PROBABILITY DISTRIBUTION FUNCTIONS OF GENERATOR PROFIT FROM SPOT MARKET

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Abstract – Generator profit from a spot market is a function of the market price of electricity. The price of electricity can be modeled as a geometric Brownian motion process or a time-varying mean reversion process. We are interested in calculating risk measures associated with profit, such as value-at-risk (VaR) or conditional value-at-risk (CVaR). The cumulative distribution function (CDF) or probability density function (PDF) of the profit as a random variable is needed. Only expected value of the profit is available through Black Sholes theory of option pricing. In this paper, we derive analytically the CDF and PDF of generator profit from a spot market when the price of electricity follows either a geometric Brownian motion process or a time-varying mean reversion process. An analytical expression for higher order moments is also derived. The expected value agrees with the result of Black Sholes theory. The comparison of accuracy and computational requirement between the analytical approach and Monte Carlo simulation are discussed.

Keywords: Valuation of generator asset, real options, option pricing, Black-Sholes theory, power system economics, generator bidding and scheduling

1 INTRODUCTION

In traditional generation planning for either long-term expansion or short-term scheduling under the regulated monopoly regime, where electricity tariff is set by the government agency to ensure a guaranteed rate of return, the objective of planning becomes simply the minimization of total cost, consisting of fixed costs and the recurrent operating costs. Standard financial tools are available for evaluating fixed costs, the cost of producing electric energy, on the other hand, depends on operational decisions taking into account engineering constraints. Probabilistic production costing methodology was developed in the 1970's, following the pioneering work of Baleriaux [1], whose mathematical theory is based on a simple model of power system dispatch. Two approaches have been widely used; an analytical approach using the equivalent load duration curves and a Monte Carlo simulation approach [2,3].

Since the middle of 1990's, worldwide trend in restructuring, or liberalization, of the electric power industry has introduced a competitive generation sector. Private generators make their own decisions on building new plants, as well as on scheduling its output to sell in a competitive market once the plant is built. Since the price of electricity is no longer set by the government; rather, it is set by the market, the profit is uncertain and has to be estimated. A landmark work in the modeling of the profit of a generator in a market environment was made by Deng [4,5] in his proposal of using spark spread options in financial option theory to calculate generator profits. Black-Shole model was used to calculate the expected profit. The significance of this work may be compared with that of Baleriaux in 1967 when the concept of probabilistic production costing was introduced.

The extremely high volatility of electricity price in the newly liberalized market has prompted a lot of research efforts in risk assessment and management. Borrowing from other commodity markets, electricity derivatives, such as futures, options, etc. have been introduced and derivative markets have been set up for risk management, with, however, varying degrees of success [6,7]. Proposals to adopt Value ar Risk (VaR) as the measure of risk assessment in electricity market have been put forward [8,9] in the context of short-term risk in an electricity spot market sells with such financial instruments as forwards, futures and options, in addition to transactions in the spot market. Conditional value at risk is shown to provide a better measure [10]. In terms of computational methods, for lack of alternatives, Monte Carlo simulations are invariably proposed.

Value-at-risk (VaR) and conditional value-at-risk (CVaR) are standard measures for risk assessment and management. In particular, CVaR is a convex function and better suited for the portfolio optimization approach to risk management. For proper evaluation of VaR and CVaR, either the cumulative distribution function (CDF) or probability density function (PDF) of the profit is needed. The objective of this paper is to derive the CDF or PDF of generator profit from a spot market. Deng [4] has shown that the generator profit has the same mathematical expression as the price of a European call option, whose value satisfies the Black-Sholes differential equation. The option price in Black Sholes theory is actually the expected value of generator profit, whereas our goal is to derive the probability distribution functions of the random variable that are directly applicable to risk assessment and management. Such derivations of analytical expressions of the CDF and PDF of option prices have not been found in the finance literature.

2 SPOT MARKET PROFIT

Power systems have to physically maintain power balance at all times. That means that if the system is operated as a market, the supply and demand of the market have to match instantaneously, or, in the parlance of market economics, the market has to be cleared at all times. This requirement can only be met with a centralized market. In most countries, the electricity market is operated as a day-ahead or hour-ahead auction market. The market operator receives bids from generators and forms an aggregate supply curve by stacking up the bids according to their bid prices. The highest bid that is needed to generate in order to meet system load demand becomes the spot price of electricity for that moment. The spot price of electricity is determined by the market as a whole. We assume that the market is operated in an ideal manner in which all generators bid their true marginal costs.

Under the simple market rule described above, from the point of view of an individual generator, if the price of electricity is above its bid price, i.e., its marginal cost, the generator gets to generate and its profit is equal to the difference between the price and its cost. If the price of electricity is below its cost, the generator does not get to generate and its profit is equal to zero. Mathematically, the profit of a generator at a future time *t* can therefore be expressed the same as the payoff from a European call option:

$$y(t) = e^{-rt}\max\left[S(t) - C(t), 0\right] \quad (1)$$

where $C(t)$ is the cost of generation and the exponential function accounts for present-valuing the profit to t=0. Future price $S(t)$ is a random variable and is assumed to follow some type of a stochastic process.

3 PRICE MODEL

Electricity prices vary with time and fluctuate. Stochastic processes, which are functions of time whose values at different times are random variables, have been used to model such functions. Like stock prices and other commodity prices, electricity prices are best modeled in terms of percentage change. Geometric Brownian motion and generalized mean reversion process have been used to model electricity prices.

The Geometric Brownian Motion (GBM) process is described by the following stochastic differential equation:

$$d\tilde{S} = \mu\tilde{S}dt + \sigma\tilde{S}d\tilde{z} \quad (2)$$

where μ is the percentage rate of return, or the growth rate, of electricity price $\tilde{S}$ and σ is its volatility. We use $\tilde{S}$ to denote random variable. $d\tilde{z}$ is a Brownian motion or Wiener process. The GBM process is commonly used to model prices of stocks, commodities, etc., including electricity.

The Mean Reversion (MR) process, described below, has the property of continuously reverting back to its mean $\bar{x}(t)$ and is suitable to model prices that exhibit daily, weekly, or seasonally variations, such as the price of electricity.

$$d\tilde{S} = k\left[g(t) - \ln\tilde{S}\right]\tilde{S}dt + \sigma\tilde{S}d\tilde{z} \quad (3)$$

It can be shown that the price of electricity, modeled either as a geometric Brownian motion process or a mean reversion process, at time *t*, $\tilde{S}(t)$, is a random variable with a lognormal distribution. The mean $\bar{x}(t)$ and variance $\sigma_x^2(t)$ of the random variable $\tilde{x} = \ln\tilde{S}$ is given in Table 1 below.

Model	GBM	MR
Distribution of $\tilde{S}$	Lognormal	Lognormal
Equation	$d\tilde{S} = \mu\tilde{S}dt + \sigma\tilde{S}d\tilde{z}$	$d\tilde{S} = k\left[g(t) - \ln\tilde{S}\right]\tilde{S}dt + \sigma\tilde{S}d\tilde{z}$
Equation in $\tilde{x} = \ln\tilde{S}$	$d\tilde{x} = \nu dt + \sigma d\tilde{z}$ $\nu = \mu - \frac{1}{2}\sigma^2$	$d\tilde{x} = k\left[g_1(t) - \tilde{x}\right]dt + \sigma d\tilde{z}$ $g_1(t) = g(t) - \frac{1}{2k}\sigma^2$
Mean $\bar{x}(t)$	$x(0) + \nu t$	$e^{-kt}x(0)$ $+\int_0^t e^{-k(t-\tau)}kg_1(\tau)d\tau$
Variance $\sigma_x^2(t)$	$\sigma^2 t$	$\frac{\sigma^2}{2k}(1 - e^{-2kt})$

Table 1: Summary of GBM process and MR process

4 PROBABILITY DISTRIBUTION OF PROFIT

We derive mathematically here the CDF and PDF of

$$\tilde{y}(t) = e^{-rt}\max\left[\tilde{S}(t) - C(t), 0\right] \quad (4)$$

where $\tilde{S}(t)$ is a lognormal distribution, i.e., $\tilde{x}(t) = \ln\tilde{S}(t)$ is a normal distribution with mean $\bar{x}(t)$ and variance $\sigma_x^2(t)$. The specific expressions of $\bar{x}(t)$ and $\sigma_x^2(t)$ for the two price models, geometric Brownian motion process or mean reversion process, can be found in Table 1.

4.1 The cumulative distribution function of profit

The CDF of $\tilde{y}(t)$ is defined as

$$F_{\tilde{y}}(y) = P\left\{\tilde{y} \le y\right\} \quad (5)$$

We consider the three intervals.

(i) $y < 0$

In this case, $F_{\tilde{y}}(y) = P\left\{\tilde{y} \le y < 0\right\}$. Since $\tilde{y}(t)$ is, by definition, non-negative, therefore $F_{\tilde{y}}(y) = 0$.

(ii) $y = 0$

In this case,

$$F_{\tilde{y}}(y) = P\left\{\tilde{y} \le y = 0\right\} = P\left\{\tilde{y} \le 0\right\} \quad (6)$$

Note that $\left\{\tilde{y} \le 0\right\}$ is equivalent to $\left\{\tilde{S}(t) \le C(t)\right\}$ or $\left\{\ln\tilde{S}(t) \le \ln C(t)\right\}$. Therefore,

$$F_{\tilde{y}}(y) = P\left\{\tilde{x}(t) \le \ln C(t)\right\} \quad (7)$$

Since $\tilde{x}(t) = \ln\tilde{S}(t)$ has a normal distribution with mean $\bar{x}(t)$ and variance $\sigma_x^2(t)$, we can write

$$F_{\tilde{y}}(y) = N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right) \quad (8)$$

(iii) $y > 0$

In this case, $F_{\tilde{y}}(y) = P\{\tilde{y} \le y\}$ and $y > 0$. If $\tilde{S}(t) < C(t),\ \tilde{y}(t) = 0$, which is the case we have considered. We need only to consider $\tilde{S}(t) > C(t)$. In this case, we can write $\tilde{y}(t) = (\tilde{S}(t) - C(t))e^{-rt}$. The condition $\{\tilde{y} \le y\}$ is equivalent to

$$\{\tilde{S}(t) \le ye^{rt} + C(t)\} \tag{9}$$

therefore,

$$P\{\tilde{y}(t) \le y\} = P\{\tilde{S}(t) \le ye^{rt} + C(t)\} \tag{10}$$

Since the logarithmic function is monotonically increasing, we may write

$$P\{\tilde{S}(t) \le ye^{rt} + C(t)\} = P\{\tilde{x}(t) \le \ln\left(ye^{rt} + C(t)\right)\} \tag{11}$$

But $\tilde{S}(t)$ is lognormal and $\tilde{x}(t)$ is normal, the CDF in this case can be expressed in terms of a normal distribution,

$$F_{\tilde{y}}(y) = N\left(\frac{\ln\left[ye^{rt} + C(t)\right] - \bar{x}(t)}{\sigma_x(t)}\right) \tag{12}$$

4.2 The probability density function of profit

In the section, we derive the probability density function of the truncated lognormal distribution $f_{\tilde{y}}(y)$ by differentiating the cumulative distribution function with respect to *y*, i.e., $f_{\tilde{y}}(y) = \frac{d}{dy}F_{\tilde{y}}(y)$. Again we consider separately the three intervals of y.

(i) $y < 0$

$$F_{\tilde{y}}(y) = 0,\quad f_{\tilde{y}}(y) = \frac{d}{dy}F_{\tilde{y}}(y) = 0.$$

(ii) $y = 0$

$$F_{\tilde{y}}(y) = N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)$$

The CDF jumps from 0 to $N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)$ at y = 0 and is not continuous. The derivative of the step function is the *delta function* $\delta(0)$. Since the magnitude of the jump of $F_{\tilde{y}}(y) = 0$ is equal to $N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)$, the delta function has to scale up to $N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)$. Thus, using the delta function, we have

$$f_{\tilde{y}}(y) = \frac{d}{dy}F_{\tilde{y}}(y) = N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)\delta(0) \tag{13}$$

(iii) $y > 0$,

$$f_{\tilde{y}}(y) = \frac{d}{dy}N\left(\frac{\ln\left[ye^{rt} + C(t)\right] - \bar{x}(t)}{\sigma_x(t)}\right)$$

Applying chain rule, we have

$$f_{\tilde{y}}(y) = \frac{d}{dx}N\left(\frac{x - \bar{x}(t)}{\sigma_x(t)}\right)\frac{d}{dy}\ln\left[ye^{rt} + C(t)\right] \tag{14}$$

where $x = \ln(ye^{-rt} + C(t))$. The first term, the derivative of CDF of a normal distribution, is the PDF of a normal distribution.

$$\frac{d}{dx}N\left(\frac{x - \bar{x}(t)}{\sigma_x(t)}\right) = \frac{1}{\sigma_x(t)\sqrt{2\pi}}\exp\left\{-\frac{[x - \bar{x}(t)]^2}{2\sigma_x{}^2(t)}\right\} \tag{15}$$

The second term can easily be calculated.

$$\frac{d}{dy}\ln\left[ye^{rt} + C(t)\right] = \frac{e^{rt}}{ye^{rt} + C(t)} \tag{16}$$

Combining the two, we obtain, for this case,

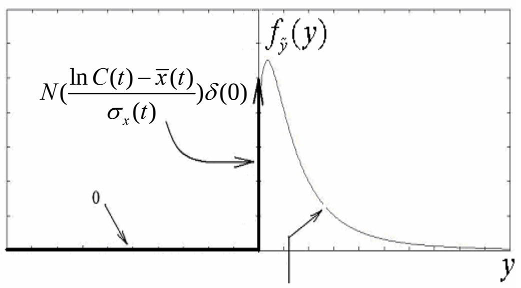
$$f_{\tilde{y}}(y) = \frac{1}{\sigma_x(t)\sqrt{2\pi}}\exp\left\{-\frac{[\ln(ye^{rt} + C(t)) - \bar{x}(t)]^2}{2\sigma_x^2(t)}\right\}\frac{e^{rt}}{ye^{rt} + C(t)} \tag{17}$$

To sum up, the CDF and PDF of a truncated lognormal distribution of the profit are summarized in Table 2 below.

y	CDF $F_{\tilde{y}}(y)$	PDF $f_{\tilde{y}}(y)$
y<0	0	0
y=0	$N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)$	$N\left(\frac{\ln C(t) - \bar{x}(t)}{\sigma_x(t)}\right)\delta(0)$
y>0	E.q.(12)	E.q. (17)

Table 2: CDF and PDF of profit from spot market

The PDF of a truncated lognormal distribution graphically sketched in figure 1.



$$\frac{1}{\sigma_x(t)\sqrt{2\pi}}\exp\left\{-\frac{[\ln(ye^{rt} + C(t)) - \bar{x}(t)]^2}{2\sigma_x^2(t)}\right\}\frac{e^{rt}}{ye^{rt} + C(t)}$$

Figure 1: The PDF of truncated lognormal distribution

4.3 Moments of profit

Once the PDF of a truncated lognormal distribution is given, higher order moments can be calculated. For the results to be more general, let us first put the PDF in (17) in a standard form as below:

$$f_{\tilde{y}}(y) = \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{y+C}\exp\left\{-\frac{[\ln(y+C)-\mu]^2}{2\sigma^2}\right\}$$

where

$$C = e^{-rt}C(t)$$
$$\sigma = \sigma_x(t)$$
$$\mu = -rt + \bar{x}(t)$$

It then follows that:

$$\ln C - \mu = \ln C(t) - \bar{x}(t)$$

(i) The jth moment about origin of $\tilde{y}(t)$

The jth moment about origin of a truncated lognormal distribution variable $\tilde{y}(t)$ is defined as

$$\int_{-\infty}^{+\infty} y^j f_{\tilde{y}}(y)dy$$

Since

$$\int_{-\infty}^{+\infty} y^j f_{\tilde{y}}(y)dy = \int_{-0}^{+0} 0^j \cdot \delta(y)\cdot N\left(\frac{\ln C-\mu}{\sigma}\right)dy + \int_{0}^{+\infty} y^j \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{[\ln(y+C)-\mu]^2}{2\sigma_x^2(t)}\right\}\frac{1}{y+C}dy$$
$$= \int_{0}^{+\infty} y^j \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{[\ln(y+C)-\mu]^2}{2\sigma^2}\right\}\frac{1}{y+C}dy$$

If we define $v = \ln[y+C]$, we obtain

$$\int_{-\infty}^{+\infty} y^j f_{\tilde{y}}(y)dy = \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\left[e^v - C\right]^j \cdot \exp\left\{-\frac{[v-\mu]^2}{2\sigma^2}\right\}dv \tag{18}$$

(ii) The mean, variance and skew of $\tilde{y}(t)$

We derive the mean, variance and skew of a truncated lognormal distribution may be expressed more concisely in terms of a parameter L_j, which is related to the jth order moment of $\tilde{y}(t)$.

Let

$$L_j(t) = \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\exp(jy)\cdot\exp\left\{-\frac{[y-\mu]^2}{2\sigma^2}\right\}dy \tag{19}$$

where $N(\cdot)$ is the CDF of the standard normal distribution..

For L_j, we have following result

$$L_j(t) = e^{-jrt}\exp\left[j\bar{x}(t) + \frac{j^2\sigma_x^2(t)}{2}\right]\cdot N\left[\frac{\bar{x}(t)+j\sigma_x^2(t)-\ln C(t)}{\sigma_x^2(t)}\right]$$
$$j = 1, 2, ... \tag{20}$$

Indeed,

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\exp(jy)\cdot\exp\left\{-\frac{[y-\mu]^2}{2\sigma^2}\right\}dy$$
$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\exp\left\{-\frac{[y-\mu]^2}{2\sigma^2} + jy\right\}dy$$
$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\exp\left\{-\left[\frac{y^2 - 2(\mu+j\sigma^2)y + (\mu+j\sigma^2)^2}{2\sigma^2}\right] + \left[\frac{2\mu\sigma^2 j + \sigma^4 j^2}{2\sigma^2}\right]\right\}dy$$
$$= \exp\left[j\mu + \frac{j^2\sigma^2}{2}\right]\cdot N\left[\frac{\mu + j\sigma^2 - \ln C}{\sigma^2}\right]$$

After substituting $C, \sigma,$ and μ back into $C(t), \sigma_x,$ and $\bar{x}(t)$, we obtain (20).

The mean of $\tilde{y}(t)$ can be derived using equation (20),

$$\bar{y}(t) = \int_{-\infty}^{+\infty} y f_{\tilde{y}}(y)dy$$

From equation (18)

$$\bar{y}(t) = \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\left[e^v - C\right]\cdot\exp\left\{-\frac{[v-\mu]^2}{2\sigma^2}\right\}dv$$
$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty} e^v \cdot\exp\left\{-\frac{[v-\mu]^2}{2\sigma^2}\right\}dv - C\frac{1}{\sigma\sqrt{2\pi}}\int_{\ln C}^{+\infty}\exp\left\{-\frac{[v-\mu]^2}{2\sigma^2}\right\}dv$$

Using L_j in (20), we obtain

$$\bar{y}(t) = \left[L_1(t) - C(t)\cdot L_0(t)\right]$$
$$= e^{-rt}\left\{\exp\left(\bar{x}(t) + \frac{\sigma_x^2(t)}{2}\right)\cdot N\left(\frac{\bar{x}(t)+\sigma_x^2(t)-\ln C(t)}{\sigma_x(t)}\right) - C(t)\cdot N\left(\frac{\bar{x}(t)-\ln C(t)}{\sigma_x(t)}\right)\right\} \tag{21}$$

The variance of $\tilde{y}(t)$ can be derived again by equation (20), together with equation (21),

$$Var\{\tilde{y}(t)\} = \overline{y^2}(t) - \left[\bar{y}(t)\right]^2$$

Using the definition of L_j, we can write:

$$Var\{\tilde{y}(t)\} = \overline{y^2}(t) - \left[\bar{y}(t)\right]^2$$
$$= \left[(L_2 - 2CL_1 + C^2L_0) - (L_1 - CL_0)^2\right]$$

Substituting L_j in (20), we obtain

$$Var\{\tilde{y}(t)\} = \overline{y^2}(t) - \left[\bar{y}(t)\right]^2$$
$$= e^{-2rt}\left[(L_2 - 2CL_1 + C^2L_0) - (L_1 - CL_0)^2\right] \tag{22}$$

The skew of $\tilde{y}(t)$ is defined as:

$$C_S\{\tilde{y}(t)\} = \frac{E\{(\tilde{y}(t)-\bar{y}(t))^3\}}{(Var\{\tilde{y}(t)\})^{3/2}}$$

and can be similarly derived.

$$C_S\{\tilde{y}(t)\} = \frac{A(t)}{B(t)} \quad (23)$$

where

$$A(t) = \left[L_3(t) - 3C(t)L_2(t) + 3C(t)^2 L_1(t) - C(t)^3 L_0(t)\right] - \left[L_1(t) - C(t)L_0(t)\right]\cdot\left\{3\left[L_2(t) - 2C(t)L_1(t) + C(t)^2 L_0(t)\right] - 2\left[L_1(t) - C(t)L_0(t)\right]^2\right\} \quad (24)$$

$$B(t) = \left\{\left[L_2(t) - 2C(t)L_1(t) + C(t)^2 L_0(t)\right] - \left[L_1(t) - C(t)L_0(t)\right]^2\right\}^{3/2} \quad (25)$$

5 NUMERICAL EXAMPLES

A numerical example is given in this section. All the computation is implemented with Matlab 7.0 on a 1.6 GHz Intel Pentium computer with 1GB RAM. The test data comes from California electricity market. A period of one year from 1998 to 1999 is used to derive the periodic characteristic of electricity price. Daily, weekly and seasonal variations are modeled as (time-varying) periodic functions. By approximating data in different time scale, the mean values of the daily, weekly and seasonally periodic functions of electricity price can be obtained.

In order to construct the long-term periodic mean, daily, weekly and seasonally fluctuations are converted to per unit with respect to the selected base price level. Thus the mean of the periodic function of long-term electricity price can be achieved, which is shown in Figure 2, and the cost function is show in Figure 3.

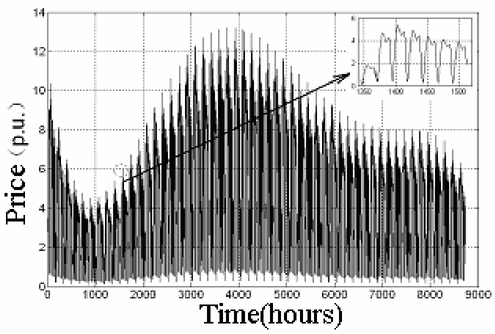


Figure 2: Long-term multi-periodic mean for electricity price

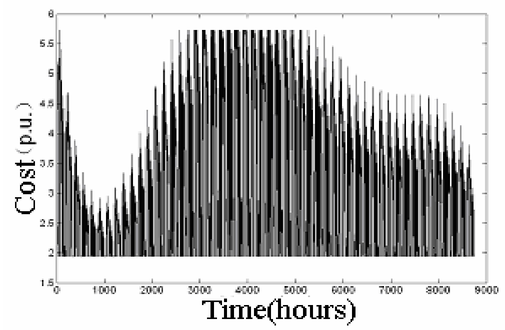


Figure 3: Cost of generator

Assuming that the volatility of electricity price $\sigma_x = 0.1$, the discount rate for one year $r = 0.1$.

The error of Monte Carlo simulation, in relation to the analytical result (3), is calculated according to:

$$\varepsilon(t)\% = \frac{V_M(t) - V_F(t)}{V_F(t)} \times 100\% \quad (26)$$

Where $V_M(t)$ is the mean value of profit at time t by Monte Carlo simulation, and $V_F(t)$ is the result of profit at time t by equation (21). After 5000 times Monte Carlo Simulation, the results of one year are shown in figure 4.

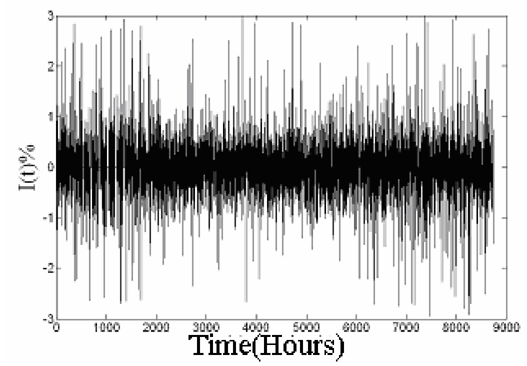


Figure 4: Simulation error in percentage

Furthermore, we obtain $\left|\overline{\varepsilon(t)}\%\right| = 0.35\%$, which shows that the results from Monte Carlo simulation and analytical approach are close.

For a selected 5 time points, the comparison of accuracy and computation times between Monte Carlo simulations with different sample sizes and our analytical approach are shown in Table 3 and 4, respectively.

Method / Time Point	Monte Carlo sample size				Analytical Approach
	500	1000	5000	10000	
1	0.670	0.641	0.646	0.659	0.658
2	1.623	1.615	1.600	1.602	1.602
3	4.02	3.953	3.815	3.811	3.810
4	2.351	2.241	2.160	2.153	2.152
5	2.200	2.103	2.011	2.010	2.009

Table 3: The comparison of acciracu (the unit of the results is p.u)

Method / Time Point	Monte Carlo sample size				Analytical Approach
	500	1000	5000	10000	
1	0.020	0.030	0.070	0.120	0.010
2	0.021	0.032	0.061	0.110	0.014
3	0.020	0.035	0.060	0.121	0.011
4	0.023	0.029	0.057	0.105	0.015
5	0.022	0.039	0.070	0.110	0.011

Table 4: The comparison of computation times(the unit of computation time is second)

Data in table 3 shows that as the sample size of Monte Carlo simulation increases from 500 to 10,000 the simulation results approach the analytical solution. The price to pay for the increased accuracy is the increased computational time, as shown in Table 4. With 500 Monte Carlo simulations, the error could be as large as 10% with twice as much computation time as the analytical solution. To have less than 2% error, one has to run 5000 simulations that costs 5 to 7 times more computation.

For one selected time point, Figure 5 shows the empirical probabilistic distribution curve of profit after 5000 times Monte Carlo simulations.

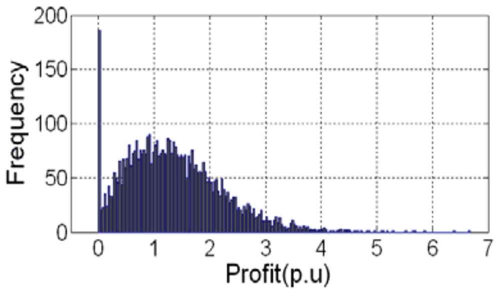


Figure 5: Empirical probabilistic density function of profit

Figure 5 shows that the empirical probabilistic distribution of profit has the same property with the analytical expressions of PDF of profit illustrated in Figure 1. Furthermore, the CDF curves of profit from analytical approach and Monte Carlo simulations are similar, which illustrated in Figure 6.

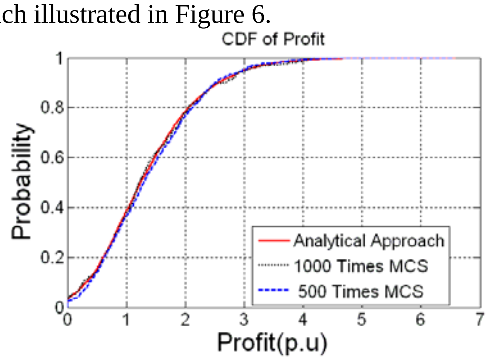


Figure 6: The CDF curves of profit from different methods

6 CONCLUSION

Analytical expressions of CDF and PDF of generator profit from a spot market when the price of electricity follows either a geometric Brownian motion process or a time-varying mean reversion process are derived. Black Sholes theory provides only the expected value of such distribution when the underlying stochastic process is a geometric Brownian motion. A time-varying mean reversion process that exhibits daily, weekly and seasonal variation is more suitable for modeling electricity prices. Once the PDF is obtained, expressions for higher-order moments can be derived. Monte Carlo simulation has been the only approach to handle asset valuation problem when the stochastic process is more complex than geometric Brownian motion. Direct application of analytical formulas makes computations more accurate and efficient. Moreover, analytical expressions can be incorporated in mathematical formulation of optimization or other problems.

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