SOLVING THE SHORT TERM OPERATING PLANNING PROBLEM OF HYDROTHERMAL SYSTEMS BY USING THE PROGRESSIVE HEDGING METHOD

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Abstract – The short-term operating planning (STOP) of hydrothermal systems aims to define generation targets that minimize the expected operating cost over the planning period. This decision problem is complex due mainly to the uncertainty of the water inflows in the reservoirs. In this sense, the optimization studies have to represent a large number of hydrological scenarios in order to evaluate the impact of an operating decision. In consequence, the STOP is modelled as a multistage stochastic optimization problem and belongs to the most difficult problems of mathematical programming. However, recent advances in stochastic programming theory and in computing technology make possible the development of new methods for solving these remarkable sized problems. Therefore, the purpose of this paper is to apply an approach based on Augmented Lagrangian, which is the Progressive Hedging - PH algorithm. We assess our approach on a realistic hydrothermal configuration extracted from Brazilian Hydrothermal power system. Practical and theoretical aspects of this approach are then compared to the most common method used to solve this class of problems: the Nested Decomposition.

Keywords: Short Term Operating Planning, Hydrothermal Systems, Stochastic Programming, Nested Decomposition, Augmented Lagrangian, Progressive Hedging.

1 INTRODUCTION

The Short-Term Operating Planning - STOP problem of hydrothermal systems consists of minimizing the expected production cost over a time period, complying with a set of technological and system constraints. This problem is quite complex due some characteristics, especially the stochastic nature of the water inflows. Solutions from models with deterministic inflow forecasts can be unsatisfactory. The decisions in such models are made under the incorrect hypothesis that a unique possible inflow is guaranteed for the upcoming weeks or months.

For example, if the average inflow scenario[1] is taken as a certain future, the resulting solutions may fail to hedge against dry inflow scenarios. As a result, if a dry scenario happens, inefficient and costly thermal plants could be urgently and unexpectedly brought on line to satisfy demand or even fail to supply the load. On the other hand, if a wet scenario happens, and the reservoir level are kept high, the reservoir capacity may be exceeded and there will be spillage in the system (meaning a waste of energy) As consequence, thermal plants could be unnecessary brought on line. In both cases, the operation costs are greater than it could be. Therefore, due to the uncertainty of the inflows this problem requires the use of stochastic programming techniques.

Like other stochastic programming problems, the STOP is a large-scale problem and it requires a high computational effort. To deal with this characteristics, some simplifications are introduced into the model. However, these simplifications must be done carefully.

The Nested Decomposition [1] is the method used to solve the optimal STOP of the Brazilian hydrothermal system. This method applies the Bender's decomposition, in this way, a cutting planes set is added to the problem every iteration to implicitly represent the connections between the consecutives time stages.

Even using the Nested Decomposition, model simplifications[2] are required to allow the problem to be solved in reasonable time. However, these simplifications can lead to inoperable dispatch or to the need for adjustments in operating scheduling. These equivocate decisions contrast to the nowadays market oriented power systems, once it requires transparent and unambiguous decisions. On this way, it is desirable to find a more efficient and precise method to solve the stochastic problem in order to avoid these simplifications.

In this context, the Augmented Lagrangian is an alternative method that can be used. Based on the duality principle, this method relaxes the subproblem coupling constraints adding specific linear and quadratic terms to the objective function. The quadratic terms lead to a differentiable problem, which make it promising to solve large-scale problems. Nevertheless it is worth to observe that, due to these quadratic terms, the Augmented Lagrangian does not allow the decomposition of the large-scale problem into smaller subproblems. The Progressive Hedging [2] is derived

[1] This is known as the Expected Value Problem.

[2] These simplifications are based on the linearization of the original problem and limitation of the number of stochastic scenarios.

from the Augmented Lagrangian method and overcomes this obstacle. The application of the Progressive Hedging to similar problems can be seen in the literature; however, no application to the STOP of hydrothermal systems has been found.

In order to reduce the linearization simplifications and increase the number of stochastic scenarios, the aim of this paper is to evaluate the application of the Progressive Hedging to the STOP problem. Comparisons between the Progressive Hedging and the Nested Decomposition (method used to solve the STOP problem in the Brazilian case) will be shown. Some characteristics of these methods will be presented and discussed in order to understand their performance. With the purpose of complement the evaluation of these methods, computational results for a large-scale problem are also presented. This problem models a realistic hydrothermal system planning problem that consists in 21 hydro and 20 thermal plants representing 66% of the Brazilian installed capacity. The stochastic model has 400 inflow scenarios.

The remainder of the paper is organized as follows. In Section 2, the stochastic programming problem representation and formulation are shown. The solution methods studied on this paper are mathematically presented and theoretically compared in Section 3. The test problem is presented in Section 4 and the results on solving this problem by the two studied methodologies are shown in Section 5. Finally, the paper conclusions are done in Section 6.

2 STOCHASTIC PROGRAMMING PROBLEMS

2.1 Uncertainty Modeling

In hydrothermal STOP problems the uncertainty is modeled as a scenario tree, as shown in Figure 1. This figure shows a three stage scenario tree. Each node (filled circle) represents a system state: a possible random realization and a set of decisions to take. Each line represents a possible state transition. A full path for stage one to stage three represents a scenario. Notice that the tree from Figure 1 has nine possible scenarios (s=1…,S=9).

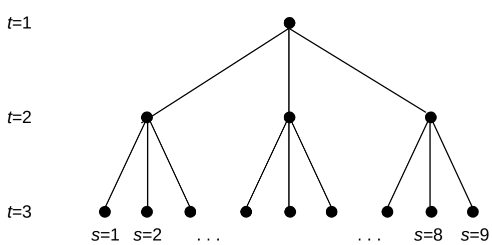

Figure 1: The uncertainty scenario tree representation.

Notice also that for each time stage there is a set of nodes, described as Ω_t, as shown in Figure 2. Also, for each node ω_t its ancestor node $a(\omega_t)$ can be defined. On the other hand, ω_t is one of the possible descendants of $a(\omega_t)$. The set of all possible descendents of a node ω_t is defined by $\Delta(\omega_t)$.

Notice that the nodes and the scenarios have related probabilities: $p_{\omega t}$ and p_s, respectively

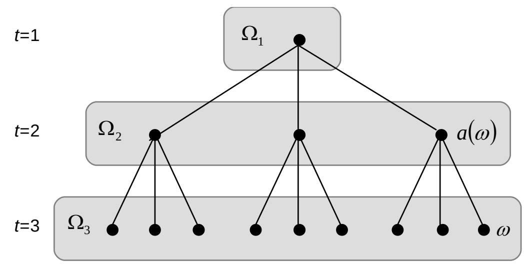

Figure 2: The random realizations on a scenario tree.

2.2 Problem Modeling

Solution methods in stochastic programming often rely on the ability to solve a *T*-stage Stochastic Linear Problem (SLP-T) on a finite scenario tree. The SLP-T can be formulated as a deterministic Linear Problem (LP) in a couple of ways, where different deterministic formulations are suitable to different solution methods.

Instead of expressing SLP-T as recursion like [3], we can enumerate all LP subproblems corresponding to each node as follows[3]:

$$\begin{aligned} &\min \sum_{t=1}^{T} \sum_{\omega_t \in \Omega_t} p_{\omega_t} c_t x_{\omega_t} \\ &\text{s.t.: } B_t x_{a(\omega_t)} + A_t x_{\omega_t} = b_{\omega_t}, x_{\omega_t} \geq 0, \text{for } t = 1,...,T \end{aligned} \quad (1)$$

The sample space stage for stage t is Ω_t and a sample point (realization) in Ω_t is ω_t. The matrix A_t is an $m_t \times n_t$ size and the remaining matrices and vectors are dimensioned to conform [4]. A stage t realization, $\xi_t(\omega_t) = b_{\omega t}$ is a m_t vector, and a probability mass function is given by $P\{\xi_t = \xi_t(\omega_t)\} = p_{\omega t}$. The formulation (1) has an interesting structure for node decomposition (e.g., Nested Decomposition).

Another particular way to model a SLP-T is to put in evidence all scenario subproblems as in (2). To express the connections between these scenario subproblems, the nonanticipativity constraints[4] are made explicit.

$$\begin{aligned} &\min \sum_{s=1}^{S} \sum_{t=1}^{T} p_s c_{s,t} x_{s,t} \\ &\text{s.t.: } B_{s,t} x_{s,t-1} + A_{s,t} x_{s,t} = b_{s,t},\ x_{s,t} \geq 0, \\ &\quad \text{for } t = 1,...,T,\ s = 1,...,S, \\ &\quad x_{s,t} \in N_t, \text{ for } t = 1,...,T-1,\ s = 1,...,S \end{aligned} \quad (2)$$

where p_s is the probability of scenario $\xi_{s,T}$, and the N_t is the nonanticipativity constraint set in the stage t.

Notice that N_t can be expressed in several ways, leading to different dual formulations. A particular nonanticipativy constraint can lead the whole model to be explored by a specific method

[3] Our model is a particular case of SLP-T, known as SLP-T with recourse.

[4] These constraints ensure that the decisions taken at stage t do not depend on a specific scenario, but on all possible scenarios. It means that all scenarios that lie on the same node must have the same decisions taken at that node.

[5]. Performing an Augmented Lagrangian with respect to the N_t it is possible to decompose (2) by scenario.

3 STOCHASTIC PROBLEMS SOLVING METHODS

3.1 Nested Decomposition

In this section we review the application of the Nested Decomposition method that decomposes the deterministic equivalent formulation (1) into subproblems by nodes. A subproblem supplies resources to its descendants and receives dual prices from its descendants to compute the optimality cuts that represent an outer linearization of recourse function, as shown in (3). By the application of the single-cut version to stage t ($1 \leq t < T$) subproblem under realization $\xi_{\omega t}$, we can express this subproblem as:

$$\min_{x_t,\alpha_t} c_t x_{\omega_t} + \alpha_{\omega_t}$$
$$\text{s.t.: } A_t x_{\omega_t} = b_{\omega_t} - B_t x_{a(\omega_t)}, x_{\omega_t} \geq 0, \tag{3}$$
$$e\alpha_{\omega_t} - \underline{G}_{\omega_t} x_{\omega_t} \geq \underline{g}_{\omega_t}$$

The rows of $\underline{G}_{\omega t}$ contain cut gradients; the elements of $\underline{g}_{\omega t}$ contain cut intercepts, and e denotes the vector of all 1s. The stage T subproblems are similar to (3) except that the cut constraints and scalar variable α_t are absent. The dual of (3) may be written as:

$$\max_{\pi_t,\alpha_t} z_{\omega_t} = \pi_{\omega_t}\left(b_{\omega_t} - B_t x_{a(\omega_t)}\right) + \theta_{\omega_t}\underline{g}_{\omega_t}$$
$$\text{s.t.: } \pi_{\omega_t} A_t - \alpha_{\omega_t}\underline{G}_{\omega_t} \leq c_t,\ e^{\mathrm{T}}\theta_t = 1,\ \theta_{\omega_t} \geq 0. \tag{4}$$

If $t=T$, $G_{\omega t}$, $g_{\omega t}$ and dual vector θ_t are absent. Let ($z_{\omega t}$, $\pi_{\omega t}$, $\theta_{\omega t}$) to be an optimal solution of (4). When the descendents $\Delta(\omega_t)$ of ω_t are solved, the particular cut gradient and scalar intercept that may subsequently be appended to ω_t are calculated by:

$$G_{\omega_t} = \sum_{\omega_{t+1}\in\Delta(\omega_t)} p_{\omega_{t+1}|\omega_t}\pi_{\omega_{t+1}}B_t,$$
$$g_{\omega_t} = \sum_{\omega_{t+1}\in\Delta(\omega_t)} p_{\omega_{t+1}|\omega_t}\pi_{\omega_{t+1}}z_{\omega_{t+1}} - G_{\omega_{t+1}}x_{\omega_t}. \tag{5}$$

The scenario tree can be traversed in different ways. The strategy used here is the fastpass tree traversing [6] in which an optimal solution from each stage is passed down to all its corresponding descendants until the last stage is reached, and then the cuts formed by the descendants at each stage are passed back up to the corresponding ancestor subproblems until the first stage is reached. For further discussion of alternate tree traversing strategies see [7].

3.2 Progressive Hedging Method

The Progressive Hedging belongs to a class of Lagrangian-based decompositions methods. The motivation of applying Lagrangian Relaxation - LR to the stochastic programming is the same as that in the deterministic mathematical programming. Specifically, the relaxation of complicating constraints decomposes the original problem into subproblems that can be solved more efficiently in separate. For example, the nonanticipativity constraints presented in (2) can be viewed as complicating constraints, and are dualized in a LR decomposition method. One computational disadvantage of LR is due to slow convergence of subgradient search algorithm when the associated dual problem (a non-smooth convex optimization problem) is optimized.

The Augmented Lagrangian contains a quadratic term at the objective function that helps to speed up the convergence. The stochastic programming methods that use Augmented Lagrangian include Progressive Hedging and Diagonal Quadratic Approximation [8]. However, these methods need to approximate the quadratic proximal term to achieve scenario decomposition.

Consider that $K_{s,t}$ is the set of all scenarios related to s at stage t by the nonanticipaticity, including itself. The Progressive Hedging models the nonanticipativity by the average value of these scenarios decisions:

$$x_{s,t} = \sum_{k\in K_{s,t}} p_k x_{k,t} \tag{6}$$

Applying the Augmented Lagrangian to the resulting model from (2) considering the (6) nonanticipativity constraints we obtain the Augmented Lagrangian function shown at (7).

$$\Lambda(x_t,\pi) = f(x_t) + \sum_{s=1}^{S}\sum_{t=1}^{T}\pi_{s,t}\left(x_{s,t} - \sum_{k\in K_{s,t}} p_k x_{k,t}\right) +$$
$$+\frac{\rho}{2}\sum_{s=1}^{S}\sum_{t=1}^{T}\left\|x_{s,t} - \sum_{k\in K_{s,t}} p_k x_{k,t}\right\|^2 \tag{7}$$
$$x_{s,t} \in X_{s,t}$$
$$t = 1,\ldots,T$$
$$s \in S$$

where $\pi_{s,t}$ are the Lagrange Multipliers from the nonanticipativity constraints and ρ is the penalty parameter.

Notice that the Augmented Lagrangian function is not decomposable in smaller subproblem. The Progressive Hedging invokes the decomposability introducing the following additional parameter (8), which is iteratively updated and treated as a constant during each iteration.

$$\bar{x}_{s,t} = \sum_{k\in K_{s,t}} p_k x_{k,t}$$
$$s \in S \tag{8}$$

The additional parameter is then placed on the Augmented Lagrangian function leading to the following scenario subproblems:

$$\Lambda_s(x_s,\bar{x}_s,\pi_s) = f_s(x) + \sum_{t=1}^{T}\pi_{s,t}\left(x_{s,t} - \bar{x}_{s,t}\right) +$$
$$+\frac{\rho}{2}\sum_{t=1}^{T}\left\|x_{s,t} - \bar{x}_{s,t}\right\|^2 \tag{9}$$

The solution of the problem by the Progressive Hedging consists in iteratively solving the dual functions (10) and the dual problem (11).

$$\psi_s(\pi_s) = \min_{x_s\in X_s}\Lambda_s(x_s,\bar{x}_s,\pi_s), s = 1,\ldots,S \tag{10}$$

$$\max \psi(\pi) = \max \sum_{s=1}^{S} \psi_s(\pi_s) \tag{11}$$

$$\text{s.t.:} \pi \in R^n$$

Since the dual problem (11) is differentiable, it can be solved by applying the gradient method (12).

$$\pi_s^{v+1} = \pi_s^v + \rho\left(x_s^v - \bar{x}_s^v\right) \tag{12}$$

3.3 Commentaries on the Methods

Notice that the Progressive Hedging has two parameter sets that are iteratively upgraded (the additional parameter and the Lagrange Multipliers). Additionally, the gradient method only uses information from the last iteration. Due to these characteristics, the Progressive Hedging is also sensitive to the use of warm start.

On the other hand, the first iterations of the Nested Decomposition process are known by its "bang-bang" behavior.

The Progressive Hedging has greater independence between the subproblems than the Nested Decomposition. Also, the Progressive Hedging only requires the interchange of information between the subproblems after an iteration, while Nested Decomposition requires it several times during an iteration. Thus, the Progressive Hedging is better fit to parallel computing than the Nested Decomposition.

Due to the optimality cuts additions to the subproblems in the Nested Decomposition, these subproblems size increases iteratively, while in the Progressive Hedging it remains the same.

As the Nested Decomposition is the most popular method used to solve linear stochastic problems, it has more available enhancements at the literature.

4 TEST PROBLEM

4.1 The Problem

The stochastic model consists in a three months planning period. The first month is discretized into five stages in which the inflows are predicted and treated as deterministic. The last two stages are monthly discretized and treated as stochastic months with 20 possible realizations each. It results in a stochastic model having 425 nodes, 400 scenarios and seven stages.

The hydrothermal test system is based on the Brazilian system and consists in 21 hydro and 20 thermal plants. The hydro installed capacity is about 47.5 GW and the thermal is 9.5 GW, which represents 66% of the Brazilian installed capacity. These plants are allocated in four different and interconnected subsystems. The total energy demand for the subsystems is 37.6 GW. Energy deficits for the different subsystems are modelled as high cost thermal plants (about three times greater than the most expensive thermal plant).

The hydro plants are physically connected in cascades, as shown in the Figure 3.

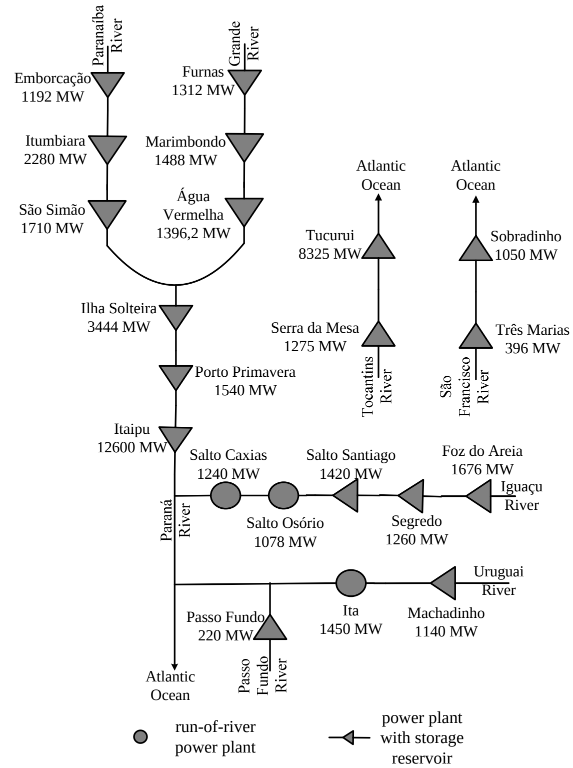

Figure 3: Test systems hydro power plants.

The hydro plant production is a function of the net head, the turbined outflow and the plant efficiency. On these paper simulations the hydro plant production was linearized in a piecewise manner resulting in three linear constraints.

In order to introduce some information from the future at the last stage, a Future Cost Function[5] was considered. This function was linearized in a piecewise manner resulting in 600 constraints.

4.2 Nomenclature

The nomenclature used to model the hydrothermal systems STOP is defined bellow. The two possible models are shown in subsections 4.3 (node subproblems) and 4.4 (scenarios subproblems).

- T total stages;
- t index of stages, so that t=1,…,T;
- E total number of energy subsystems;
- e index of subsystems., so that e=1,…,E
- R_e total number of hydro plants at subsystem e;
- r index of hydro plants, so that r=1,…,R_e;
- Q_r turbined outflow in hydro plant r;
- sp_r spillage in the reservoir of the hydro plant r;
- ph_r power output in hydro plant r;
- y_r inflow into reservoir of hydro plant r;
- M_r total number of upstream reservoirs from reservoir r;
- m index of upstream reservoirs, so that m=1,…, M_r;

[5] This function was obtained from the solution of the immediate longer term problem [9].

v_r volume of water into the reservoir of hydro plant r at the end of stage t;
I_e total number of thermal plants at e subsystem;
i index of thermal plants, so that i=1,…,I_e;
pt_i power generated in thermal plant i;
ct_i production cost of thermal plant i;
d_e energy deficit at subsystem e;
cd deficit cost;
L_e energy demand at subsystem e;
Int_{le} energy interchange from subsystem l to subsystem e.

To differentiate the node and the scenario formulations there are additional notations defined at Subsection 2.1.

For the node formulation used by the Nested Decomposition there are the following specific notations:

Ω_t, set of nodes on stage t;
ω_t a specific node in stage t, so that $\omega_t \in \Omega_t$,
$a(\omega_t)$ the ancestor node of ω_t.

For the scenario formulation used by the Progressive Hedging there are the following specific notation:

S total number of scenarios;
s index of scenarios, so that s=1,…,S.

4.3 Node Problem

The subproblem for a particular node ω belonging to the stage t is formulated as in (13). This model is the one used by the Nested Decomposition.

$$F_{\omega\in\Omega_t} = \min \sum_{i\in I} ct_i\left(pt_{\omega_t,i}\right) + \sum_{e\in E} cd\left(d_{\omega_t,e}\right)$$

s.t.:

The demand supplying equation:

$$\sum_{i\in I_e} pt_{\omega,t,i} + \sum_{r\in R_e} ph_{\omega_t,r} + \sum_{l\in E} Int_{\omega_t,l,e} + d_{\omega_t,e} = L_{t,e}$$

The stream-flow balance equation:

$$v_{\omega_t,r} + Q_{\omega_t,r} + sp_{\omega_t r} - \sum_{m\in M_r}\left(Q_{\omega_t,m} + sp_{\omega_t,m}\right) = y_{\omega_t,r} + v_{a(\omega_t),r}$$

The hydro plants production function:

$$ph_{r\omega_t} = \Im\left(v_{\omega_t,r}, v_{a(\omega_t),r}, Q_{\omega_t,r}, sp_{\omega_t,r}\right)$$

Box-constraints: (13)

$$0 \le pt_{\omega_t,r} \le pt_r^{\max}$$
$$0 \le ph_{\omega_t,r} \le ph_r^{\max}$$
$$-Int_{\omega_t,e,l}^{\max} \le Int_{\omega_t,l,e} \le Int_{\omega_t,l,e}^{\max}$$
$$v_r^{\min} \le v_{\omega_t,r} \le v_r^{\max}$$
$$0 \le sp_{\omega_t,r} \le sp_r^{\max}$$
$$0 \le Q_{\omega_t,r} \le Q_r^{\max}$$
$$0 \le d_{\omega_t,e} \le L_{t,e}$$
$$e = 1,\dots,E$$

For the test problem, each node subproblem has 113 variables and 89 constraints, except for the last stage that has 114 variables and 689 constraints due to the Future Cost Function.

4.4 Scenario Problem

The subproblem for a particular scenario s is modeled as in (14). This is the model used by the Progressive Hedging.

$$F_{s\in S} = \min \sum_{t=1}^{T}\sum_{i\in I} ct_i\left(pt_{s,t,i}\right) + \sum_{e\in E} cd\left(d_{s,t,e}\right)$$

s.t.:

$$\sum_{i\in I_e} pt_{s,t,i} + \sum_{r\in R_e} ph_{s,t,r} + \sum_{l\in\Omega_e} Int_{s,t,l,e} + d_{s,t,e} = L_{t,e}$$
$$v_{s,t,r} - v_{s,t-1,r} + Q_{s,t,r} + sp_{s,t,r} - \sum_{m\in M_r}\left(Q_{s,t,m} + sp_{s,t,m}\right) = y_{s,t,r}$$
$$ph_{rts} = \Im\left(v_{s,t,r}, v_{s,t-1,r}, Q_{s,t,r}, sp_{s,t,r}\right)$$
$$0 \le pt_{s,t,i} \le pt_i^{\max}$$
$$0 \le ph_{s,t,r} \le ph_r^{\max} \qquad (14)$$
$$-Int_{s,t,l,e}^{\max} \le Int_{s,t,l,e} \le Int_{s,t,e,l}^{\max}$$
$$v_r^{\min} \le v_{s,t,r} \le v_r^{\max}$$
$$0 \le sp_{s,t,r} \le sp_r^{\max}$$
$$0 \le Q_{s,t,r} \le Q_r^{\max}$$
$$0 \le d_{s,t,e} \le L_{t,e}$$
$$e = 1,\dots,E$$
$$t = 1,\dots,T$$

For the test problem, each scenario subproblem has 792 variables and 1223 constraints (1079 ordinary constraints, taking into account the Future Cost Function, and 147 stage coupling constraints)

5 COMPUTATIONAL RESULTS

In order to study a variety of warm-starts (which are particularly important to Progressive Hedging) some different routines were implemented. These routines are explained bellow:

- PH: Progressive Hedging using starting point at zero for the additional parameters (8) and for the Lagrange Multipliers (12);
- PH WS-EV: Progressive Hedging using the Expected Value Problem solution as a warm-start for the additional parameters. The Lagrange Multipliers were started with zero;
- PH WS-SP -10%: Progressive Hedging using the solution of a similar problem (demand 10% decreased) as warm-start for the additional parameters and the Lagrange Multipliers;
- PH WS-SP +10%: Progressive Hedging using the solution of an similar problem (demand 10% increased) as warm-start for the additional parameters and the Lagrange Multipliers;
- NE: Nested Decomposition without a starting point;
- NE WS-EV: Nested Decomposition using the optimality cuts from the solution of the Expected Value Problem.

The simulations have been done over a Pentium 4, 3.2 GHz with 1GB of RAM. The linear and the quadratic problems were solved using CPLEX 7.1.0.

The stopping criterion was the end of the first iteration that reaches a total running time equal to one hour.

The final objective value for each routine was compared to the deterministic equivalent LP solution and shown in Table 1. This table also shows the number of iterations and the method running time.

Method	Objective Function	Iteration	Running Time
PH	228.8%	7	60.45 min
PH WS-EV	-0.3%	7	63.70 min
PH WS-SP -10%	8.5%	7	62.17 min
PH WS-SP +10%	-0.1%	7	63.62 min
ND	0.8%	36	60.70 min
ND WS-EV	0.0%	31	61.14 min

Table 1: Methods comparisons: objective function, number of iterations and running time.

In Table 1 it can be seen that a warm start is decisive for the Progressive Hedging success: the PH with starting solution at zero did not converge in the specified time while the routines that used warm start did. The ND WS EV is the unique routine that reaches the solution with zero deviation at the precision. The ND, the PH WS EV and the PH WS-SP +10% reach deviations of 0.8%, 0.3% and 0.1% from the Deterministic Equivalent solution, respectively, which are also good results. These results lead to the conclusion that the use of warm-start is decisive for the Progressive Hedging and that this method can be competitive when compared to the Nested Decomposition. Additionally, it can be seen that the use of warm-start is more effective to the Progressive than the Nested Decomposition.

It can also be seen in Table 1 that the Nested Decomposition routines had about 4.5 times more iterations than the Progressive Hedging.

The primal feasibility to the Progressive Hedging is measured by the parameterized average deviation of the nonanticipativity variables, as shown in Table 2. Once again, it can be seen that the use of the warm-start is decisive for the Progressive Hedging method.

Method	Primal Feasibility
PH	0.11%
PH WS-EV	0.02%
PH WS-SP -10%	0.11%
PH WS-SP +10%	0.02%

Table 2: Primal feasibility measurement on Progressive Hedging.

The Nested Decomposition duality gaps[6] are shown in Table 3. The performance enhancement by using the Expected Value solution to perform a warm-start for the Nested Decomposition is also confirmed.

Method	Duality Gap
ND	0.83%
ND WS-EV	0.03%

Table 3: Duality gap on Nested Decomposition.

The methods convergence is shown in Figure 4. It can be seen that the Nested Decomposition has an oscillatory behavior even on the expected solution neighboring.

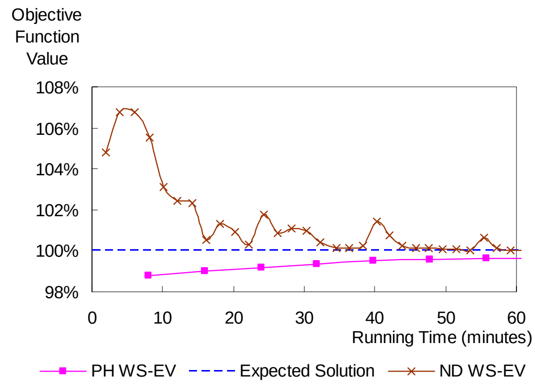

Figure 4: Methods convergence.

6 CONCLUSIONS

The hydrothermal systems Short-Term Operating Planning - STOP is an optimization problem that deals with the water inflow uncertainty. On this way, it is treated as a stochastic programming problem. As many others stochastic programming problems, the STOP has remarkable sizes. Therefore, simplifications are done intending to solve it within a reasonable time. However, these simplifications can induce equivocate or unpractical results. In order to solve the STOP problem on a more efficient way, intending to avoid simplifications, two solving methods were studied: the Nested Decomposition, method used to solve the Brazilian STOP, and the Progressive Hedging, that is a promising method.

The Progressive Hedging is better fit to parallel processing than the Nested Decomposition. Another advantage of the Progressive Hedging is that it subproblems have fixed sizes during the iterative solution process, while the Nested Decomposition iteratively adds constraints to the subproblems.

The Progressive Hedging is more sensitive to the use of warm-start and its success depends on it. Actually, the Progressive Hedging becomes very competitive to Nested Decomposition when using a good warm-start.

[6] The duality gap measures the precision of the recourse function shown in (3).

The computational results show that the running time and the final solution precision of both methods are quite equivalent.

Finally, the paper shows that the Progressive Hedging is a competitive method to solve the hydrothermal systems STOP.

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