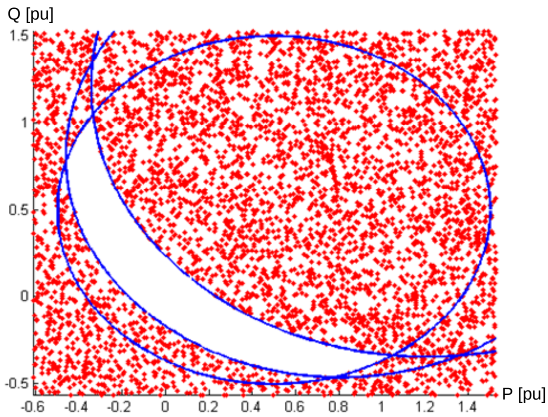

Figure 5: Subspace of unfeasible power injections

It can be observed that the calculated boundaries correctly localize the subspace of feasible power combinations for this basic example.

In the following section the calculation method for a network with n nodes an m lines will be introduced.

8 NUMERICAL CALCULATION

In this section the convex sets $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_{I\max}$ are calculated for a given network topology with n nodes and m lines, which will then be translated to the domain of complex nodal power.

8.1 Calculating the body of feasible line currents

In order to calculate the set $\mathbf{I}_{feas}$ according to (21) the first step in the calculation is the determination of the four subspaces $CR(\underline{Y}_{line})$, $RR(\underline{Y}_{line})$, $NS(\underline{Y}_{line})$ and $LN(\underline{Y}_{line})$ and their representation based on orthonormal column vectors spanning the respective subspace. The set of orthonormal column vectors will be denoted $\underline{CR}(\underline{Y}_{line})$, $\underline{RR}(\underline{Y}_{line})$, $\underline{NS}(\underline{Y}_{line})$ and $\underline{LN}(\underline{Y}_{line})$, respectively.

The Singular Value Decomposition (SVD) of the matrix $\underline{Y}_{line}$ provides the necessary information in order to obtain the required sets of orthonormal column vectors spanning the four subspaces in the domain and codmain of $\underline{Y}_{line}$

The SVD decomposes a matrix $\underline{A}$ into the three matrices $\underline{C}$, $\underline{W}$ and $\underline{D}$ according to (30).

$$\underline{A} = \underline{C} \cdot \underline{W} \cdot \underline{D}^* \tag{30}$$

These three matrices have the following dimensions and properties:

- $\underline{C}$ is a $(m \times m)$-dimensional matrix that contains m m-dimensional column vectors that form a orthonormal basis spanning the entire codomain of $\underline{A}$
- $\underline{D}$ is a $(n \times n)$-dimensional matrix containing n n-dimensional column vectors forming a orthonormal basis spanning the entire domain of $\underline{A}$
- $\underline{W}$ is a $(m \times n)$-dimensional diagonal matrix that contains the singular values of $\underline{A}$ in the main diagonal in descending order.

Special attention is required for the matrix $\underline{W}$. (31) depicts the basic structure of $\underline{W}$, where r is the rank of matrix $\underline{A}$.

$$\underline{W} = \begin{bmatrix} \overline{w}_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \overline{w}_r & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \quad (31)$$

With (31) it can be seen that $\underline{W}$ links the first r base vectors of $\underline{C}$ and $\underline{D}$ to each other. These base vectors span the column range $CR(\underline{Y}_{line})$ and row range $RR(\underline{Y}_{line})$ of $\underline{A}$. The last $(n-r)$ and $(m-r)$ vectors of $\underline{D}$ and $\underline{C}$, respectively, form the nullspace $NS(\underline{Y}_{line})$ and left nullspace $LN(\underline{Y}_{line})$ of $\underline{A}$.

In case of $\underline{Y}_{line}$ the nullspace in the domain of $\underline{Y}_{line}$ is one-dimensional and represents the direction in which the voltage vector may be shifted without changing the distribution of line currents. The column range of $\underline{Y}_{line}$ in the codomain of $\underline{Y}_{line}$ is r-dimensional and spans the subspace of all combinations of complex line currents that can occur simultaneously according to Kirchhoff's mesh law.

As the set $\mathbf{I}_{feas}$, formulated in (21) is a convex set, it can be described by a convex hull. In order to translate this hull, the intersection of (32) and (33) has to be calculated. In a numerical calculation this has to be done iteratively.

$$\left|\overline{I}_i\right| = I_{i,\max}; \forall i \in \{1,\ldots,m\} \quad (32)$$

$$\underline{I} \in \left\{\underline{I} = \overline{\alpha}_1 \cdot \underline{C}_1 + \ldots + \overline{\alpha}_r \cdot \underline{C}_r ; \overline{\alpha}_i \in \mathbb{C}\right\} \quad (33)$$

The iterative intersection with l points gives a set of points $\underline{I}_{\max,1\ldots l}$ lying in the rim of the analytical intersection of (32) and (33).

After the intersection was calculated it can be translated point wise to the domain of complex nodal voltages using (34) for the o-th vector of $\underline{I}_{\max,1\ldots l}$.

$$\underline{V}_o = \underline{RR}(\underline{Y}_{line}) \cdot \underline{W}^{-1}_{(r\times r)} \cdot \underline{CR}(\underline{Y}_{line})^* \cdot \underline{I}_{\max,o} \quad (34)$$

To further translate this set to the domain of complex nodal power (34) the solution of (34) has to be shifted according to (35) in order to meet the reference node voltage.

$$\underline{V}_{\mathrm{Imax},o} = \underline{V}_o + \frac{V_{ref} \cdot e^{j0°} - \underline{V}_{o,1}}{NS(\underline{Y}_{line})_1} \cdot NS(\underline{Y}_{line}) \quad (35)$$

The vectors calculated in (35) can then be translated vector wise to the domain of complex nodal power using (23) forming a set of points in the hull (called vertices) of the set of complex power combinations that do not result in line overloads. This set of points is a so called vertex-polytope (v-polytope) representation of a subspace. The construction of the final (and more manageable) representation as a halfspace-polytope (or h-polytope) will be discussed later in this section.

8.2 Calculating the body of feasible nodal voltages

Starting with the definitions of sets $\mathbf{V}_1$ in (13) and $\mathbf{V}_{2,i}$ in (14) the corresponding sets in the domain of complex nodal power have to be constructed by sampling the hull of the respective sets and translate them using (23).

As the sets $\mathbf{V}_{I\max}$, $\mathbf{V}_1$ and $\mathbf{V}_{2,i}$ are convex and, under the assumption that (23) preserves their convexity, their calculated v-polytope representation can be transformed into an equivalent h-polytope representation of each respective set (see **figure 6**). The transformation of a v-polytope into an h-polytope is known as the *facet enumeration* problem [8, 9, 10].

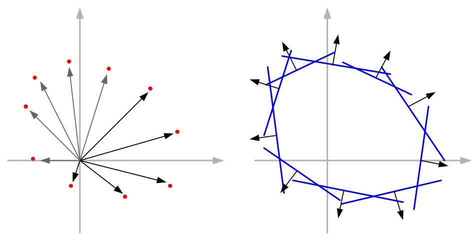

Figure 6: Concept of v- and h-Polytope

An h-polytope is represented by halfspace definitions. Each halfspace definition represents a $(n-1)$-dimensional hyperplane. The hyperplanes cut out a subset of operational states. The intersections of all halfspaces describe the h-polytope. Each of the hyperplanes is a so called facet of the h-polytope and can be represented by an inequality in the Hessian normal form according to:

$$\underline{n}_{facette,i} \cdot \underline{S} \le d_i \quad (37)$$

where $\underline{n}_{facette,i}$ is the normal vector of the i-th facet. With this representation it is possible to determine whether or not a certain point of operation $\underline{S}$ is element of the before calculated images of the sets $\mathbf{V}_{I\max}$, $\mathbf{V}_1$ and $\mathbf{V}_{2,i}$. Thus the decision whether an operational state $\underline{S}$ is feasible or not is equivalent to checking $\underline{S}$ against a finite number of inequalities in Hessian normal form. The computational time of this decision is only dependent on the number of halfspace definitions and can easily be distributed among parallel machines.

9 DISCUSSION

The basic idea of the presented approach is to localize the subspace of feasible operating states by tracing the boundaries imposed by stability constraints from their origin to their position in the domain of complex nodal power. As these initial boundaries can all be described using convex sets, the solution space is described using convex subspaces, so that the feasibility decision can be made directly in the domain of complex nodal power.

These convex subspaces can then be specified using so called h-polytopes, represented by a finite set of inequalities, which have a lot of advantages in terms of formal representation and computation in information systems.

Once the subspace of feasible complex power combination can be described using h-polytopes it is possible to:

- Evaluate a point of operation in constant predetermined time.
- Introduce the set of inequalities as inequality constraints into optimization problems and thus improve optimization results.
- Determine on-line the distance to unfeasible operational states and provide a coordinator with improvement suggestions.

With the help of the calculated subspace and its boundaries it is possible to establish a network control system for distribution networks basing on autonomous software agents. These agents would have the capability of determining the feasibility of power configuration, independent from the energy management system structure, under real-time conditions and thus reliably avoid violation of operational limits. Furthermore they can calculate the distance to the feasibility boundary in order to assign maximum nodal power limits to certain nodes within the network. The possible benefits of such system is the possibility to increase power injections of DER according to the actual network state and to dynamically assign maximum charging power to electric cars connected to the grid.

Future work will have to focus on the behaviour of the power flow equations that map the sets, defined in the domain of complex nodal voltage to the domain of complex nodal power. Especially the impact on the feasibility expression should receive special attention.

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