

CAPACITOR PLACEMENT IN UNBALANCED POWER SYSTEMS

P. Varilone and G. Carpinelli
Dipartimento di Ingegneria Industriale
Universita degli Studi di Cassino
03043 Cassino, Italy
varilone@unicas.it, carpinelli@ing.unicas.it

A. Abur
Department of Electrical Engineering
Texas A&M University
College Station, Texas, U.S.A
abur@ee.tamu.edu

Abstract – This paper presents a capacitor placement method for three phase unbalanced power systems. The method aims to minimize not only the power losses and capacitor costs, but also the distortions due to harmonics present in the power system. The proposed method is capable of accomplishing this objective for both balanced and unbalanced operating conditions. One of the objectives of this paper is to demonstrate the significant differences in the results of capacitor placement studies when unbalanced systems are approximated by their positive sequence single phase equivalents. Furthermore, the effects of taking harmonic distortions into account during the capacitor placement procedure are also demonstrated. Numerical examples on a distribution test power system are provided to illustrate the method.

Keywords: *Optimization, capacitor placement, loss minimization, harmonic distortion, unbalanced operation, distribution systems.*

1 INTRODUCTION

Power distribution systems contain shunt capacitors at various strategic locations in order to maintain a desired voltage profile, correct power factor and reduce power losses along feeders. When dealing with a large scale distribution system containing several feeders and their laterals, deciding on the best locations and sizes of these capacitors becomes a complicated optimization problem. In addition to the scale of the problem, there are other issues such as the discrete nature of capacitor sizes, operational limits on voltages and feeder loadings, that need to be addressed. Effective solution algorithms for balanced distribution feeders have been developed [1,2]. These solutions mainly utilize the positive sequence network model and the associated power flows in formulating the problem. Hence, the results do not directly apply for systems containing feeders with missing phases, unevenly loaded feeders or shunt capacitors on single or double phase feeders. Three phase unbalanced distribution systems are later studied in [3,4] where simulated annealing and genetic algorithms are respectively used to solve this more complicated problem. A simplified formulation and the MINOS optimization package are used to solve the same problem in [5]. Recently, distribution systems are populated with nonlinear loads or control devices that generate

unwanted harmonics in the systems. Harmonics are known to cause overvoltages under certain network configurations involving shunt capacitors. This issue is raised and a solution is proposed for minimizing harmonic overvoltages in [6]. A practical method that avoids convergence problems and incorporates discrete nature of capacitor banks along with the voltage distortions due to the installed capacitors, is developed and presented in [7]. This method is based on the balanced three phase operation and therefore analyzes the positive sequence network only.

In this paper, the work reported in [7] will be extended to the more general case of the three phase unbalanced operation. Several distribution feeders are known to have line sections carrying a mixture of single, double or three phase loads. Such systems as well as those having full three phase but unbalanced loads can be studied using the presented method in this paper. In addition to the cost of losses and capacitor installations, it is possible to associate a cost with the harmonic distortions as discussed in [8-9]. Hence, the problem is formulated in such a way that both network losses and harmonics are minimized along with the cost of capacitors placed for this purpose.

The paper is organized such that the problem description is presented first. This is followed by the sections describing the details of the three phase power flow and linear harmonic analysis modules, which make up the main computational engines of the overall algorithm. Simulation results obtained using the developed program and a test system will be presented next. Conclusions and suggestions on future work will be presented in the final section.

2 DESCRIPTION OF THE PROBLEM

The objective of the presented method is to determine the best locations and sizes of shunt capacitors for each phase, so that the total cost of the capacitors, of the total power losses of the network and the harmonic distortion of the network are minimized. This objective should be met subject to the network three-phase power flow equations as well as the limits on the bus voltage and current magnitudes, harmonic indices (HI) and the total number of capacitor units to be installed. Hence, it

can be formulated as the following optimization problem:

$$\begin{array}{ll} \text{Minimize} & J(X,U) = J_C + J_L + J_{IH} \\ \text{Subject to} & \text{Three-phase PF Eq.s at} \\ & \text{fundamental and harmonics} \\ & V, I - \text{limits} \\ & HI \text{ limits} \\ & \text{Limits on number of cap.s} \end{array}$$

where the terms of the objective function and constraint list are defined below.

2.1 The Objective function, J

The objective function assumes a given loading level for the system. While it is possible to extend this function to a sum of similar functions, so that a number of loading levels can be accounted for, this will not be done here. All simulation results and the discussion of the proposed algorithm will be based on single loading assumption. Such an extension can however be incorporated into the presented algorithm without much difficulty. The three terms that make up the objective function are described below:

2.1.1 Cost of Capacitors, $J_C = C_C^T \cdot U$

C_C : is the cost vector for capacitor units at each bus.
 U : is the vector of capacitor units placed at each bus.

2.1.2 Cost of Losses, $J_L = C_L \cdot (P_G - P_D)T$

C_L : is the cost of a per unit energy loss
 P_G : is the total generation at (X,U)
 P_D : is the total load
 T : is the loss duration
(X,U): is the power flow solution corresponding to the installed capacitor vector of U.

The cost of demand lost can be also incorporated without difficulty.

2.1.3 Cost of Distortions, J_{IH}

The evaluation of the cost of the distortions is studied first by the authors in [8, 9]. These costs are assumed to be the sum of the operating costs and the aging costs. The operating costs refer to the costs of the incremental losses caused by the harmonics and the aging costs refer to the incremental costs due to the premature aging of the components caused by the harmonics. Details can be found in [8, 9].

2.2 The Constraints

Each solution should satisfy the three-phase power balance equations at the fundamental frequency. Thus, a full three-phase power flow solution is to be run to verify this constraint. This solution will also be used to check the limit violations at fundamental on bus voltages and line currents. The effect of each capacitor unit on the distortions of the bus voltages will have to be also checked. This is accomplished by solving a linear three-phase harmonic analysis problem where all nonlinear loads are represented by their harmonic cur-

rent injections. Details of this computation are discussed in a later section.

3 SOLUTION METHOD

The above described optimization problem is solved by using a simple yet effective procedure, which is developed and successfully employed in solving the single phase problem in [7]. The main idea behind the proposed procedure is based on sequential placement of discrete capacitor units of incremental size. It is assumed that each phase of any bus can be placed an incremental discrete capacitor at each optimization step (three-phase bank in case of three-phase bus, single-phase bank otherwise). If capacitor installations are prohibited for any of the buses, then this condition can easily be enforced by leaving out that bus from the bus list. Furthermore, if different buses have different sizes of unit capacitors, then placement of unit capacitors can be accordingly modified to suit each bus. Therefore, the physical constraints that relate to the discrete nature of capacitor installations as well as the non-homogeneity of the discrete units available for installations at different buses, are naturally accounted for without any complex logic.

At each step of the optimization procedure, the three phase power flow solution as well as the harmonics of the bus voltages corresponding to the incremental addition of unit capacitors at each possible bus, will have to be computed. Note that, the purpose of these computations is to obtain and compare the values of the objective function for all possible incremental installations and choose the one that reduces the objective function the most. It is also noted that for purposes of comparing effects of unit capacitor installations at different buses on the objective function, approximate three-phase power flow solutions can be used. Once a candidate is selected, then an accurate solution can be obtained for the selected configuration only. A fast but approximate three phase power flow solution is implemented for the first part. A three phase linear harmonic analysis is carried out for the calculation of distortions. These are discussed in more detail below.

3.1 Fast Three-phase Power Flow Update

The three-phase power flow solution is affected incrementally each time a unit capacitor is added at a system bus. This incremental change can be captured by expressing the power flow solution by its first order approximation. Consider the three phase power flow equations given by:

$$f(X,U) = 0 \quad (1)$$

The first order Taylor approximation will be:

$$[F_x(X_0, U_0)] [dX] = - [F_u(X_0, U_0)] du_i \quad (2)$$

Note that du_i is the unit capacitor added at node i , one phase of a bus and the functions F_x and F_u represent the gradient of $f(X,U)$ evaluated with respect to X and U respectively. At each optimization step, the most recent operating point will be denoted by (X_0, U_0) which will depend upon the way the optimization proceeded up till that point. The updated power flow solution will then be obtained as:

$$X' = X_0 + dX \quad (3)$$

This procedure will be repeated as many times as the number of nodes excluding those where no capacitors are allowed to be placed. Upon computing the objective function $J(X',U)$ for all cases, the solution which yields the smallest objective function will be selected. A full three phase AC power flow solution will be obtained and all operating constraints will be checked for this case. In case the solution violates any one of the limits, the same procedure will be repeated for the second best solution. This procedure will continue until a feasible solution is obtained. If no solution can be found satisfying all the constraints, then the optimization procedure will be terminated. Otherwise, it will proceed to the next optimization iteration.

3.2 Calculation of distortions

In parallel with the fundamental frequency power flow solution, a solution for each dominant harmonic present in the network will have to be obtained so that the corresponding effects of the power system harmonics can be evaluated. This is accomplished by solving the network's linear harmonic equations given by:

$$[Y_n][V_n] = [I_n] \quad (4)$$

where $[Y_n]$, $[V_n]$ and $[I_n]$ are the three phase network admittance matrix, bus voltage vector and independent current source vector evaluated at the n -th harmonic frequency. The admittance matrix is modified each time a unit capacitor is added at a bus. Also note that, the harmonic injections of nonlinear devices are modeled according to the recommendations of the IEEE PES Working Group on Harmonics Modeling and Simulation [10-11] where it is suggested that the phase angles of the injected harmonic current sources be adjusted according to the phase angle of the fundamental with respect to reference.

A Matlab-based program is developed in order to evaluate the above described procedure for general three phase unbalanced system operation. At this time, the program is implemented for a single loading level and therefore the capacitor placement can be tested only for fixed capacitor types. However, it can be easily modified to account for seasonal or daily load variations of the bus loads. A flow chart of the procedure implemented by this program is shown in Figure 1. Next

section will present several representative cases which are simulated using the developed program.

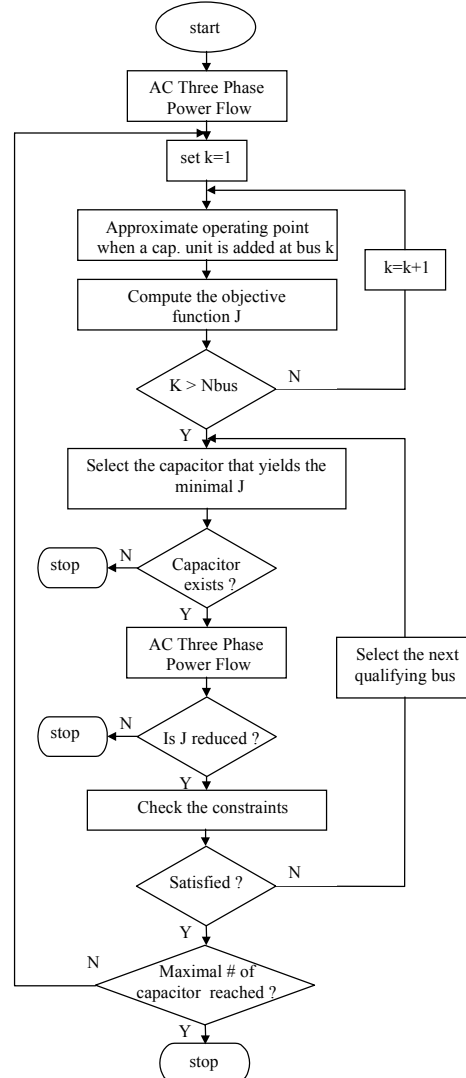


Figure 1: Flowchart of the proposed procedure

4 SIMULATION RESULTS

The simulations are run using the three phase unbalanced IEEE 13-bus test system shown in Figure 2 [11]. This system contains a mixture of single, double and three phase lines and loads, making it quite suitable for evaluating the proposed method. Note that the system lines 684-653 and 684-611 are single phase; lines 671-684, 632-645, 645-646 are two phase and the remaining lines are three phase. The complete network data and parameters can be found in [11].

The system contains loads that are linear, non linear, and/or unbalanced. The harmonic producing loads include fluorescent lighting, adjustable speed drives for

heat pumps and non-specific sources such as PCs, TVs, etc. The load data reported in [11] are reduced by 50%, since the voltage distortions are found to exceed the IEEE-519 recommended limits [12] before the placement of any capacitors.

Unit capacitors available at any bus are assumed to come in discrete sizes of 50 kVAR. The unit cost of capacitors are reported in [13] while the unit cost of energy are reported in [14].

The constraint limits are assumed as 105% for the voltages at fundamental, 3% for the single voltage harmonics and 5% for the total harmonic distortion.

Several cases are simulated. The following results will be presented as representative cases.

4.1 Case 1

In this case the sensitivity of capacitor placement to the objective function structure is investigated. The objective function is first considered as the sum of the losses and the capacitor costs only. The harmonic costs are then introduced into the objective function, to observe the significance of this consideration in the outcome of the optimization process. In order to simplify the analysis, the harmonic index is assumed to be simply the total harmonic distortion and its associated cost is evaluated so as to have the same cost of losses and

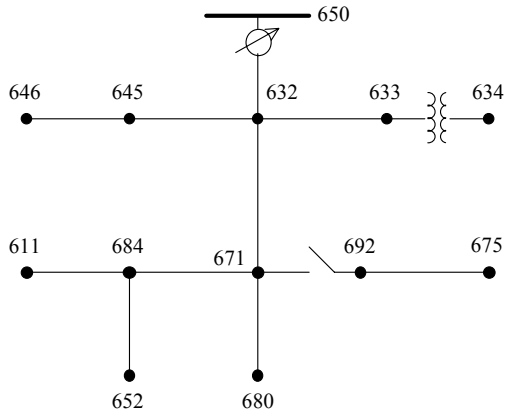


Figure 2: Network Diagram for the 13-bus test system

harmonics in the system without capacitors.

The locations and sizes of the capacitors that yield the minimum objective function in both cases are listed in Table 1. As evident from the table, two unit capacitor banks are located at the most distant three phase load bus (note that bus #680 is unloaded) and two other unit banks are located at a single phase bus. On the other hand, the capacitor banks at the single phase bus are no longer allocated when the objective function takes into account the THD cost.

Fig. 3 shows the variation of the two objective functions one with and one without the THD cost term, as

function of capacitor bank additions. The objective functions are normalized to make the comparison easy. It should be noted that in the case without THD term the placement procedure is terminated at the fifth step due to a constraint violation. The violated constraint is the value of the fundamental voltage at bus #675, which exceeds the specified limit of 1.05 p.u., when an additional unit capacitor is attempted to be placed after the fourth step.

Note that, in Fig.3, the solid lines represent the variation of the objective function J until the optimal solution is reached. The dashed lines indicate how J would have varied, had the optimization procedure not been terminated. In the top curve, the procedure terminates because the minimum is reached. In the bottom curve, it is terminated by the constraint violation after step 4 and no other solution yielding a lower J within the constraints can be found at the fifth step.

4.2 Case 2

In this case the sensitivity of the capacitor placement procedure to the specified constraints is considered. Initially, constraints identical to those used in Case 1, are considered. These limits are subsequently relaxed (110% for the voltages at fundamental, 5% for the single voltage harmonics and 8% for the total harmonic distortion), in order to observe their significance on the

Table 1: Capacitor Placement Results of Case 1

| Bus code | 675 | | | 611 | | |
|-------------------------|------|------|------|-----|---|------|
| | 1 | 2 | 3 | 1 | 2 | 3 |
| Without THD cost (kVAr) | 2x50 | 2x50 | 2x50 | | | 2x50 |
| With THD cost (kVAr) | 2x50 | 2x50 | 2x50 | | | |

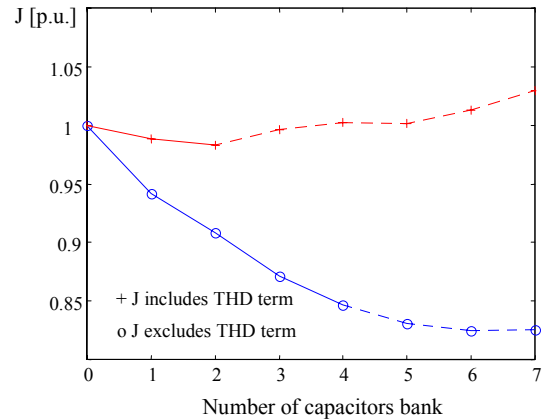


Figure 3: Variation of objective functions versus number of capacitors bank

Table 2: Capacitor Placement Results of Case 2

| Bus code | 675 | | | 611 | | | 645 | | | 646 | | |
|----------------------------|------|------|------|-----|---|------|-----|---|------|-----|---|------|
| | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Without THD cost (kVAr) | 2x50 | 2x50 | 2x50 | | | 2x50 | | | 1x50 | | | 1x50 |
| With THD cost (kVAr) | 2x50 | 2x50 | 2x50 | | | | | | | | | |

Table 3: Capacitor Placement Results of Case 3

| Bus code | 675 | 611 | 692 | 671 | 652 | 645 | 646 |
|----------------------------|-------|-------|-------|-------|-------|-------|-------|
| Without THD cost (kVAr) | 2x150 | 1x150 | 2x150 | 1x150 | 1x150 | 1x150 | 1x150 |
| With THD cost (kVAr) | 2x150 | 1x150 | 2x150 | 1x150 | 1x150 | 1x150 | 1x150 |

outcome of the optimization process. The locations and sizes of the capacitors that yield the minimum objective function in case of relaxed limits are listed in Table 2.

It follows from the analysis of the results in Tables 1 and 2 without THD cost that the same capacitor banks are located in the three phase bus #675; two more banks are located at the single phase busses #645 and #646. When the objective function takes into account the THD cost, none of the capacitor banks in single phase busses are allocated.

4.3 Case 3

In the absence of a three phase capacitor placement program such as the one described here, an approximate solution may be attempted by using a single phase program that works for balanced systems. This means that all unbalanced loads and structurally non-symmetric lines will be replaced by balanced components and hence the solution will be different. This case investigates the errors involved in such an approximation.

The case compares the capacitor placement results of Case 1 with that of a fully balanced version of the same system. Thus, the procedure of Section 3 is applied to a balanced system which is created by approximating the unbalanced system of Case 1 by a balanced version. This is accomplished by balancing all the lines (all lines are assumed three phase and structurally symmetric) and loads (all loads are assumed three phase balanced loads) of the unbalanced system. Moreover, harmonics of order 5, 7, 11 and 13 only have been taken into account with consequent reduction of harmonic distortion levels.

The locations and sizes of the capacitors (all 150 kVAR three phase banks) that yield the minimum objective function for this fully balanced case are listed in Table 3. Results shown in the table indicate that in both cases capacitor banks are located in the same three phase buses #675, #611, #692, #671, #652, #645, #646.

A comparison of the results shown in Tables 1 and 3 implies that in the case of the balanced system a much larger number of capacitors are required to achieve the minimum J. This is due to both the reduced level of voltage harmonics and the larger amount of total reactive load in the balanced system.

5 CONCLUSIONS

This paper presents a capacitor placement procedure for three phase unbalanced power systems. The proposed procedure not only accounts for system losses and capacitor costs, but also for harmonic distortion of bus voltages and the effects of unbalanced operation. The recent proliferation of nonlinear loads in distribution systems makes the issue of harmonic pollution an important one to be considered in any of the distribution automation functions such as capacitor sizing and location. This paper presents a simple yet effective solution to the problem. Numerical examples demonstrating the effectiveness of the procedure under both balanced and unbalanced operating conditions are provided.

Future work will involve more realistic representation of the harmonic cost in the objective function.

REFERENCES

- [1] M. Baran, F.F. Wu, "Optimal Capacitor Placement on Radial Distribution systems", IEEE Trans. on Power Delivery, Vol.4, No.1, Jan. 1989, pp.725-734.
- [2] J. J. Grainger, and S. H. Lee, "Optimum Size and Location of Shunt Capacitors for Reduction of Losses on Distribution Feeders", IEEE Trans. on PAS, Vol.100, Mar.1981, pp.1105-1118.
- [3] H-D. Chiang, J-C. Wang, J. Tong and G. Darling, "Optimal Capacitor Placement, Replacement and Control in Large-Scale Unbalanced Distribution

- Systems: System Modeling and A New Formulation”, IEEE Trans. on Power Systems, Vol.10, No.1, February 1995, pp.356-362.
- [4] C.S. Chen, C.T. Hsu and Y.H. Yan, “Optimal Distribution Feeder Capacitor Placement Considering Mutual Coupling Effect of Conductors”, IEEE Trans. on Power Delivery, Vol.10, No.2, April 1995, pp.987-994.
- [5] K.N. Miu, H-D. Chiang and G. Darling, “Capacitor Placement, Replacement and Control in Large-Scale Distribution Systems by a GA-Based Two-Stage Algorithm”, IEEE Trans. on Power Systems, Vol.12, No.3, August 1997, pp.1160-1166.
- [6] Y. Baghzouz, S. Ertem, “Shunt Capacitor Sizing for Radial Distribution Feeders with Distorted Substation Voltages”, IEEE Trans. on Power Delivery, Vol.5, April 1990, pp.650-657.
- [7] Bei Gou and A. Abur, “Optimal Capacitor Placement for Improving Power Quality”, Paper SM 011. Proceedings of IEEE/PES Summer Meeting, July 18-22, 1999, Edmonton, Canada.
- [8] P. Caramia, G. Carpinelli, E. Di Vito, A. Losi and P. Verde, “Probabilistic Evaluation of the Economical Damage due to the Harmonic Losses in Industrial Energy Systems”, IEEE Trans. on Power Delivery, Vol.11, No.2, 1996, pp.1021-1031.
- [9] P. Caramia, G. Carpinelli, A. Russo, P. Varilone, P. Verde: “An Integrated Probabilistic Harmonic Index”, IEEE PES 2002 Winter Meeting, New York (USA), 27-31 January 2002
- [10] IEEE Task Force on Harmonics Modeling and Simulation, “Modeling and Simulation of the Propagation of Harmonics in Electric Power Networks – Part I: Concepts, Models and Simulation Techniques”, IEEE Trans. on Power Delivery, Vol. 11, No. 1, 1996, pp. 452-465
- [11] IEEE Task Force on Harmonics Modeling and Simulation, “Modeling and Simulation of the Propagation of Harmonics in Electric Power Networks – Part II: Sample Systems and Examples”, IEEE Trans. on Power Delivery, Vol. 11, No. 1, 1996, pp. 466-474
- [12] IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems, IEEE Std 519-1992, IEEE, New York, NY, April 1993.
- [13] A.E. Emanuel, C. Kawann, “Passive Shunt Harmonic Filters for Low and Medium Voltage: A Cost Comparison Study”, IEEE Trans. on Power Systems, Vol. 11, No. 4, 1996, pp. 1825-1831
- [14] T.T. Chang, H. Chang, “Application of Differential Evolution to Passive Shunt Harmonic Filter Planning”, 8th IEEE ICHQP, Athens (Greece), 1998, pp. 149-153