Optimal Zero Injection Considerations in PMU Placement: An ILP Approach

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Abstract - This paper addresses various aspects of optimal Phasor Measurement Unit (PMU) placement problem. Zero injection busses, which are analogous to transshipment nodes have the potential to reduce the number of PMUs to obtain the complete system observability. Hitherto, modeling of zero injection constraints had been a challenge due to intrinsic nonlinearities, which increases the complexity of the discrete optimization problem. We show that zero injection constraints can also be modeled as linear constraints in an Integer Linear Programming (ILP) framework. Further, we observe that the minimum PMU placement problem has multiple solutions with the same cost. We propose two indices, viz.: the Bus Observability Index (BOI) and the System Observability Redundancy Index (SORI), to further rank these multiple solutions. BOI gives a measure of the number of PMUs observing a given bus and the SORI is sum of all BOI for a system. Results on the IEEE 118 bus system are presented. The results indicate that: (1) the proposed method of modeling zero injection constraints improves computational performance, and, (2) BOI and SORI help in improving the quality of PMU placement. We show that the solution with maximum SORI outscores other optimal solutions. Subsequently, we also develop a simple and elegant method to handle single PMU outage as well as different system topologies arising due to single line outage in the system.

Keywords - Integer Linear Programming (ILP), Optimal Phasor Placement (OPP), Phasor Measurement Unit (PMU), Zero Injection

1 Introduction

PHASOR measurement units (PMUs) provide time synchronized phasor measurements in a power system [1]. Synchronicity in PMU measurements is achieved by time stamping of voltage and current waveforms using a common synchronizing signal available from the Global Positioning System (GPS). The ability to calculate synchronized phasors makes PMU one of the most important measuring devices in the future of power system monitoring and control.

Throughout the paper we presume that a PMU placed on a bus measures the following parameters:

1. Voltage magnitude and phase angle of the bus;
2. Branch current phasor of all branches emerging from the bus.

PMU placement at all substations allows direct measurement of the state of the network. However, PMU placement on each bus of a system is difficult to achieve either due to cost factor or due to nonexistence of communication facilities in some substations. Moreover, as a consequence of Ohm's Law, when a PMU is placed at a bus, neighboring busses also become observable [2, 3]. This implies that a system can be made observable with a lesser number of PMUs than the number of busses.

Ref [4] has shown that the minimum PMU placement problem is NP-complete. This implies that no polynomial time algorithm can be designed to solve the problem exactly. Work on optimal PMU placement using an ILP approach has been pioneered by Ali Abur [5], [6]. For an n bus system, if the *PMU placement vector* $\mathbf{x}$ having elements x_i defines possibility of PMUs on a bus, i.e.

$$x_i = \begin{cases} 1 & \text{if a pmu is installed at bus } i; \\ 0 & \text{otherwise;} \end{cases}$$

then the minimum PMU placement problem can be defined as the following optimization problem:

Formulation: OPP

$$\min \sum_{i=1}^{n} x_i \tag{1}$$

subject to the constraints:

$$\mathbf{Ax} \geq \mathbf{e} \tag{2}$$

where $\mathbf{e}$ is a unit vector of length n, i.e.

$$\mathbf{e} = [1\,1\ldots1]^T$$

and

$$\mathbf{x} = [x_1\;x_2\ldots x_n]^T$$

and $\mathbf{A}$ is the *binary connectivity matrix* of the system, i.e.

$$a_{i,j} = \begin{cases} 1 & \text{if either } i=j \text{ or if } i \text{ and } j \\ & \text{are adjacent nodes;} \\ 0 & \text{otherwise;} \end{cases}$$

Ref [7] proposed an approach for PMU placement which requires complete enumeration of trees. Formulation based on meta heuristics like simulated annealing (SA), genetic algorithm, tabu search, etc have been considered in [8, 9, 10]. A genetic algorithm method using adaptive clonal algorithm has been proposed in [11].

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Beyond the number of PMUs required to make a system observable, a good PMU placement algorithm must also consider the following additional issues:

1. Loss of a PMU or communication line;

2. Modeling of zero injection busses;

When a system is made observable with minimum number of PMUs, lack of communication channels or a PMU outage itself will lead to unobservable busses in the system. Hence loss of PMU has to be considered in design stage. In particular, references [3, 6, 12] consider the requirement that the system should remain observable even with one PMU outage/branch outage.

Zero injections busses, which are analogous to transshipment nodes, have the potential to reduce the number of PMUs required for complete system observability. Ref. [6] considers modeling of zero injection constraints in an otherwise ILP framework. In the resulting formulation, observability constraints arising out of zero injection busses turn out to be non-linear. This increases the complexity of the discrete optimization problem.

We show that modeling of zero injection busses in the optimal PMU placement problem can be achieved by using linear constraints. This is a noteworthy contribution of this paper. This implies that the optimal PMU placement problem with zero injection busses can, *now*, be solved by standard ILP solvers. Subsequently, we also develop a simple and an elegant methodology to handle single PMU outage as well as different system topologies arising due to single line outage in the system.

Our investigations show that for the optimal PMU placement problem, multiple solutions with the same cost exist. To compare these solutions qualitatively, we introduce a performance index SORI. If a bus i is observed by n_i number of PMUs, then *system observability redundancy index* (SORI) is given by $\gamma_i = \sum n_i$. If in a system, multiple optimal solutions exist, then it is worthwhile to choose that optimal solution which further maximizes observability redundancy index. This important objective is also introduced in this paper.

This paper is organized as follows:- Section II deals with modeling of zero injection busses and section III explains maximizing redundancy in observability. In section IV we present case studies involving IEEE 14, 57 and 118 bus systems. Section V concludes the paper.

2 Modeling of Zero Injection Busses

Zero injection busses are the busses from which no current is being injected into the system. If zero injection busses are also modeled in the PMU placement problem, the total number of PMUs can be further reduced. To understand this issue, consider the four bus example shown in Fig. 1. Fig. 1(a) depicts the system with injections in all the busses. Fig. 1(b) shows a similar system with zero injection in bus 2 and injections in bus 1, 3 and 4. For system (a), it can easily be seen that a minimum of two PMUs are required to make the system completely observable. These can be placed on any two of the four busses. For example, if a PMU is placed on bus 1, another PMU is required to make observable bus 4. In contrast consider system (b). For a PMU at bus 1, the current in branch 2-4 becomes known as bus 2 is a zero injection bus. i.e. $I_{24} = I_{12}$. Hence knowing the line parameters, the voltage at bus 4 can be calculated [1] as:

$$V_4 = V_2 - I_{12} z_{24}$$

Hence a separate PMU is not required at bus 4 for (b). Therefore it is seen that presence of zero injections can help in reducing total number of PMUs required to observe the system.

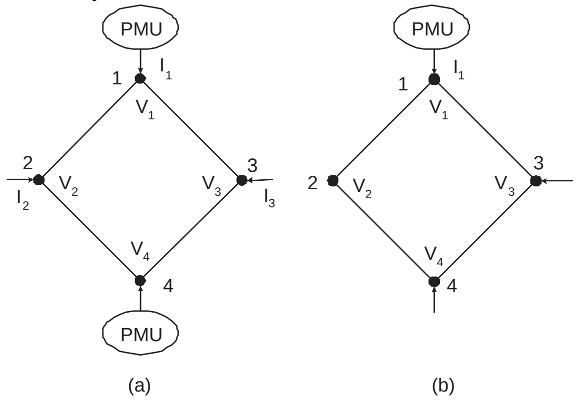

Figure 1: A 4-bus system considering busses 2 and 3 as (a) injection busses and (b) zero injection busses

Modeling of zero injection busses in ILP framework has remained a challenge. Ref. [6] follows an approach requiring nonlinear framework. We now propose a method to model these constraints within a linear framework.

Consider a zero injection bus as shown in Fig. 2. If busses 1 to $(m-1)$ are observable, i.e. their voltage phasors are known, then either current $I_{i,1}$ is available directly from a PMU or it can be calculated as follows:

$$I_{i,1} = y_{i,1}[V_i - V_1]$$

where $y_{i,1}$ is the line admittance between bus 1 and bus i. Consequently, bus m can also be made observable by calculating the bus voltage as follows:

$$V_m = V_1 - z_{1,m} \sum_{i=2}^{m-1} I_{i,1}$$

where $z_{1,m}$ is the line impedance between busses 1 and m. Every zero injection node leads to one additional constraint. Hence, in the best case, the minimum number of PMUs required to observe the system can be reduced by the total number of zero injection busses in the system.

For a zero injection bus i, let $\mathcal{A}_i$ indicate the set of busses adjacent to bus i. Let $\mathcal{B}_i = \mathcal{A}_i \cup \{i\}$. Let the number of zero injection busses in a system be given by z and the set of zero injection busses be indicated by $\mathcal{Z}$.

The way to model zero injection busses in the ILP framework is to *selectively* allow for existence of some unobservable busses. However, we impose additional constraints that:

[1]For simplicity, we have assumed series branch model. However the description applies equally for π model of transmission line.

1. Such busses must be adjacent to zero injection busses,

2. Number of unobservable busses in the set $\mathcal{B}_i$ (i.e. a zero injection bus i and its adjacent busses) is at most one.

Therefore the problem is modeled in a linear form as follows:

Model: OPP-Z

$$\min \sum_{i=1}^{n} x_i \tag{3}$$

subject to:

$$\mathbf{Ax} \geq \mathbf{u} \tag{4}$$

and

$$u_j = 1 \; \forall \, j \notin \mathcal{B}_1 \cup \mathcal{B}_2 ... \cup \mathcal{B}_z \tag{5}$$

and

$$\sum_{k \in \mathcal{B}_i} u_k \geq |\mathcal{A}_i| \;\; \forall \, i \in z \tag{6}$$

or

$$\mathbf{a}_i \mathbf{u} \geq |\mathcal{A}_i| \tag{7}$$

where $|\mathcal{A}_i|$ represents the cardinality of set $\mathcal{A}_i$. $\mathbf{a}_i$ represents the i^{th} row, i.e. row corresponding to the zero injection busses of binary connectivity matrix $\mathbf{A}$. Constraint (5) ensures that busses which are not adjacent to zero injection busses are definitely made observable.

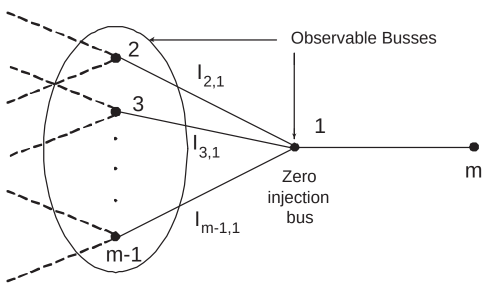

Figure 2: Modeling of zero injection busses

Example: Here we consider IEEE 14-bus system (Fig. 3). This system has bus 7 as zero injection bus. Thus, $\mathcal{Z} = \{7\}$, set $\mathcal{A}_7 = \{4, 8, 9\}$ and set $\mathcal{B}_7$=$\{4, 7, 8, 9\}$. Thus (6) becomes:

$$u_4 + u_7 + u_8 + u_9 \geq 3$$

i.e. out of the four busses 4, 7, 8 and 9, we must have, at least, three busses observable.

Again, for this system, constraint (5) becomes:

$$u_j = 1 \; \forall \, j \in \{1, 2, 3, 5, 6, 10, 11, 12, 13, 14\}$$

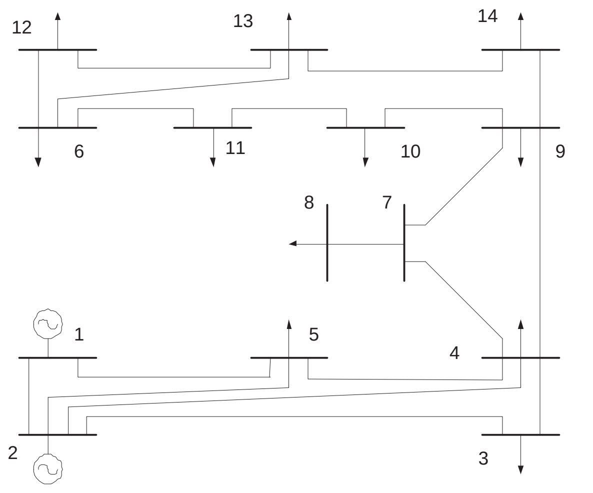

Figure 3: IEEE 14-Bus Test System (7^{th} bus being zero injection bus)

Solving the above formulation in ILP solver leads to an optimal number of 3 PMUs for making the system observable, with location of PMUs being on busses 2, 6 and 9. Hence, it is seen, in comparison with minimum number of PMUs calculated as per formulation **OPP** (without considering zero injections), here complete system observability is achieved with lesser number of PMUs.

3 Maximizing Redundancy in Observability

If the formulation-**OPP** has multiple number of optimal solutions, then the question of superiority of a particular solution vis-a-vis other optimal solution arises. In this section, we propose the *Bus Observability Index* (BOI) as a performance indicator on quality of the optimization. Let us define BOI for bus i (β_i) as the number of PMUs which are able to observe a given bus. Consequently maximum bus observability index is limited to *maximum connectivity* (η_i) of a bus plus one i.e.

$$\beta_i \leq \eta_i + 1$$

Now we define *System observability redundancy index* (γ) (SORI) as the sum of bus observability for all the busses of a system.

$$\gamma = \sum_{i=1}^{n} \beta_i$$

Consider the 6-bus system shown in Fig. 4. It is seen that a minimum of two PMUs are required to ascertain system observability. Consider two such optimal solutions shown in Fig. 4.

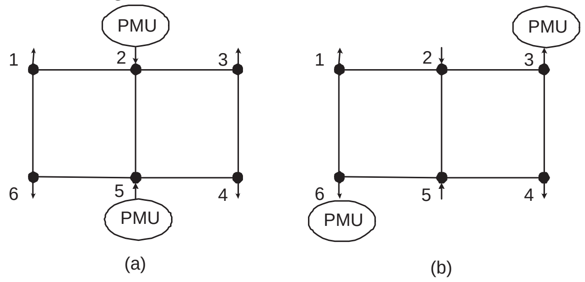

Figure 4: A 6-bus system with system observability (a) 6 and (b) 8

For the PMU placement as given in Fig. 4(a), BOI (β_i) for busses 1 to 6 are 1, 2, 1, 1, 2, and 1 respectively.

This makes SORI, $\gamma_a = 8$. Alternatively, for PMU placement in Fig. 4(b), BOI for busses 1 to 6 are unity, making $\gamma_b = 6$. Hence, the PMU placement with maximum SORI in Fig. 4(a) should be chosen for final placement.

Maximizing SORI has the advantage that a larger portion of the system will remain observable in case of a PMU outage. For example, in Fig. 4(a), one PMU outage will result in loss of observability of 2 busses, as against 3 busses remaining unobservable for loss of single PMU for system in Fig. 4(b). It can be seen that:

$$\gamma = \mathbf{e}^T \mathbf{A}\mathbf{x}$$

To solve the problem of maximizing SORI while guaranteeing system observability with minimum number of PMUs, we solve the following *slave* problem:

Formulation: Max Obs

$$\max \mathbf{e}^T \mathbf{A}\mathbf{x} \tag{8}$$

subject to the constraints:

$$\sum_{i=1}^{n} x_i = a \tag{9}$$

$$\mathbf{A}\mathbf{x} \geq \mathbf{e} \tag{10}$$

where a is the minimum number of PMUs obtained for complete observability as per *master* problem-**OPP**.

3.1 PMU Outage

To enhance the reliability of system monitoring, if a bus is observed by at least two PMUs instead of one, the loss of one will still keep the system observable. This can be modeled by modifying the constraints given by equation (2) to:

$$\mathbf{A}\mathbf{x} \geq 2\mathbf{e} \tag{11}$$

4 Case Studies

Case studies for zero injection considerations of PMU placement have been carried out for IEEE14-bus system, IEEE 57-bus system and IEEE 118-bus system. Tomlab's ILP solver [13] has been used for this purpose. The simulations are carried out for various scenarios as summarized in Fig. 5.

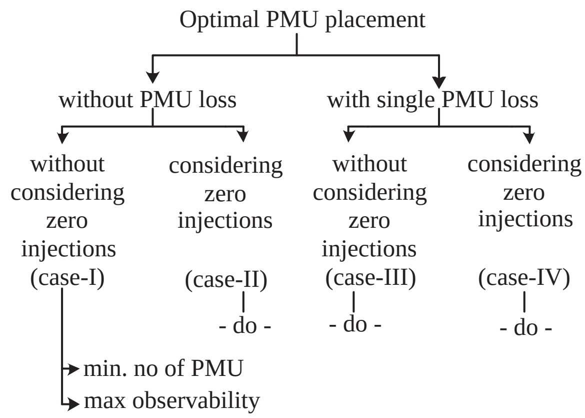

Figure 5: Four cases resulting from the formulated problems

4.1 Modeling of Zero Injection Busses

Zero injection busses considered for various systems are:

$$\begin{aligned}
\mathcal{Z}_{14\ bus} &= \{7\} \\
\mathcal{Z}_{57\ bus} &= \{4, 7, 11, 21, 22, 24, 26, 34, 36, 37, \\
&\quad 39, 40, 45, 46, 48\} \\
\mathcal{Z}_{118\ bus} &= \{5, 9, 30, 37, 38, 63, 64, 68, 71, 81\}
\end{aligned}$$

Table 1: Results-**OPP**: Minimum No of PMUs

S. No.	IEEE System	No PMU Outage		Single PMU Outage	
		Without Modeling Zero Inj	Modeling Zero Inj	Without Modeling Zero Inj	Modeling Zero Inj
1.	14 Bus	4	3	9	7
2.	57 Bus	17	14	33	29
3.	118 Bus	32	29	68	64

Table 1 brings out results of formulation-**OPP** under the backdrop of zero injections and PMU outages. It is seen that the number of PMUs almost doubles if the system observability is to be maintained after single PMU loss. Considering zero injections with no PMU outage scenario, the number of PMUs required for system observability reduces, at most, by the number of zero injection busses (Table-1, s.no-1, column-3 and 4). Similarly, considering zero injections while maintaining system observability for a single PMU outage, the number of PMUs required reduces, at most, by twice the number of zero injection busses (Table-1, s.no-1, column-5 and 6).

Table 2 shows results after solving the slave problem-**Max Obs**. It compares the results with those obtained from master formulation-**OPP**. It is seen that the redundancy in system observability is enhanced significantly by solving the slave problem.

Table 2: Results-**Max Obs**: system observability redundancy index (initial SORI → final SORI)

S. No.	IEEE System	No PMU Outage		Single PMU Outage	
		Without Modeling Zero Inj	Modeling Zero Inj	Without Modeling Zero Inj	Modeling Zero Inj
1.	14 Bus	16→19	15→15	34→39	31→33
2.	57 Bus	67→72	58→61	126→130	111→113
3.	118 Bus	156→164	148→152	298→309	290→297

Fig. 6 brings out the results of formulation of single PMU outage. Each of the three bars correspond to SORI busses ($\Sigma\beta_i$), total number of observable busses (Σu_i) and total number of busses that are observable from at least two nodes.

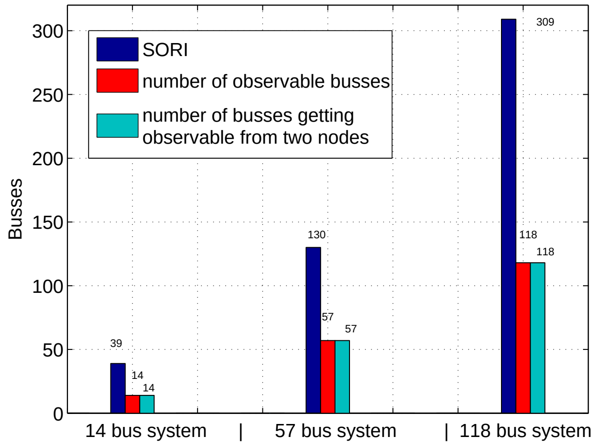

Figure 6: Results- considering single PMU outage

4.2 Computational Evaluation

The simulations have been run on a computer having the following configuration:

CPU	–	Pentium(R) IV 3.00 GHz
Level L2 Cache	–	2 MB
System Memory	–	1 GB RAM

Table 3 gives the CPU time for various formulations on IEEE 118-bus system.

Table 3: Computational Time (in sec): IEEE 118-Bus System

S. No	IEEE System	No PMU Outage		Single PMU Outage	
		Without Modeling Zero Inj	Modeling Zero Inj	Without Modeling Zero Inj	Modeling Zero Inj
1.	OPP	0.0156	-	0.0156	-
2.	OPP-Z	-	0.0312	-	0.0156
3.	Max Obs	0.0156	0.0468	0.0156	0.0468
Total Time Taken		0.0312	0.078	0.0312	0.0624

For the sake of comparative evaluation, the approach presented in [6], which requires nonlinear constraints, has been implemented using Tomlab optimization toolbox. Both, our proposed method and the method proposed in [6] require the same number of PMUs for system observability in the presence of zero injection constraints, validating the proposed method. For IEEE 118-bus system, the CPU time taken is 1.1250 sec as against 0.0312 sec (Table-3, s.no. 2, column 4) for the proposed formulation. Thus, it is seen that modeling of zero injection as linear constraints (formulation-**OPP-Z**) has reduced the computational burden by 36 times.

Finally, Fig. 7 illustrates placement of PMUs on IEEE 57-bus system. For the zero injection bus 4, currents $I_{6,4}$, $I_{5,4}$ and $I_{3,4}$ are known as a consequence of PMUs on busses[2] 6 and 15. Consequently, current $I_{4,18}$ is calculated using Kirchhoff's current law, thereby making bus 18 observable. Note that neither bus 18 nor its adjacent busses 4 or 19 have a PMU. This shows how zero injection busses can extend observability to a neighboring bus.

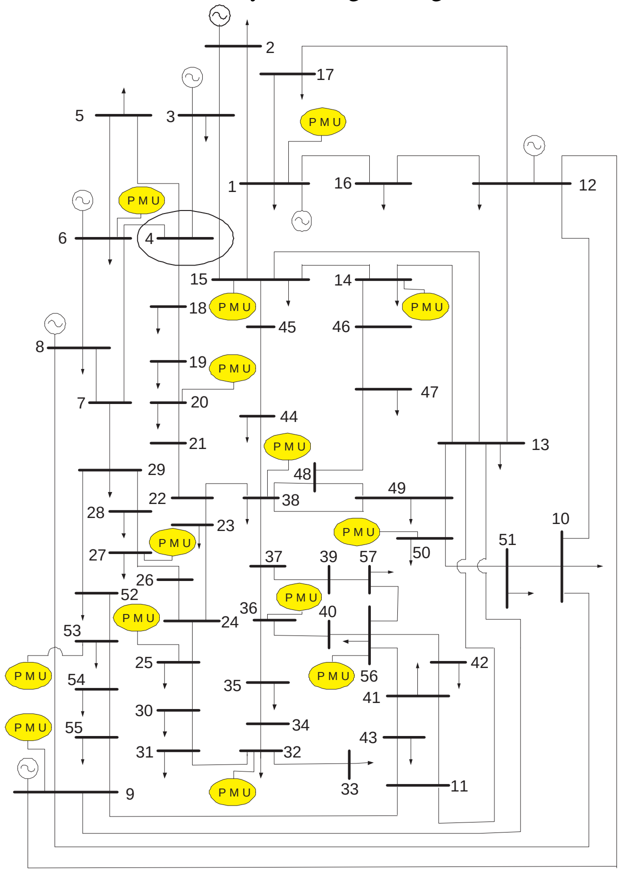

Figure 7: IEEE 57-Bus System: Location of 14 PMUs for complete system observability (considering zero injection busses)

5 Conclusions

The salient contributions of this paper are the following:

1. Zero injection constraints have the capability to further reduce PMU requirement. We develop linear model for zero injection constraints.

2. Optimal PMU placement has multiple solutions. We propose Bus Observability Index (BOI) and System Observability Redundancy Index (SORI) and show that solution with maximum SORI outscores other optimal solutions.

Results on IEEE 14, 57 and 118-bus systems demonstrate the claim made. The proposed model is free of unwarranted complexities. This claim is vindicated by the excellent computational performance of the algorithm.

A Line Outage

If two PMUs are observing a bus, then a related line outage will not affect the node observability. Hence, the problem of single line outage is a subset of the problem of single PMU loss considered above. However, if a user wants to model different topologies arising out of contingencies, but does not want to guarantee observability in

[2] Due to PMUs at busses 6 and 15, voltage phasors at busses 3, 4, 5 and 6 are known. Hence respective branch current can be computed.

the event of PMU outage, then we write constraint equations as follows:

$$\mathbf{a}_i\mathbf{x} \geq 1$$

for bus i which is not affected by contingency, else:

$$\mathbf{a}_i\mathbf{x} \geq 2$$

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