

# A Mathematical Framework for Unit Commitment and Operating Reserve Assessment in Electric Power Systems

M. Fotuhi-Firuzabad  
maf290@engr.usask.ca

R. Billinton  
R\_Billinton@engr.usask.ca

Power Systems Research Group  
University of Saskatchewan  
Saskatoon, Canada

**Abstract--A mathematical framework is presented in this paper for operating reserve assessment based on a hybrid deterministic/probabilistic approach. The approach known as system well-being embeds the conventional deterministic criteria into a probabilistic method. The proposed mathematical framework includes the computational requirements in both spinning and supplemental reserves. Interruptible load is also included in the framework as part of the operating reserve. The impact of ramp-rate limits on economic load dispatch is incorporated in the overall framework. The computational tools required to incorporate factors such as load forecast uncertainty and postponable outages are also presented.**

**Keywords:** power system reliability, unit commitment, spinning reserve, operating reserve, well-being approach

## 1. INTRODUCTION

Operating reserve requirements in today's electric power industry are usually determined using deterministic criteria or rule-of-thumb methods. The essential disadvantage of deterministic criteria is that they do not respond to the many factors that influence the actual risk in the system. Utilization of probabilistic techniques will permit the capture of the random nature of system component and load behavior in a consistent manner. Despite the obvious disadvantages of deterministic approaches, there is considerable reluctance to apply probabilistic techniques to assess spinning reserve requirements [1-3]. This reluctance dictates a need to create a bridge between the deterministic methods and the prevalent probabilistic techniques. This formulation, designated as system well-being, in the form of system health, margin and at risk, can play an important role in providing system operators with a better understanding of operating reliability. In the well-being approach, the probability of the system being in the healthy state together with the conventional risk index can be used as system operating criteria [4-6].

An overall mathematical framework is presented in this paper for determining the reserve requirements based on the well-being approach. The framework includes the computational requirements in both spinning and supplemental reserves. Non-spinning reserves such as rapid start gas turbine units and hot reserve units are assessed using the concepts of area risk curves where the associated lead times play a key role in the analysis. Interruptible load is also included in the framework as part of the operating reserve depends on the time of

interruption [1,7,8]. Once the required amount of operating reserve is determined, the impact of ramp-rate limits on economic load dispatch is incorporated. Inclusion of all these facilities in unit commitment and operating reserve assessment requires different techniques and computational tools. Factors such as load forecast uncertainty and postponable outages are also included in the framework. The presented mathematical modeling is easy to program and there are no restrictions on the size of the system.

## 2. WELL-BEING MODEL

In order to incorporate practical security considerations in operating reserve assessment, the system performance can be divided into operating states classified in terms of the degree to which the adequacy and security constraints are satisfied. The system states, designated as healthy, marginal and at risk are shown in Figure 1.

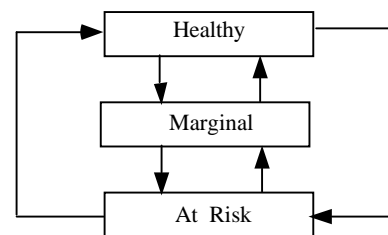


Figure 1. Well-being model.

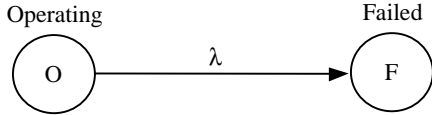
The system operates within the specified limits in both the healthy and marginal states. In the healthy state all the equipment and security constraints are within limits while supplying the total system demand. In this state, there is sufficient reserve margin such that accepted deterministic criterion, such as any single contingency, can be tolerated without violating the limits. In the marginal state, the operating constraints are within limits, but some specific single contingencies will result in a limit being violated due to insufficient reserve margin. The system is, therefore, on the edge of being in trouble in this state. The operating constraints are violated in the risk state and the system may be required to shed load in this state. The probabilities associated with the healthy and risk states can be considered as operating reserve criteria.

A contingency enumeration technique is used to calculate the well-being indices, i.e. healthy, marginal and risk state probabilities. For a given contingency, the security constraints must be tested before it is judged to be a successful state (healthy or marginal) or a failed state

(risk). The decision on whether a given successful contingency belongs to the healthy or marginal states cannot be made correctly unless all possible next level outages involving that particular outage are examined. A comprehensive mathematical framework is presented in the following sections for various aspects of operating reserve well-being analysis.

### 3. GENERATING UNIT AND INTERRUPTIBLE LOAD MODELS

Operating reserve can be generally divided into the two classes of unit reserve and system reserve. Unit reserve may be in the form of spinning or stand-by units. Figure 2 shows a modified two-state model [7] used in operating reserve assessment for spinning units. It is assumed that the system lead time is relatively short and therefore the probability of repair occurring during the small lead time is negligible. Under this condition the time dependent probabilities of the operating and failed states for a unit can be approximated by Equations 1 and 2, respectively, at a given delay time of T.



**Figure 2.** Two-state model of a generating unit used in operating reserve evaluation.

$$P(\text{failed}) \cong \lambda T = \text{ORR} \quad (1)$$

$$P(\text{operating}) \cong 1 - \lambda T \quad (2)$$

Where  $\lambda$  is the unit failure rate and ORR is the outage replacement rate [7].

The two-state model can further be modified by including postponable outages. In this case the total unit failure rate is reduced by the degree of postponability  $\beta$ . The probability of finding the unit in the failed state at a given time T in the future can be obtained using Equation 3 [7].

$$P(\text{failed}) \cong (\lambda - \beta\lambda)T = (1 - \beta)\lambda T \quad 0 \leq \beta \leq 1 \quad (3)$$

Rapid start and hot reserve units are represented by four and five state models as shown in Figures 3.a and 3.b respectively [9]. The time dependent state probabilities are evaluated using a matrix multiplication technique.

$$[P(t)] = [P(t_0)] [P]^m \quad (4)$$

Where

$[P(t)]$  = vector of state probabilities at time t,

$[P(t_0)]$  = vector of initial probabilities,

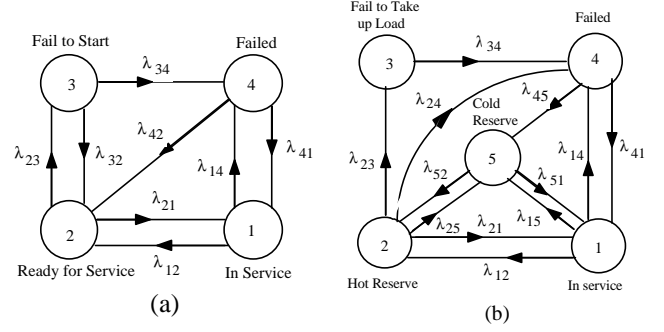
$[P]$  = stochastic transitional probability matrix,

$m$  = number of time steps used in the discretization process.

$t_0 = t_r$  lead time of rapid start units

$t_0 = t_h$  lead time of hot reserve units

The vector of initial probabilities for the rapid start and hot reserve units are given in Equations 5 and 6 respectively.



**Figure 3:** Four and five state models for rapid start and hot reserve units respectively.

$$[P(t_r)] = [P_1(t_r) \ 0 \ 0 \ P_4(t_r)] \quad (5)$$

$$[P(t_h)] = [P_1(t_h) \ 0 \ 0 \ P_4(t_h) \ 0] \quad (6)$$

$$P_4(t_r) = \frac{\lambda_{23}}{\lambda_{21} + \lambda_{23}} \quad (7)$$

$$P_1(t_r) = 1 - P_4(t_r) \quad (8)$$

$$P_4(t_h) = \frac{\lambda_{23} + \lambda_{24}}{\lambda_{21} + \lambda_{23} + \lambda_{24}} \quad (9)$$

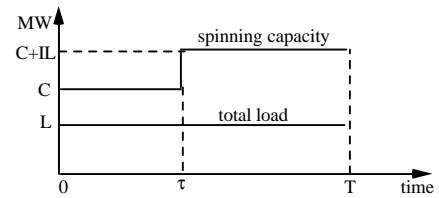
$$P_1(t_h) = 1 - P_4(t_h) \quad (10)$$

The probabilities of finding the rapid start and hot reserve units on outage given that a demand has occurred are given by Equations 11 and 12 respectively. The unit availabilities are calculated using the complementary values of Equations 11 and 12.

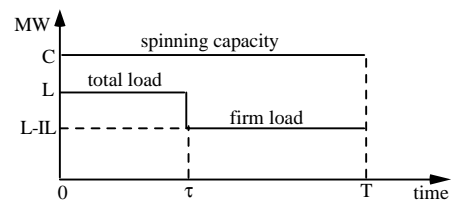
$$P(\text{Down}) = \frac{P_3(t) + P_4(t)}{P_1(t) + P_3(t) + P_4(t)} \quad (11)$$

$$P(\text{Down}) = \frac{P_3(t) + P_4(t) + P_5(t)}{P_1(t) + P_3(t) + P_4(t) + P_5(t)} \quad (12)$$

Interruptible load can be modeled as an equivalent generating unit with zero failure rate or considered as a load variation as shown in Figures 4 and 5 respectively where  $\tau$  is the load interruption time. In this paper, the load variation model is used for the purpose of computational analysis.



**Figure 4.** Equivalent unit approach model for interruptible load.



**Figure 5.** Load variation approach model for interruptible load.

#### 4. MATHEMATICAL EXPRESSIONS

The symbols used in this section are defined as follows:

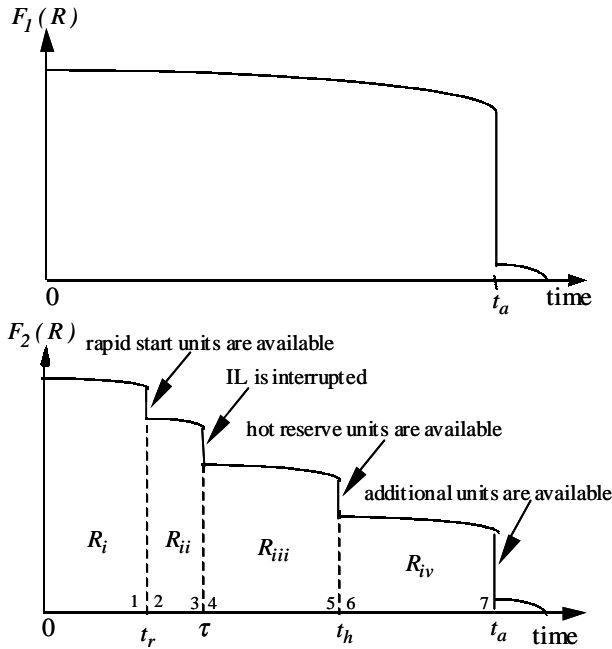
$SP_h, SP_r$	specified healthy and risk state probabilities,	$P_{r5}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units and load demand of ( $L - IL$ ) at time $t_h$ ,
$P_h, P_m, P_r$	actual healthy, marginal and risk state probabilities using the area risk curve $F_2(R)$ ,	$P_{r6}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units + <i>NHR</i> hot reserve units and load demand of ( $L - IL$ ) at time $t_h$ ,
$P_{lh}, P_{lm}, P_{lr}$	calculated healthy, marginal and risk state probabilities considering only spinning capacity (area risk $F_1(R)$ ),	$P_{r7}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units + <i>NHR</i> hot reserve units and load demand of ( $L - IL$ ) at time $t_a$ ,
$R_i, R_{ii}, R_{iii}, R_{iv}$	partial risks	$L$	system load demand,
<i>NRS</i> , <i>NHR</i>	number of rapid start and hot reserve units,	$\tau$	interruption time of interruptible load,
$P_h(i), P_m(i), P_r(i)$	healthy, marginal and risk state probabilities respectively, associated with the $i^{th}$ step load level in the normal distribution,	<i>IL</i>	interruptible load,
$R_i^k, R_{ii}^k, R_{iii}^k, R_{iv}^k$	partial risks associated with the $i^{th}$ step load level in the normal distribution,	$t_a$	lead time of additional generating units in the system,
$RP_h, RP_m, RP_r$	response healthy, marginal and risk state probabilities,	$t_r, t_h$	lead time of rapid start and hot reserve units,
$RP_h^k, RP_m^k, RP_r^k$	response healthy, marginal and risk state probabilities given that $CAP_{rsu}^k$ response is available from rapid start units,	<i>SD</i>	Standard deviation in the normal distribution,
$SRP_h, SRP_r$	specified response healthy and risk state probabilities,	$A_j(i)$	probability of the load being at the $i^{th}$ class interval in the normal distribution,
$F_1(R)$	area risk function considering only spinning capacity,	$s$	number of steps in the normal distribution (an integer odd number)
$F_2(R)$	area risk function including rapid start, hot reserve and interruptible load,	$L^k$	load associated with the $k^{th}$ class interval in the normal distribution
$F_i$	operating cost of the $i^{th}$ committed unit,	$RS^k$	total available response associated with the $k^{th}$ class interval in the normal distribution,
$A_i, B_i, C_i$	cost parameters of the $i^{th}$ committed unit,	$SOC^k$	total system operating cost associated with the $k^{th}$ class interval in the normal distribution,
$\Delta L$	incremental load,	<i>MT</i>	margin time,
$\Delta F$	incremental cost,	<i>RRM</i>	required regulating margin,
$ARES_c$	total available response at contingency $c$ ,	<i>TRS</i>	total available response at the margin time of <i>MT</i> ,
$P_{rk}$	risk of $k^{th}$ period in the area risk curve $F_2(R)$ ,	<i>TSOC</i>	total system operating cost,
$TD_r$	total decreased risk due to the inclusion of stand-by units and interruptible loads,	$LC_i$	loading capacity of unit $i$ ,
<i>NO</i>	number of on-line committed units,	$SRC_i$	spinning reserve capacity of unit $i$ ,
$G_i$	maximum capacity of unit $i$ ,	$RC_{sru}^i$	response capability of the $i^{th}$ spinning unit,
$P_{r1}$	risk calculated for the on-line units <i>NO</i> and load demand of $L$ at time $t_r$ ,	$RES_{sru}$	response from spinning units,
$P_{r2}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units and load demand of $L$ at time $t_r$ ,	$RES_{rsu}$	response from rapid start units,
$P_{r3}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units and load demand of $L$ at time $\tau$ ,	$RES_{int}$	response from interruptible loads,
$P_{r4}$	risk calculated for the <i>NO</i> on-line units + <i>NRS</i> rapid start units and load demand of ( $L - IL$ ) at time $\tau$ ,	$RES_{sru}^i$	response from the $i^{th}$ spinning unit,
		$RES_{rsu}^i$	response from the $i^{th}$ rapid start unit,
		$GRS_i$	maximum capacity of the $i^{th}$ rapid start unit,
		$RR_{sru}^i$	response rate of the $i^{th}$ spinning unit,
		$RR_{rsu}^i$	response rate of the $i^{th}$ rapid start unit,
		$\lambda_{sru}^i$	failure rate of the $i^{th}$ spinning units,
		<i>NC</i>	number of capacity states in the COPT,
		$CAP_{sru}^j$	response capacity of the $j^{th}$ state in the COPT associated with the on-line spinning units,
		$PC_{sru}^j$	probability of the $j^{th}$ capacity state in the COPT associated with the on-line spinning units,
		$RC_{rsu}^i$	response capability of the $i^{th}$ rapid start unit within the margin time of <i>MT</i> ,
		$CAP_{rsu}^j$	response capacity of the $j^{th}$ state in the COPT associated with rapid start units,

$PC_{rsu}^j$  probability associated with the  $j^{th}$  response capacity state in the COPT of rapid start units,  
 $P_c$  probability of contingency c,

The unit commitment scheduling is started by committing a number of units starting with the most economic unit. The initial number of committed units is therefore the minimum number of units required to satisfy Equation 13.

$$\sum_{i=1}^{NO} G_i > L \quad (13)$$

The upper curve in Figure 6 shows a possible area risk curve for a system with no stand-by units and interruptible load. This area risk curve represents the behavior of the system when only spinning and synchronized units (on-line units) are considered in the reserve calculations. A typical area risk curve for a system with rapid start, hot reserve units and interruptible load is shown in the lower curve of Figure 6.



**Figure 6.** Area risk curves with and without stand-by units and interruptible loads.

The system risk can be calculated by simulating all possible contingencies. Evaluation can require a considerable computation time specially for systems with a large number of committed units. In order to decrease the computational time the partial system risk at each period is determined using a capacity outage probability table (COPT) [7]. Using the initial number of committed units the system risk is calculated in the presence of rapid start units, interruptible load and hot reserve units.

$$P_r = R_i + R_{ii} + R_{iii} + R_{iv} \quad (14)$$

$$R_i = \int_0^{t_r} F_2(R) dt = P_{r1} - P_{r0} \quad (15)$$

$$R_{ii} = \int_{t_r}^{\tau} F_2(R) dt = P_{r3} - P_{r2} \quad (16)$$

$$R_{iii} = \int_{\tau}^{t_h} F_2(R) dt = P_{r5} - P_{r4} \quad (17)$$

$$R_{iv} = \int_{t_h}^{t_a} F_2(R) dt = P_{r7} - P_{r6} \quad (18)$$

It is assumed that the system is not in trouble at the decision time of  $t=0$  and therefore  $P_{r0}$  is zero. In addition, it is also assumed that there is enough capacity available after the system lead time  $t_a$  such that the risk after this time is negligible.

The calculated system risk is compared with the specified risk as shown below.

$$P_r \leq SP_r \quad (19)$$

If Equation 19 is not satisfied, an additional unit is added to the already on-line committed units and the above procedure is continued until the system risk is satisfied. Once the system risk is satisfied, the healthy and marginal state probabilities are calculated as follows.

$$TD_r = \int_0^{t_a} F_1(R) dt - \int_0^{t_a} F_2(R) dt \quad (20)$$

$$TD_r = P_{1r} - P_r \quad (21)$$

$P_{1r}$  is determined from a COPT made of  $NO$  on-line committed units. The COPT represented in a descending order is shown in Table 1. Each unit is represented by the two-state model shown in Figure 2. The failed and operating state probabilities are calculated as follows.

$$P(\text{failed}) \cong \lambda_{rsu}^i \times t_a \quad (22)$$

$$P(\text{operating}) \cong 1 - \lambda_{rsu}^i \times t_a \quad (23)$$

$$P_{1r} = \sum_{i=1}^{2^{NO}} P^i \times Q_i \quad (24)$$

$$Q_i = \begin{cases} 0 & \text{if } L < C^i \\ 1 & \text{if } L \geq C^i \end{cases} \quad (25)$$

**Table 1.** COPT of the  $NO$  on-line spinning units.

Capacity in service	Individual probability
$C^1$	$P^1$
$C^2$	$P^2$
$\vdots$	$\vdots$
$0$	$P^{2^{NO}}$

The healthy state probability,  $P_{1h}$ , cannot be calculated using the COPT and is determined using a contingency enumeration technique. For a given contingency c it is assumed that  $m_1$  set of units are in service and  $m_2$  set of units are out of service.

$$m_1 + m_2 = NO \quad (26)$$

$$P_c = \prod_{i=1}^{m_1} (1 - \lambda_{sru}^i \times t_a) \prod_{j=1}^{m_2} (\lambda_{sru}^j \times t_a) \quad (27)$$

$$\left( \sum_{i=1}^{m_1} G_i \right) - G_k > L \quad k \in m_1 \text{ set of units} \quad (28)$$

$$P_{1h} = \left( \sum P_c \mid \text{if Equation 28 is satisfied} \right) \quad (29)$$

$$P_{1m} = 1 - P_{1h} - P_{1r} \quad (30)$$

Comparing the two area risk curves  $F_1(R)$  and  $F_2(R)$  :

$$P_r = 1 - P_{lh} - P_{lm} - TD_r \quad (31)$$

$$P_r = 1 - P_{lh} - P_{lm} - TD_r + (P_{lh} + P_{lm})TD_r - (P_{lh} + P_{lm})TD_r \quad (32)$$

$$P_r = 1 - [P_{lh}(1 + TD_r) + P_{lr} \times TD_r] - [P_{lm}(1 + TD_r)] \quad (33)$$

$$P_r = 1 - P_h - P_m \quad (34)$$

$$P_h = P_{lh}(1 + TD_r) + P_{lr} \times TD_r \quad (35)$$

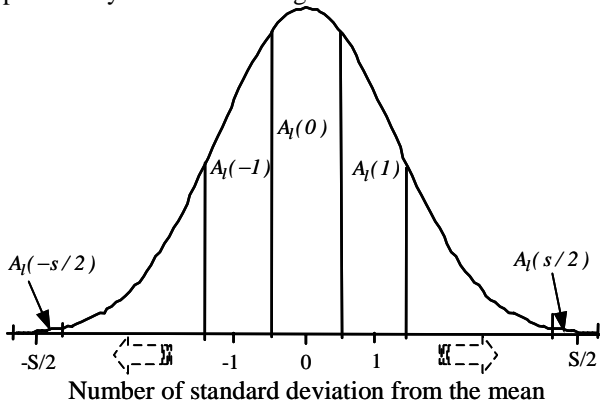
$$P_m = P_{lm}(1 + TD_r) \quad (36)$$

If the unit commitment criterion is to satisfy a single risk criterion, the procedure is stopped here. Otherwise the health criterion is checked as shown below:

$$P_h \geq SP_h \quad (37)$$

If the health criterion is not satisfied, one more unit is committed in addition to the already on-line committed units and the procedure is continued until the criterion is satisfied.

Load forecast uncertainty can be included in the well-being analysis as some deviation always exists between the forecast and the actual loads. The uncertainty is described by a normal distribution in which the distribution mean is the forecast load  $L$  and the standard deviation is obtained from previous forecasts [7]. The normal distribution can be divided into class intervals as shown in Figure 7 whose number depends upon the accuracy required. The area of each class represents the probability of the load being at the class interval mid value. The operating state probabilities for each load level  $L^k$  are first calculated and then weighted by the probability of the load being in each level



**Figure 7.** Class intervals of normal distribution.

$$L^k = L(1 + SD \times k) \quad k = -\frac{s}{2}, \dots, 0, \dots, +\frac{s}{2} \quad (38)$$

$$P_r(k) = R_{ii}^k + R_{iii}^k + R_{iv}^k \quad (39)$$

$$P_r = \sum_{i=-s/2}^{s/2} P_r(i) \times A_l(i) \quad (40)$$

$$\left[ \sum_{i=-s/2}^{s/2} P_r(i) \times A_l(i) \right] \leq SP_r \quad (41)$$

$$P_h = \sum_{i=-s/2}^{s/2} P_h(i) \times A_l(i) \quad (42)$$

$$P_m = \sum_{i=-s/2}^{s/2} P_m(i) \times A_l(i) \quad (43)$$

$$\left[ \sum_{i=-s/2}^{s/2} P_h(i) \times A_l(i) \right] \geq SP_h \quad (44)$$

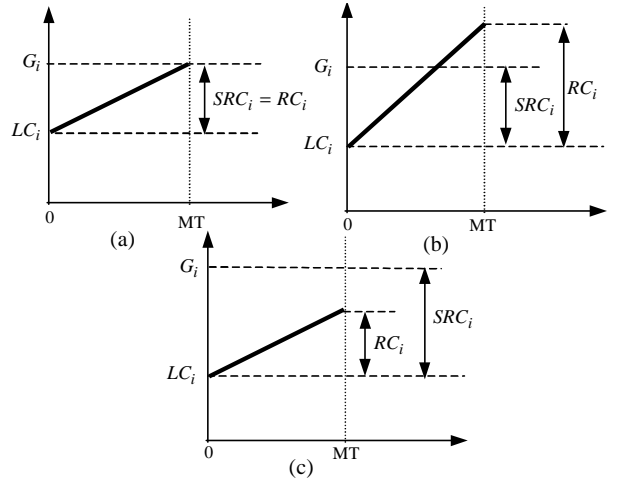
The healthy, marginal and risk state probabilities associated with each load level are used to compute the overall health, margin and risk indices. The overall values are compared with the criterion values.

Once the number of committed units are determined for a specified unit commitment criterion, the next step is to determine the well-being indices with respect to the response capability of the generating system. The actual operating ranges of all the committed units are restricted by their ramp rate characteristics [10-11]. The total response output can be provided by spinning units, rapid start units and interruptible loads. Rapid start units and interruptible loads can contribute to the response output if Equations 45 and 46 are satisfied.

$$t_r < MT \quad (45)$$

$$\tau \leq MT \quad (46)$$

$$TR = R_{sru} + R_{rsu} + R_{int} \quad (47)$$



**Figure 8.** Response characteristics of spinning units.

The response characteristics of the spinning units can be categorized into three classes as shown in Figure 8.

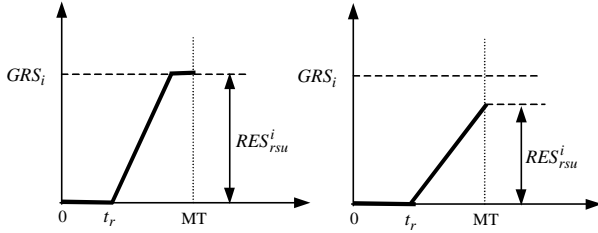
The three classes are defined as follow:

$$\left\{ \begin{array}{lll} SRC_i = RC_i & \text{Figure 8.a} & \text{Group I} \\ SRC_i < RC_i & \text{Figure 8.b} & \text{Group II} \\ SRC_i > RC_i & \text{Figure 8.c} & \text{Group III} \end{array} \right\} \quad (48)$$

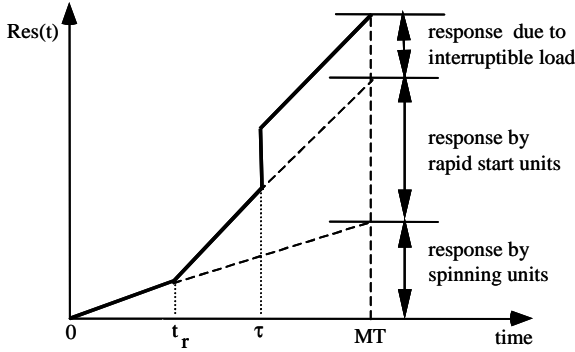
The response characteristic of a rapid start unit is represented in Figure 9. The overall system response characteristic is shown in Figure 10, in which the total response in the system is provided by on-line spinning units, rapid start units and interruptible load.

Similar to the case of unit commitment, the response risk is first determined for each load dispatch using a capacity outage probability table (COPT). The COPT is obtained

by combining all the spinning committed units. The capacity associated with each unit is the available response capacity of that unit.



**Figure 9.** Response characteristics of rapid start units.



**Figure 10.** Overall system response characteristic.

$$RES_{sru}^i = \text{Min}\{G_i - LC_i, RC_{sru}^i\} \quad (49)$$

where

$$RC_{sru}^i = RR_{sru}^i \times MT \quad (50)$$

Each unit is represented by a two-state model where the failed and operating state probabilities are calculated as follows.

$$P(\text{failed}) \cong \lambda_{sru}^i \times MT \quad (51)$$

$$P(\text{operating}) \cong 1 - \lambda_{sru}^i \times MT \quad (52)$$

The COPT is represented by an equivalent multi-state unit model as shown in Table 2. In this case, the number of units contributing to create the COPT is not necessarily  $NO$  as the response from some of the units which are operating at their maximum capacity is zero.

**Table 2.** COPT of the available response of the on-line spinning units.

Response capacity	Individual probability
$CAP_{sru}^1$	$PC_{sru}^1$
$CAP_{sru}^2$	$PC_{sru}^2$
.	.
0	$PC_{sru}^{NC}$

The available response from rapid start gas turbine units at the margin time of  $MT$  can also be represented in the form of a COPT as shown in Table 2. The capacity associated with each unit is the available response capacity of a rapid start unit at the margin time of  $MT$ . The associated probability at  $MT$  is calculated using the complement of Equation 11. The available response

associated with a rapid start unit at the margin time of  $MT$  is calculated as follows.

$$RES_{sru}^i = \text{Min}(GRS_i, RC_{sru}^i) \quad (53)$$

where

$$RC_{sru}^i = RR_{sru}^i \times (MT - t_r) \quad (54)$$

The total number of states in the COPT is  $2^{NRS}$  where  $NRS$  is the total number of rapid start units.

Both COPTs are arranged in descending order, i.e.

$$CAP_{sru}^j > CAP_{sru}^{j+1} \quad \text{for } j = 1, 2, \dots, NC - 1 \quad (55)$$

$$CAP_{sru}^j > CAP_{sru}^{j+1} \quad \text{for } j = 1, 2, \dots, 2^{NRS} - 1 \quad (56)$$

**Table 3.** COPT of the available response of the rapid start gas turbine units.

Response capacity	Individual probability
$CAP_{sru}^1$	$PC_{sru}^1$
$CAP_{sru}^2$	$PC_{sru}^2$
.	.
0	$PC_{sru}^{2^{NRS}}$

The contribution of rapid start gas turbine units to response capability is considered using the conditional probability approach.

$$RP_r = \sum_{k=1}^{2^{NRS}} RP_r^k \times PC_{sru}^k \quad (57)$$

for the  $k^{\text{th}}$  state of the rapid start units COPT (Table 3), the total available response is equal to:

$$CAP_{sru}^k + RES_{int} + \text{response of spinning units}$$

$$RP_r^k \text{ is found from Table 2 as shown below.}$$

$$RP_r^k = \sum_{i=1}^{NC} PC_{sru}^i \times Q_i \quad (58)$$

$$Q_i = \begin{cases} 0 & \text{if } RRM < CAP_{sru}^i + RES_{int} + CAP_{sru}^k \\ 1 & \text{if } RRM \geq CAP_{sru}^i + RES_{int} + CAP_{sru}^k \end{cases} \quad (59)$$

The calculated response risk is compared with the specified response risk as shown below.

$$RP_r \leq SRP_r \quad (60)$$

If Equation 60 is not satisfied, the load dispatch must be readjusted to increase the total response from the on-line committed units. This is done by taking away an incremental load from a unit in Group II whose incremental running cost is the highest and giving to a unit in Group III whose incremental running cost is the lowest.

If the cost model is defined as Equation 61, the incremental running cost of unit  $k$  for an increasing load of  $\Delta L$  and that of unit  $j$  for a decreasing load of  $\Delta L$  are calculated using Equations 62 and 63 respectively.

$$F_i = A_i + B_i \times LC_i + C_i \times LC_i^2 \quad (61)$$

$$\Delta F_k = \Delta L [B_k + C_k (2LC_k + \Delta L)] \quad (62)$$

$$\Delta F_j = \Delta L [B_j + C_j (2LC_j - \Delta L)] \quad (63)$$

The total running cost increases by  $(\Delta F_k - \Delta F_j)$ .

The response risk is calculated after each reloading step and is continued until Equation 59 is satisfied. Once the response risk criterion is satisfied, the next step is to calculate the response healthy and marginal state probabilities as follows.

$$RP_h = \sum_{k=1}^{2^{NRS}} RP_h^k \times PC_{rsu}^k \quad (64)$$

$RP_h^k$  is calculated using a contingency enumeration technique. For a given contingency the on-line committed units are arranged into two sets, where in set 1  $m_1$  units are in service and in set 2,  $m_2$  units are on outage. The probability associated with the contingency  $c$ ,  $P_c$ , is calculated using Equation 2.

$$m_1 + m_2 = NO \quad (65)$$

$$P_c = \prod_{i=1}^{m_1} (1 - \lambda_{sru}^i \times MT) \prod_{j=1}^{m_2} (\lambda_{sru}^j \times MT) \quad (66)$$

$$\sum_{i=1}^{m_1} LC_i + ARES_c > (RRM + L) \quad (67)$$

$$ARES_c - RES_{sru}^j > LC_j \quad j \in m_1 \text{ set of units} \quad (68)$$

$$ARES_c = \left( \sum_{i=1}^{m_1} RES_{sru}^i \right) + RES_{int} + CAP_{rsu}^k \quad (69)$$

If both Equations 67 and 68 are satisfied, the contingency lies in the healthy state. By varying  $m_2$  from 0 to  $NO$ , all possible contingencies are considered.

$$RP_h^k = \left\{ \sum P_c \mid \text{contingency } c \text{ belongs to the healthy state} \right\}$$

$$RP_m = 1 - RP_h - RP_r \quad (70)$$

The calculated response healthy state probability is compared with the specified response health as shown below.

$$RP_h \geq SRP_h \quad (71)$$

If Equation 71 is satisfied, the final load dispatch is considered as an acceptable and optimum one. The total system operating cost and the expected system response are given by Equations 72 and 74 respectively.

$$TSOC = \sum_{i=1}^{NO} F_i \quad (72)$$

$$TRS = \text{response from spinning units} + \text{response from rapid start units} + \text{interruptible load} \quad (73)$$

$$TRS = \sum_{i=1}^{NO} RES_{sru}^i + \left( \sum_{k=1}^{2^{NRS}} CAP_{rsu}^k \times PC_{rsu}^k \right) + RES_{int} \quad (74)$$

If the response criterion is not satisfied, the load dispatch must be readjusted or more unit(s) should be committed to provide additional response.

The impact of load forecast uncertainty can also be included in the ramp rate constraint economic load dispatch similar to the unit commitment situation discussed earlier. The expected system operating cost and

system response are given by:

$$TSOC = \sum_{k=-s/2}^{s/2} SOC^k \times A_l(k) \quad (75)$$

$$TRS = \sum_{k=-s/2}^{s/2} RS^k \times A_l(k) \quad (76)$$

Acceptable system operating requires that the system healthy state probabilities are maintained at acceptable levels in both the unit commitment and response analysis while minimizing the system operating cost.

## 5. CONCLUSION

An overall mathematical framework is presented in this paper for determining operating reserve requirements based on the well-being approach. The framework includes the respective mathematical formulations required for both spinning and supplemental reserves. Non-spinning reserves such as rapid start gas turbine units and hot reserve units are assessed using the concept of area risks. Interruptible load is also included in the framework as part of the operating reserve. The mathematical formulations required to incorporate the ramp-rate constraints in economic load dispatch is also presented. The incorporation of load forecast uncertainty and postponable outages are also included in the framework. The mathematical models presented can be extended to operating reserve requirements in interconnected systems.

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