Advantages of power system state estimation using Phasor Measurement Units

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Abstract - An important tool for Energy Management Systems (EMS) is state estimation. Based on measurements taken throughout the network, state estimation gives an estimation of the state variables of the power system while checking that these estimates are consistent with the measurements. Traditionally, input measurements have been provided by the SCADA system (Supervisory Control and Data Acquisition). A disadvantage is that the measurements are not synchronised, which means that state estimation is not very precise during dynamic phenomena in the network. With the advent of real-time Phasor Measurement Units (PMU’s), synchronised phasor measurements are possible which allows monitoring of dynamic phenomena. In this paper, the effect of the asynchronicity of measurements on the accuracy of the state estimation is analysed. Also, the possibility of using PMUs for state estimation and the effect of using PMU measurements as well as SCADA measurements is explored.

Keywords - Energy management system, power system state estimation, phasor measurement unit, Global Positioning System.

1 Introduction

Over the past decade, dynamic phenomena in power systems have made it ever more urgent to have a reliable tool to monitor the systems. An essential tool for monitoring of power systems is state estimation. The state of an electrical power system can be estimated using a set of measurements taken throughout the network at a certain time. Based on these measurements, state estimation allows estimation of the state of the power system while checking that these estimates are consistent with the measurements. An important aspect of the state estimator will be the speed it needs to process the data coming from the measurement devices. Traditionally, input measurements have been provided by the SCADA system (Supervisory Control and Data Acquisition), which is designed to capture only quasi-steady state operating conditions, preventing the monitoring of transient phenomena [4]. Synchronicity of the electrical measurements cannot be guaranteed when using the SCADA system. This means that during a dynamic event, the measurements provided to the state estimator by the SCADA system will not allow an accurate estimation of state variables. In actual operation, the snapshot is collected over a few seconds. Therefore, a time skew error on the measurements occurs during dynamic phenomena [5]. The lack of synchronisation means it is likely that among the measurements processed by the state estimator, some are taken before the change and others after. This is a type of coordinated noise because of the interdependent nature of the errors. For instance, a load change at one node, or a topology change means there is a redistribution of the load on the surrounding lines. With the advent of real-time Phasor Measurement Units (PMU’s), fast transients can be tracked at high sampling rates. The sampling frequency depends on an external clock signal (GPS receiver) with absolute time reference. Typically, phase measurements are updated 20 times per second [6].

The type of measurements needed by the state estimator will depend on the numerical method implemented. The complexity of the numerical method allowing state estimation will depend on the functions between the measurements and the state variables. While classical state estimation uses active and reactive power measurements as well as voltage measurements, state estimation using PMUs requires voltage and current phasors. Classical state estimation provides a state estimation solution by iterations, necessary because of the non-linear equations between the measurements and the state variables. As in most state estimation iterative solution methods, it is assumed that the changes between two operating points are small [5]. However, transactions at the transmission level and abrupt flow changes in the open and liberalised electricity market mean that this assumption is no longer correct.

Simplicity of relations between the phasor measurements and the state variables means that state estimation using PMUs is much quicker than classical state estimation [1]. Integration of these devices will not be instantaneous because of their cost and it is unlikely that PMUs will replace the SCADA system for state estimation and other EMS functions soon. It is quite useful therefore to enable the use of PMUs to improve classical state estimation since initially there will not be enough PMUs for state estimation using their measurements only. In this paper the effect of PMU measurements on the accuracy of the state estimation is considered.

One of the essential functions of a state estimator is to detect measurement errors and to identify and eliminate them if possible [3]. The post-estimation tests, carried out after the state estimation, try to detect and eliminate the bad data from the measurement set and improve state estimation parameters. The existing post-estimation methods will be applied to state estimation using measurement sets with a time skew error when necessary.
In this paper, the effect of asynchronicity in the measurement set and of additional PMU measurements on the accuracy of the state estimation is analysed. The first section describes the measurements used by a state estimator. Subsequently the various state estimators are described. The third section describes the bad data identification and detection methods. Finally, simulations analysing the effect of time skew errors when using a weighted least squares state estimator are discussed.

2 Measurements

The output of a state estimator is the value of the state variables at a given time, deduced from a set of measurements taken on the network. If \(z\) is the measurement vector then the model used by power system state estimation is [7]:

\[
z = h(x) + e, \tag{1}
\]

where \(x\) is the current state vector, \(h\) is the vector of nonlinear measurement functions between \(x\) and \(z\) and \(e\) is the measurement noise vector. Currently, measurements are provided by the SCADA system. The SCADA system includes three components: the Remote Terminal Units (RTU), the communication links between RTUs and master stations or SCADA front end computers. The main function of SCADA is receiving and processing tele-information, forming the real-time bases and archives, displaying the information, documenting the data and finally solving the dispatching tasks. The RTUs collect various types of measurements from the field and are responsible for transmitting them to the control center [3]. In this case the measurements are analog and represent voltage amplitudes, injected power at a node or line power flows. With the advent of real-time Phasor Measurement Units (PMU’s), fast transients can be tracked at high sampling rates. PMUs are actually numeric devices allowing synchronised phasor measurements. A GPS clock provides a synchronised phasor measurements. A GPS clock provides the input measurement, \(h(x)\) is the estimated measurement and \(R\) is the diagonal covariance matrix associating a variance to each measurement. In other words, the method ensures that the error between the measurements and the estimations of these measurements when using the state variables, is minimised.

Until recently, state estimation had to rely on non-phasor measurements. As a result, most work on state estimation [3, 7] has been about interpreting the classical measurements made available by the SCADA measurement system. Classical WLS state estimation involves the processing of those measurements in order to obtain the best fit or estimation for the state variables by minimisation of the residuals or estimation errors. The main part is to evaluate the derivatives of each measurement with respect to the state variables in order to calculate the jacobain matrix \(H\).

When solving for \(x\) we get

\[
x = (H^T R^{-1} H)^{-1} H^T R^{-1} z. \tag{5}
\]

3 State estimation

Let us suppose that a vector \(z\) of \(m\) measurements is available as an input for state estimation at the control center. All measurements can be evaluated using the state variables which are to be determined as in equation 1.

The Weighted Least Squares (WLS) estimator minimises the performance index or sum of the squared residuals [7, 3]

\[
J(x) = (z - h(x))^T R^{-1} (z - h(x)), \tag{4}
\]

where \(z\) is the input measurement, \(h(x)\) is the estimated measurement and \(R\) is the diagonal covariance matrix associating a variance to each measurement. In other words, the method ensures that the error between the measurements and the estimations of these measurements when using the state variables, is minimised.

\[
m_2 = (1 - p) m_{tot} \tag{3}
\]

where \(m_1\) is the number of measurements taken before the dynamic event, \(m_2\) is the number of measurements taken after the change and \(m_{tot}\) is the total number of measurements \((m_{tot} = m_1 + m_2)\). Because the measurements are taken over a period of a few seconds, the rate of asynchronicity of the measurement set will depend on the time when the dynamic event takes place. To get an idea of the extent asynchronicity can have on the accuracy of state estimation results, the dynamic model of the electrical network is ignored so both states considered are stable.

The measurements should also be well distributed throughout the network so that there exists a unique solution to the state estimation problem. When PMUs provide the measurements, the observability algorithm [2] can be used: If a PMU is placed at node \(v\), then all the buses incident to \(v\) are observed.
relationship between a current flow between two adjacent buses $i$ and $j$ is shown in equation 6:

$$ I_{ij} = y_{ij,sh} V_i^r + y_{ij} V_i^r - V_j, $$  

(6)

where $V_i$ is the voltage phasor at bus $i$, $V_j$ is the voltage phasor at bus $j$, $y_{ij}$ is the series admittance of the line and $y_{ij,sh}$ is the lumped shunt admittance of line $ij$ connected to bus $i$. Finally, the classical state estimator can be extended so it is able to process PMU measurements as well. Voltage and current phasor measurements are added to the measurement vector $z$ in equation 1. The equation between a current magnitude measurement and that the real and complex part of the current phasor can be measured and used for evaluating the derivatives in the Jacobian matrix. The equations between the phasor measurements are linear as shown in equations 7 and 8,

$$ I_{ij} = -Bc_{ij} V_i^r + G_{ij} V_i^r - G_{ij} V_j^r $$

(7)

$$ I_{ij} = Bc_{ij} V_i^r + B_{ij} V_i^r - G_{ij} V_i $$

(8)

where $I_{ij}$, $V_i$, $V_j$ are the real and complex parts of the current and voltage phasors, $G_{ij}$ and $B_{ij}$ are the real and complex parts of the bus admittance matrix and $Bc_{ij}$ is the lumped shunt admittance of line $ij$ connected to bus $i$. Because $V_i = V_i \cos \theta_i$ and $V_i = V_i \sin \theta_i$, the equation between the measurements and state variables is nonlinear.

For benchmarking the performance of the WLS state estimation, three parameters are used:

1. The performance index with respect to the estimated measurements:

$$ J = Y^T R^{-1} Y $$

(9)

where $Y = z - h(x)$ is the error between the actual measurement and the estimated measurement

2. The performance index with respect to the correct measurements

$$ J_b = Y_b^T R^{-1} Y_b $$

(10)

where $Y_b = z_{base} - h(x)$ is the error between the exact measurements and the estimated measurements.

3. Variance of the estimations: Represents the error between the estimated value and the correct value calculated by the load flow:

$$ P = \frac{1}{m_{tot}} (x_{corr} - x)(x_{corr} - x)^T $$

(11)

where $x_{corr}$ is the correct state vector, $x$ is the estimated state vector and $m_{tot}$ the number of measurements

4 Bad Data detection and identification

One of the essential functions of a state estimator is to detect measurement errors and to identify and eliminate them if possible [3]. The Post-estimation tests, carried out after the state estimation, try to detect and eliminate the bad data from the measurement set and improve state estimation parameters.

4.1 Post-Estimation tests

When using the WLS estimation method, detection and identification of bad data is done only after the estimation process by processing the measurement residuals. The analysis is essentially based on the properties of these residuals, including their expected probability distribution. Bad data can be detected by the Chi-squares test described below. Upon detection of bad data, identification of the bad measurements can be accomplished by further processing of the residuals as in the Largest Normalized Residual (LNR) and Hypothesis Testing Identification (HTI) methods [3]. Both methods use the residual sensitivity matrix $S$ which represents the sensitivity of the measurement residuals to the measurement errors.

4.1.1 Chi-squares test

Intuitively, it is to be expected that bad data should cause an unexpectedly large value of $J(x)$ [9]. Assuming that the errors due to the state estimation are normally distributed ($e_i \sim N(0,1)$) it can be shown that the performance index $J$ has a $\chi^2$-distribution with $m_{tot} - n$ degrees of freedom, where $m_{tot}$ is the number of measurements and $n$ is the number of state variables [3]. After state estimation is performed, bad data will be suspected if

$$ J(x) \geq \chi^2_{m_{tot} - n, p}, $$

(12)

where $p$ is the detection confidence level.

4.1.2 Largest Normalised Residual

In this method, normalised residuals are used to devise a test for identifying and subsequently eliminating bad data [3]. After state estimation has been performed, an error vector $e$ subsides between the actual measurements $z$ and the estimated measurements $h(x)$. The normalised residuals are computed for each measurement as

$$ r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}}, $$

(13)

where $\Omega$ is the residual covariance matrix ($\Omega = SR$) and $r$ is the measurement residual. If

$$ r_i^N \geq c, $$

(14)

where $c$ is a chosen identification threshold, then the $i^{th}$ measurement will be suspected as bad data. At each stage the bad measurement with the largest normalised residual is removed from the measurement set before estimating the state again. This will be repeated until the performance index has an acceptable value as determined by the Chi-squares test.
4.1.3 Hypothesis Testing Identification

The HTI method uses the properties of the measurement residual to find all the bad measurements in one iteration. The measurement residual $r$ can be expressed as

$$r = S e,$$  \hspace{1cm} (15)

where $S$ is the residual sensitivity matrix and $e$ is the measurement noise vector as in equation 1. Whereas the LNR method is based on the residuals, which may be strongly correlated, the Hypothesis Testing Identification (HTI) method estimates the measurement errors directly. Knowing the residual vector $r_S$ we intend to determine an estimate $\hat{r}_S$ of some measurements using the linear relationship 15

$$\hat{r}_S = S_{SS}^{-1} r_S,$$  \hspace{1cm} (16)

where $r_S$ is the measurement residual associated with the selected suspect measurements and $S_{SS}$ is the submatrix of sensitivity matrix $S$ relative to the selected measurements. These measurements are chosen among the suspected ones. This selection of suspected measurements is crucial for the method. First, the measurements with a normalised residual $r_i$ greater than a critical value are selected as in equation (14) [10]. The measurements are then sorted by decreasing order of $|r_i|$. For each measurement, a test has to be carried out in order to verify if adding the measurement to the set of suspect measurements will not make $S_{SS}$ matrix singular. For each suspect measurement $|r_{Si}|$ is calculated

$$|r_{Si}| = \frac{|\hat{r}_{Si}|}{\sigma_i \sqrt{T_{ii}}}$$  \hspace{1cm} (17)

where $T = S_{SS}^{-1}$. Since the normalised variable will have a standard Normal distribution, a proper cut-off value $N_{1-\frac{z}{2}}$ can be looked up from a Standard Normal distribution table [11].

5 Simulations

The 14 bus IEEE test network in Figure 1 is used for the simulations.

![Figure 1: Effect of power increase on performance index](image)

Simulation of asynchronicity of measurements can be done by randomly taking measurements before and after an event. Let us suppose that the measurements taken before the event belong to $S_1$ and the measurements taken after belong to $S_2$. The dynamic event can be a load or a topology change. The measurement set obtained after this process is supposed to represent the input of a state estimator provided by a SCADA system during an event. Measurements taken before and after the tripping of the line are mixed with a certain asynchronicity rate $p$ as defined in section 2. No noise is added to the input measurement as the main effect analysed is asynchronicity. Two types of events are considered: a load change at one of the buses or a topology change. The effect of additional PMUs on the accuracy of the state estimation is analysed as well. A PMU is placed at the slack bus in order to measure the reference phasor. The four cases considered are therefore: No PMUs in the network, PMUs at buses 1 and 9, PMUs at buses 1, 9 and 6, PMUs at buses 1, 9, 6 and 5.

5.1 Load change

When analysing the effect of a load change on the accuracy of the state estimation, two variables can be considered: the amplitude of the load change at the bus or the asynchronicity ratio of the measurement set. In this case, three successive variations of the load by 10% of the initial load is analysed.

5.1.1 Amplitude of the load change

In this paragraph, the effect of the amplitude of the load change on the state estimation parameters is verified. The asynchronicity ratio $p$ is supposed fixed and is equal to 0.5.

![Figure 2: Effect of power increase on performance index $J$](image)

Figure 2 represents the influence of the amplitude of the load change on the performance index $J$. When PMU measurements are added to the input of the state estimator, the performance index goes up which can be understood when looking at the definition of the performance index $J$. It is defined as the sum of the residuals, adding more measurements means that there are more residuals, which increases the performance index.
Figure 3: Effect of power increase on performance index \( J_b \)

Figure 3 represents the influence of the amplitude of the load change on the performance index \( J_b \). Here also, the performance index \( J_b \) is lower when PMU measurements are added. Parameters in Figures 2 and 3 are proportional to the square of the error because the performance index depends on the squared residuals. The linear relation between the square of the amplitude of the load change and the performance indexes is due to the greater error caused as the change gets bigger.

Figure 4: Effect of power increase on performance index \( P \)

Figure 4 represents the influence of the amplitude of the load change on the performance index \( P \). In this case as well, the presence of PMUs clearly has an effect, maintaining the variance at a low value.

5.1.2 Asynchronicity ratio

In this paragraph, the effect of the asynchronicity ratio \( p \) on the state estimation parameters is verified. In this case, the branch between nodes 2 and 4 is removed.

Figure 5: Effect of the asynchronicity ratio on performance index \( J \)

Figure 5 represents the effect of the asynchronicity ratio on the performance index \( J \). When no PMUs are placed in the network, the performance index when asynchronicity ratio \( p \) is 0 or 1 is very small. In that case, all the measurements are taken either before or after the change and the measurements are close to the estimated measurements. When PMUs are present in the network, this is not the case because it is supposed that PMU measurements are synchronised, and are therefore taken at the same time, after the load change in this case.

Figure 6: Effect of the asynchronicity ratio on performance index \( J_b \)

Figure 6 represents the effect of the asynchronicity ratio on the performance index \( J_b \). Like in section 5.1, the presence of PMUs has a good effect on parameter \( J_b \).

Figure 7: Effect of the asynchronicity ratio on variance \( P \)

Figure 7 represents the effect of the asynchronicity ratio on the performance index \( P \). The parameter indicates a lower sensitivity to the asynchronicity as PMUs are added.

5.2 Topology change

In this section, the asynchronicity error due to a topology change is analysed. In this case, the line between buses 2 and 4 is supposed to have tripped. The figures below represent the effect of the asynchronicity ratio and the number of PMUs on the state estimation parameters.
In Figure 8, the effect of the asynchronicity ratio on the performance index is represented. The effect of PMU measurements in the input measurement set is also observed. The performance index increases as the asynchronicity ratio \(p\) goes up, which means that the proportion of measurements belonging to \(S_1\) increases. In the case when all the measurements belong to \(S_1\), the performance index is not good because the information on the topology corresponds to the measurements belonging to \(S_2\).

Figure 9 represents the effect of the asynchronicity ratio and number of PMUs on state estimation parameter \(J_b\). This parameter goes down as PMUs are added, which means that the estimation is more precise.

Figure 10 represents the effect of the asynchronicity ratio and number of PMUs on state estimation parameter \(P\). The PMUs have a beneficial effect on the parameter and limit the increase of \(P\) as the asynchronicity rate \(p\) goes up. Because of the high performance index shown in Figure 8, it might be useful to use the post-estimation methods described in section 4 to try to bring down the performance index to an acceptable level.

Figure 11 represents the performance index \(J\) obtained after post-estimation. Results show that in presence of asynchronous measurements, the LNR method clearly gives better results than the HTI method, especially when more than half of the total number of measurements \(m_{sett}\) are taken before the change (\(p \geq 0.5\)). Using the LNR method, the performance index is brought down to an acceptable level.

As a rule, bad measurements should be eliminated. Figure 12 represents the proportion of eliminated measurements belonging to \(S_1\) or \(S_2\) when using the LNR method. For small values of the asynchronicity \(p\), most of the eliminated measurements belong to \(S_1\) which does not provide many measurements for the asynchronous set. For high values of \(p\) the eliminated measurements still belong to the same set since the LNR method uses the topology after elimination of branch 2-4 to identify bad measurements so \(S_1\) measurements are bad for LNR.
Figure 13: Eliminated measurements using HTI

Figure 13 represents the proportion of eliminated measurements belonging to $S_1$ or $S_2$ when using the HTI method. In this case, most of the eliminated measurements belong to $S_2$ for low values of the asynchronicity while for a high asynchronicity the measurements belong to $S_1$. What is more, most of the measurements eliminated when using HTI come from the set which provides the most measurements whereas most of the measurements LNR eliminated belong to $S_1$ for all values of asynchronicity.

6 Conclusion

The aim of the paper was to show the importance of PMUs for power system state estimation. The PMUs will be most useful during dynamic events when SCADA measurements are asynchronous and do not allow an accurate state estimation. An important property of PMUs is that all measurements are synchronised and therefore are not asynchronous. The dynamic model of the network is ignored when a change occurs in the network. Two types of events were considered here, a load change and a topology change. In both cases, asynchronicity of measurements was shown to have an important effect. However, the topology change is shown to cause much more inaccuracies in the state estimation result. Because of the detection of bad data by the chi-squares test in that case, post-estimation methods using LNR or HTI were compared. The HTI method is generally not as good as the LNR method, especially when measurements at the input of the state estimator present a high asynchronicity rate. In all cases considered, PMUs have a beneficial effect on the accuracy of the state estimation. Further studies could consider the dynamic model of the network. An important result, nevertheless, is that asynchronicity of the input measurements has a bad effect on the accuracy of the state estimation. This is why it is important that the measurements are synchronised so they allow an accurate visualisation of the network and a better quality of the state estimation.

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