OPTIMAL PLACEMENT OF PHASOR MEASUREMENTS FOR ENHANCED STATE ESTIMATION: A CASE STUDY

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Abstract – **This paper describes a case study where an optimal strategy is developed and applied to a large utility system for placing phasor measurements in order to improve bad data processing capability and to maintain observability during anticipated contingencies. Assuming that a limited number of phasor measurements will be placed in the system, the developed method aims to ensure bad data detectability for all conventional as well as phasor measurements during normal operation. It also ensures full network observability during expected contingencies. Results of applying this method to a large utility system are illustrated and then verified by simulating bad data and contingencies on the same system.**

Keywords: State estimation, phasor measurements, network observability, meter placement, contingencies, critical measurements.

1 INTRODUCTION

As more and more utility companies are considering placement of phasor measurements, the issue of optimally locating these measurements becomes important. Since phasor measurements can be utilized for a number of applications, it is difficult to find locations that will satisfy optimality conditions for all the applications considered. This study focuses on the state estimation function only.

Phasor measurement units (PMU) are manufactured by various companies and therefore they come in many different types. Some of these have several channels which can accept voltage and current signals. Others have a pair of channels, allowing one voltage and one current signal to be processed by the unit. This paper will consider the phasor measurement units of the latter variety.

It is well known that having sufficiently large number of PMUs will allow the state estimation problem to be formulated as a linear problem and thus the solution can be obtained by direct solution of normal equations if weighted least squares method is employed. While PMUs are rapidly populating power systems, their numbers are still too low to implement such a solution in any existing utility system. It is however possible to implement a hybrid solution where the nonlinear problem is solved first by iterative methods and then PMU measurements are processed using linear recursive methods in order to improve the estimate.

Given the rapidly growing numbers of PMUs, it is pertinent to investigate requirements on the number and location of these devices to make the system fully observable. This paper is concerned about optimal placement of PMUs for the transmission system that is operated by the Entergy Corporation.

The first part of the paper investigates the problem of placing PMUs at the right locations in order to make a utility system fully observable using the minimum possible number of PMUs. This study accounts for any existing passive buses, where there is no load or generation, and therefore the net real and reactive power injection is zero.

Next, the study looks at the current situation where the system is already observable by the existing conventional measurements. Despite full observability, the measurement system is vulnerable against bad data in certain parts of the system where bad data in a subset of measurements can not be detected. Optimal placement of few PMUs may eliminate this weakness by providing the necessary redundancy to ensure detectability of errors which may appear in any one of the existing measurements.

Finally, the paper investigates the network observability problem under a set of topology changes which represent possible line outage contingencies. An optimal placement strategy is developed to make sure that the system remains fully observable during any one of these topology changes.

2 OBSERVABILITY PROBLEM

The problem of optimal PMU placement has been investigated earlier in [1,2]. These studies are based on the assumption that PMUs will be associated with buses where they will monitor the bus voltage as well as currents along the branches (lines and transformers) that are incident to this bus. Results of such studies indicate that the entire system can be made observable by placing PMUs at roughly 1/3 of the buses in the system [2]. This number will decrease if existing zero injection buses are taken into account during the placement process.

For an N-bus system, the PMU placement problem can be written as follows [2]:

$$\min \sum_{i}^{N} W_i \cdot y_i \qquad (1)$$

$$s.t. \quad T \cdot Y \geq \hat{1} \qquad (2)$$

where,

N is the number of buses in the system

W_i is the installation cost for PMU i

y_i is the binary variable which is 1 or 0 depending on whether a PMU is placed at bus i or not respectively.

T is the binary bus to bus connectivity matrix, which is defined as:

$$T_{ij} = \begin{cases} 1 & \text{if i = j} \\ 1 & \text{if bus i is connected to bus j} \\ 0 & \text{otherwise} \end{cases}$$

$\hat{1} = [111\ldots1]^T$

This formulation assumes that each PMU will have infinite number of channels to handle as many incident branch currents as necessary. Solution of the optimization problem (1) and (2) will require placement of such PMUs at the designated buses. In this study, the existing PMUs owned and planned to be installed by Entergy are not of this type. They are capable of measuring voltage and current phasors at the sending end of a single line. Hence, the above formulation needs to be revised to customize the solution for the so called "branch" (as opposed to "bus") PMUs. This will be illustrated in the next section.

2.1 Branch PMU placement:

This section will describe the formulation of the optimal placement problem for PMUs which are capable of monitoring only a single line, namely the sending end bus voltage and the sending end current flow towards the receiving end bus. The objective of the optimization problem remains the same, i.e. to determine the optimal location and number of such PMUs in order to make the entire system observable based on these PMU measurements only. Unlike the formulation given above, PMUs will now be associated with system branches rather than buses or nodes.

Consider a power system containing L branches. The optimal PMU placement problem can be defined as:

$$\min \sum_{i}^{L} C_i . x_i \qquad \text{s.t.} \quad A \cdot X \geq \hat{1} \tag{3}$$

where,

L is the number of branches in the system.

x_i is the binary variable which is 1 or 0 depending on whether a PMU is placed on branch i or not respectively.

C_i is the cost of installation of PMU on branch i.

A is the bus to branch connectivity matrix, which is defined as:

$$A_{ij} = \begin{cases} 1 & \text{if branch } j \text{ is incident to bus } i \\ 0 & \text{otherwise} \end{cases}$$

$\hat{1} = [111\ldots1]^T$

The 5-bus system shown in Figure 1 can be used to illustrate the PMU placement problem.

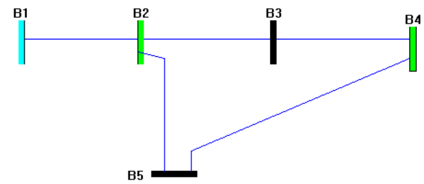

Figure 1: 5-bus system network diagram.

Forming the bus to branch connectivity matrix for this system will yield:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

One solution for the optimization problem in (3) will be $X = [1 \;\; 0 \;\; 1 \;\; 1 \;\; 0]^T$. The solution is not unique since all installation costs are assumed to be the same for PMUs.

Note that the minimum number of required PMUs to make a given power system fully observable without using any other virtual or pseudo measurements is equal to the next integer larger than (N-1)/2. In case of zero injections, this number will be smaller by taking advantage of the extra equations representing these zero injection measurements.

This method is applied to the IEEE 14-bus system which is shown in Figure 2 as well as the IEEE 30-bus system described in [3]. In the 14-bus system, bus 7 is a passive bus, with no generation or load, i.e. it is a zero injection bus. Such zero injection buses provide the opportunity to reduce the required number of PMUs to be placed in the system for full system observability. Zero injections are incorporated into the optimization problem formulation by modifying the system model before solving the problem (3).

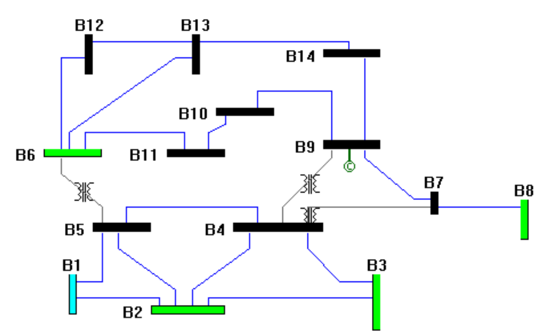

Figure 2: 14-bus system network diagram.

Consider a zero injection at bus k with M_k neighboring buses. In this set of $(M_k + 1)$ unknown bus voltages, it will be possible to solve for the remaining one voltage if any subset of M_k voltages are already available. Hence, we can merge bus *k* with one of its neighbors, e.g. for the 14-bus example, bus 7 and 8 can be merged into one superbus, before solving the optimization problem given by (3). Regretfully, in this particular example, having this zero injection does not lead to any reduction in the required number of PMUs due to the special topology of the system around bus 7.

Results of applying the method to the 14-bus and 30-bus systems are shown in Tables 1 and 2 respectively. Note that for both cases, zero injections are ignored.

PMU	From	To
1	2	1
2	3	4
3	5	6
4	7	8
5	9	14
6	10	11
7	12	13

Table 1: Optimal locations of PMUs for the 14 bus system.

PMU	From	To
1	2	1
2	4	3
3	5	7
4	6	8
5	9	11
6	10	20
7	12	13
8	14	15
9	16	17
10	18	19
11	21	22
12	23	24
13	25	26
14	27	28
15	29	30

Table 2: Optimal locations of PMUs for the 30 bus system.

Finally, the same optimization problem is solved for the Entergy's system. This system has 2285 buses and the optimal number of PMUs is found to be 1291. Note that this number is larger than the minimum limit of (2285-1)/2 = 1142, illustrating the fact that topology of the system may force the optimimal number of required PMUs to be much larger than this theoretical lower limit.

3 BAD DATA PROBLEM

It is well documented in the literature that bad data detection on analog measurements is only possible if these measurements are not critical [4]. Critical measurements are those measurements whose removal from the measurement set will cause unobservability. Hence, their measurement residuals are identically equal to zero irrespective of their accuracy and hence their errors can not be detected by the commonly used largest normalized residual test.

In this part of the study, the objective is to determine the best locations for placing PMUs in order to create a measurement set that does not include any critical measurements. The utilized approach is an extension of the one described in [5] which involves two stages. The first stage is determination of candidate PMU locations that will transform each identified citical measurement in the system into a redundant measurement. The second stage takes these candidates and makes an optimal selection out of these to find the optimal solution.

The method that is employed in order to determine the critical measurements and the candidate measurements to make these redundant can be implemented as summarized below:

1- Form the measurement jacobian matrix, *H*.

2- Augment *H* by adding all the candidate PMUs at the bottom. Let the submatrix representing the rows corresponding to the candidate measurements be denoted by H_c.

3- Perform a rectangular LU factorization on *H*. This can be accomplished by using Peters-Wilkinson method described in [6]. Row pivoting during this factorization should be limited to the rows corresponding to the existing (not any of the candidate) measurements. If such pivoting proves inadequate, then it implies an unobservable system. In that case, row pivoting will be extended to the rows of candidate measurements so that a minimally observable set of measurements can be found to make the system observable first. The following steps assumes that the system is observable (either initially or made observable via this extension) and will identify critical measurements within this set.

4- Note that after row pivoting, top n rows of H will represent the minimally observable set of measurements. Let the LU factors of *H* be written as:

$$H = \begin{bmatrix} H_0 \\ H_r \\ H_c \end{bmatrix} = \begin{bmatrix} L_0 \\ M_r \\ M_c \end{bmatrix} \cdot [U] \qquad (4)$$

where:

H_0: whose rows correspond to minimally observable measurement set of "*n*" (number of columns in *H*) measurements.

H_r: whose rows correspond to redundant existing measurements.

H_c: whose rows correspond to candidate PMU measurements.

Calculate the test matrix T defined as:

$$T = \begin{bmatrix} T_r \\ T_c \end{bmatrix} = \begin{bmatrix} M_r \\ M_c \end{bmatrix} [L_0]^{-1} \qquad (5)$$

If column k in T_r is null, it will correspond to a critical measurement. Then, the rows having nonzero entries in column k of T_c will correspond to the candidates that will transform this critical measurement into a redundant one.

The above procedure can be illustrated using the 6-bus system shown in Figure 3. Considering the existing measurements as shown in the figure and assuming candidate phase angle (PMU) measurements at every bus, the following results will be obtained by applying the procedure described above:

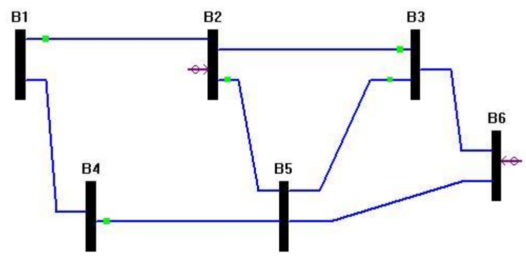

Figure 3: 6-bus system used for bad data problem.

$$H_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_r = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$H_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & -0.33 & 0.33 & 0 & 0 & 1 \end{bmatrix}$$

$$M_r = \begin{bmatrix} 0 & 0.33 & -0.33 & 0 & 0.5 & 0.5 \\ 0 & -0.33 & -0.66 & 0 & 0.5 & -0.5 \end{bmatrix}$$

$$M_c = \begin{bmatrix} 0 & 0.33 & -0.33 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0.5 & -1.5 \\ 0 & 0 & 0 & 1 & -0.5 & -1.5 \\ 0 & 0 & 0 & 0 & -0.5 & -1.5 \\ 0 & 0 & 0 & 0 & 0 & -1.5 \end{bmatrix}$$

The critical measurements can be identified by checking the null columns of T_r given below:

$$T_r = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 & 0.5 \\ 0 & -0.5 & 0 & 0 & 0.5 & -0.5 \end{bmatrix}$$

Column 1 which corresponds to the phase angle (PMU) measurement at bus 1 will be ignored since it serves as the reference bus. That leaves columns 3 and 4 corresponding to the injection at bus 6 and the flow from 4 to 5. The candidates which can transform these two measurements into redundant ones can then be found by calculating T_c which is given below:

$$T_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & -0.5 & 0 & 0 & 0.5 & -1.5 \\ 1 & -0.5 & 0 & 1 & -0.5 & -1.5 \\ 1 & -0.5 & 0 & 0 & -0.5 & -1.5 \\ 1 & -0.5 & 0.5 & 0 & 0 & -1.5 \end{bmatrix}$$

Hence, phase angle measurements at bus 3 and 4 will be identified as the candidate measurements.

Typically, this step will be followed by an optimization procedure, where an optimal subset of the identified candidate measurements will be determined. This is accomplished by solving another integer programming problem which is formulated as follows:

$$\min \sum_{i}^{L} C_i \, . x_i \qquad s.t. \quad Q \cdot X \geq \hat{1} \tag{6}$$

where:

X is a binary array whose entries correspond to the candidate measurements (in this example, they are the phase angle measurements at buses 2 through 6) and they will be 1 or 0 depending on whether they are selected or not respectively.

C_i is the installation cost for PMU i .

Q represents the critical measurement to candidate measurement incidence matrix, defined as follows:

$$Q_{ij} = \begin{cases} 1 & \text{if meas } j \text{ is a candidate for critical meas } i \\ 0 & \text{otherwise} \end{cases}$$

For this simple 6-bus example, since there are only two non-overlapping candidate measurements, Q matrix will be trivial, so will be the optimal solution, i.e. both of the candidate measurements will be selected.

Another case can be illustrated by considering the 14-bus system with its measurement configuration shown in Figure 4, where all measurements are critical. It can be shown that, by applying the above procedure, a single PMU measurement at bus 6 (e.g. monitoring branch 6-11) will eliminate all critical measurements. While this is an extreme case, it illustrates the power of placing few PMUs to make significant improvements in bad data detection capability.

Next, the same procedure is applied for a practical case of the Entergy Corporation's power system. This system has 165 critical measurements 111 of which are power flows and 54 are power injections. It is found that by placing PMUs on 135 strategic branches, all of these critical measurements will become redundant measurements, allowing detection of errors if they occur in these measurements.

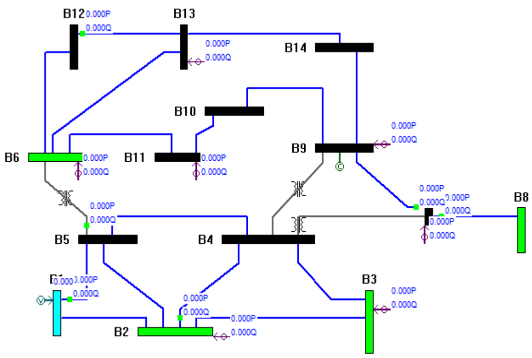

Figure 4: 14-bus system measurement configuration.

4 ROBUSTNESS AGAINST CONTINGENCIES

Network observability depends on types and locations of measurements as well as network topology. Anytime a change occurs in one of these factors, observability analysis ought to be repeated to ensure that state estimation problem can still be solved.

While it is not possible to design the measurement system so that it remains robust against any sort of switching that the system may experience, measurement design can take into account most likely contingencies that involve topology changes. This is quite valuable since it is during these contingencies that the assistance of the state estimator will be most needed. So, in this section, the problem of strategic placement of PMUs against topology changes associated with expected contingencies will be investigated. The approach will be similar to the ones used for the solution of the above two problems. The first stage involves determining whether the considered topology change actually causes any change in network observability. If the system remains observable after the topology change, then no action will be necessary. Else, a set of candidate PMUs will be identified such that placing any one of them will restore observability.

Identification of candidate PMUs will be done as follows:

1. Form the augmented jacobian H which contains all existing measurements followed by the candidate PMU measurements. For each contingency, remove the line which is opened during the contingency and modify the relevant entries in H.
2. Factorize H by limiting the row pivoting to the rows associated with the existing measurements only. If the contingency causes unobservability, a zero pivot will be encountered.
3. Trace the column below the zero pivot in the lower triangular factor and the candidate PMU measurements will be given by those with nonzero entries in this column.

In order to illustrate this approach, consider the 6-bus system measurement configuration given in Figure 5.

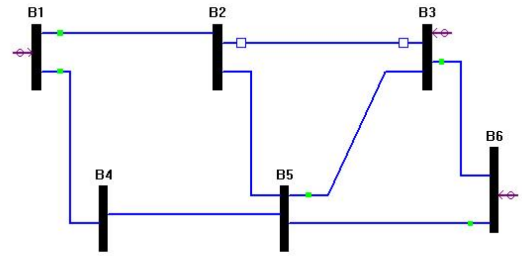

Figure 5: Contingency case: line 2-3 is out of service.

In addition to the given measurements, every bus will be assumed to have a candidate PMU. Line 2-3 outage will be used as a contingency example. Note that the system will become unobservable if line 2-3 is taken out of service.

Modified H after the removal of line 2-3 will be:

$$H = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Its lower triangular factor L will then be given by:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 1 & 0 \\ 0 & 0 & -0.5 & 0 & -3 & 0 \\ 0.5 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0.5 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Tracing the column under the zero pivot (column 6), PMU measurements at buses 3,5 and 6 are identified as candidates for recovering observability during this contingency.

Repeating this procedure for each contingency, a set of candidates will be determined for each contingency and a contingency to candidate PMU incidence matrix will be formed. This matrix will replace the matrix Q in (6) and solution of (6) will then yield the optimal set of candidate PMUs for handling all contingencies.

For the case of the Entergy system, the list of top 20 contingencies is given in Table 3. Each of these contingencies is tested for network observability. The results reveal that only two of them, which are shown in Table 4, will yield an unobservable system. Applying the factorization based procedure above to these two contingency cases yields the results shown in Table 4. Due to the small number of cases, there is no overlap between the candidates and therefore both candidates will have to be installed for handling the two contingencies in this particular case.

From	To
1089	1081
821	801
713	1640
1007	1783
1046	1035
642	588
1529	1501
168	406
2873	2875
333	332
304	442
14	464
8	238
405	326
1705	1779
2831	2817
304	300
301	300
302	300
134	2975
1728	1779

Table 3: Top 20 contingencies for the Entergy system.

Contingency	PMU Location
1640-713	1639
134-2975	3412

Table 4: PMU placement for contingencies in the Entergy system.

5 REMARKS AND CONCLUSIONS

This paper describes procedures which are used in order to determine the best locations to place phasor measurement units for different purposes in a large utility system. The paper first investigates the long term planning for PMU placement where the entire system is to be fully observed by PMUs only. Then, it considers shorter term plans where PMUs are optimally placed in order to improve bad data detection capability of the state estimator by targeting the critical measurements and transforming them into redundant measurements. Finally, robustness against a set of specified line outage contingencies is improved by way of strategically placing PMUs. All the described techniques are tested on small examples as well as on a large utility system measurement set. As PMUs continue to populate power systems, studies such as this are expected to assist utilities who are considering investing in this new technology.

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