

A PARAMETRIC SENSITIVITY FORMULATION FOR POWER SYSTEM ANALYSIS

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Abstract -The paper presents the development of a Parametric Sensitivity approach based on the Modified Nodal Formulation (MNF) for Power System Analysis. This technique is called the Modified Nodal Formulation Sensitivity Analysis (MNFSa). The algorithm and development of the mathematical formulations for the MNFSa that was built into a computer program is illustrated.

The developed program offers a new tool to help in analyzing the quality of power system design by studying the effect of adding, removing and changing the characteristics of different power system equipment.

The paper lists the MNFSa capabilities in identifying system problems, the parameters that are causing these problems, the interaction between the capacitive and reactive system parameters, and in devising solutions and mitigation methods.

The MNFSa can be applied to assist in identifying power quality issues related to harmonic over voltages and resonance in power systems. This application includes identifying the route cause of problems such as resonance phenomena and harmonic amplification and assets in finding solutions to these problems.

Index Terms-- Power System Sensitivity Analysis, Modified Nodal Formulation, Parametric Analysis.

I. OVERVIEW

One of the primary objectives of power system analysis is to study the impact of changes in power system parameters on system behavior. For example, an operation engineer may change the value of a system parameter, such as reactive power injection at a bus, in order to improve the voltage profile of certain busses or to avoid voltage collapse of the system. For the same reason, the engineer may also change the system topology by switching ON/OFF a component such as a capacitor bank or a transmission line. In every case it is important to know the effect of these changes on the system state of interest. Therefore, studying the effect of changes in power system parameters on the relevant aspects of the system response forms an important tool to both power system designers and operation engineers [1-3]. This effect can be referred to, in general term, as power system parametric sensitivities. Sensitivity analysis is a mathematical process widely applied in engineering problems, medical, economics and many other applications to analyze models and data. The sensitivity analysis is used to validate parameter estimates obtained in model fitting along with their significance. Parameter sensitivity can be defined as the effect of parameter

changes on the dynamics of a system such as time or frequency response, state, transfer function or any other quantity characterizing the dynamics of the system [4, 5, 6]. Parameter sensitivity analysis is a mathematical tool that is becoming widely applied in engineering problems where mathematical models are used for the purposes of analysis and synthesis [7, 8, 9]. Formulating the power system parametric sensitivities in an efficient algorithm will provide power systems engineers with a valuable tool to study certain power system phenomena and identify effective means of dealing with these events. A power system parametric sensitivity formulation based on the well-known Modified Nodal Formulation (MNF) [10,11] is proposed. The paper presents and proposes a novel Modified Nodal Formulation Sensitivity Analysis (MNFSa) method, detailed development into a computer program. The method can be used to calculate sensitivity of system response to changes in any system parameters using the MNF.

II. THE MODIFIED NODAL FORMULATION

The Modified Nodal Formulation (MNF) is a circuit analysis technique that models each circuit parameter with its own contribution as an individual entity. This technique can directly compute both branch current and node voltages as a primary state variable and consequently the matrices forming MNF equations may contain admittances and/or impedances. Having the ability to incorporate any node voltage and branch current in the MNF equations makes it possible to directly obtain sensitivities of any circuit state (voltage or current) to any circuit parameter. Another advantage of the MNF is the ability to model inductive elements as impedances and capacitive elements as admittances; this allows RLC circuits to be modeled as first order differential equations instead of integro-differential equations. The MNF can easily model circuit elements that cannot be represented in the Nodal Admittance Formulation (NAF), such as ideal voltage source, controlled voltage sources, short circuit. In addition to the modeling capabilities, a stamp defines each type of circuit elements in the MNF. Element stamp is a systematic approach used to easily adopt the MNF into a set of equations. Therefore, sequentially reading the circuit elements and augmenting element stamps into the system equations easily formulate system equations. The MNF formulations were described by Ho *et al* (1975) and named the Modified Nodal Formulation method or (MNF).

The paper objective is to present the development of a sensitivity analysis technique based on the well known modified nodal formulation power systems analysis. The mathematical formulation used in the development of the technique is modular and general to permit the incorporation of the parameters of any power system element in the sensitivity analysis. By using the MNF the formulation of state-equations were avoided and hence the system equations can be solved much faster by solving a set of first order differential equations.

The modified formulation was introduced to overcome modeling capabilities and better computational efficiency than the conventional NAF. It is now the most common method used in circuit analysis programs. It has also become a technique for explaining general ideas in circuit theory [12]. The following are the two basic MNF equations:

$$T \cdot x = W \quad (2.1)$$

$$T = \begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{I} \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} \mathbf{J} \\ \mathbf{E} \end{bmatrix} \quad (2.2)$$

Where

\mathbf{G} is the reduced nodal conductance matrix whose size is $n \times n$. Its elements are the derivatives of the nodal KCL equations with respect to the node voltages.

\mathbf{B} is an $n \times m$ matrix where m is the number of new variables (currents). Its elements are the derivatives of the nodal KCL equations with respect to the new variables.

\mathbf{C} is an $m \times n$ matrix whose elements are obtained as derivatives of the new equations with respect to the node voltages.

\mathbf{D} is an $m \times m$ matrix with elements representing the derivatives of the new equations with respect to the new variables. \mathbf{B} and \mathbf{C} may have only 0, 1 and -1 as matrix elements.

\mathbf{v} is the vector of n node voltages.

\mathbf{I} is the vector of m new branch currents.

\mathbf{J} is the vector of node current-excitations. The k^{th} element of \mathbf{I} is the sum of all independent current sources flowing into the node k .

\mathbf{E} is the vector of branch voltage sources. An element of \mathbf{e} will be non-zero if an ideal voltage source is described by the corresponding equation.

III. DEVELOPMENT OF MODIFIED NODAL SENSITIVITIES ANALYSIS (MNFSA)

The proposed method is based on developing a parametric sensitivity utilizing the MNF to perform power system sensitivity analysis. The proposed formulation shall be able to determine the sensitivity of any power system state (node voltage or branch current), due to changes in any system parameters. This is due to the

ability of the MNF to model both voltages and currents as system variables. Therefore, it is possible to obtain the sensitivity of node voltages and branch currents with respect to any circuit parameters (inductances, resistances, capacitance, etc).

The system equation of the MNF can be written as

$$T(h_i) \cdot x(h_i) = W(h_i) \quad (3.1)$$

Where h_i is the parametric value of arbitrarily chosen system element i , i.e. h_i can be resistor (R_i), inductor (L_i), capacitor (C_i) or conductance (G_i)

Taking the derivative of Equation (3.1) with respect to parametric value h_i

$$\frac{\partial T}{\partial h_i} x + T \frac{\partial x}{\partial h_i} = \frac{\partial W}{\partial h_i} \quad (3.2)$$

Rearranging equation (3.2)

$$T \frac{\partial x}{\partial h_i} = \frac{\partial W}{\partial h_i} - \frac{\partial T}{\partial h_i} x \quad (3.3)$$

$$\frac{\partial x}{\partial h_i} = T^{-1} \left(\frac{\partial T}{\partial h_i} x - \frac{\partial W}{\partial h_i} \right) \quad (3.4)$$

Define a scalar variable f , where f is the j^{th} unknown variable in the vector \mathbf{x} , this is to deal with one system variable at a time; then f can be mathematically defined as

$$\mathbf{j} = \mathbf{d}^t x \quad (3.5)$$

Where \mathbf{d} is a constant vector with one at the j^{th} element and zero in the rest.

Take the derivative of equation 3.5 with respect to h_i

$$\frac{\partial \mathbf{j}}{\partial h_i} = \mathbf{d}^t \frac{\partial x}{\partial h_i} \quad (3.6)$$

Substitute equation (3.4) into equation (3.6), then

$$\frac{\partial \mathbf{j}}{\partial h_i} = \mathbf{d}^t \left\{ T^{-1} \left(\frac{\partial T}{\partial h_i} x - \frac{\partial W}{\partial h_i} \right) \right\} \quad (3.7)$$

Define an adjoint vector x^a so that

$$(x^a)^t = \mathbf{d}^t T^{-1} \quad (3.8)$$

Rewrite (3.8)

$$T^t x^a = \mathbf{d} \quad (3.9)$$

Substituting equation (3.9) into equation (3.7), we get

$$\frac{\partial \mathbf{j}}{\partial h_i} = (x^a)^t \frac{\partial T}{\partial h_i} x - (x^a)^t \frac{\partial W}{\partial h_i} \quad (3.10)$$

Table 4.1 MNFSA Input Data For The Sample System

Section I: General data								
NEL	NODES	INIT_FREQ	END_FREQ	STEP_FREQ	OUTNOD	REFNODE	TYP	VBASE
6	4	0	400	50	2	0	2	13800

Section II: System parameters data				
ETYP	N1	N2	VAL	SC
L1	1	2	0.0016	0
L2	2	4	0.03056	1
R1	3	0	5.36	0
R2	4	0	4.4	1
C1	2	3	0.207E-3	1
E1	1	0	13800.0	0

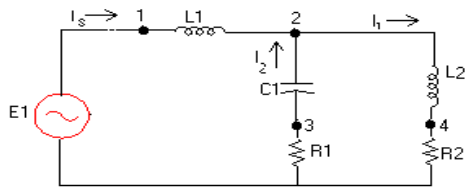


Figure 4.1 Sample System

Where

NEL: Number of power system elements involved in the power system model.

NODES: Number of the power system nodes

INIT_FREQ: The initial frequency for the study

FINAL_FREQ: Final frequency

STEP_FREQ: Simulation frequency step

OUTNOD: Output variable; defines the output node voltage or branch current of interest

REFNOD: Defines the reference node (0 means ground)

TYP: Define the type of analysis (TYP=1 frequency scan, TYP=3 parametric sensitivity, TYP=4 frequency sensitivity)

BV: Base voltage. The program uses this value to normalize the parametric sensitivity.

ETYP: Type of element (L Inductance, R resistance, C capacitance, G Conductance, E Ideal voltage source, J Ideal current source)

N1: The "From" node or terminal

N2: The "To" node or terminal

VAL: Actual value of the parameter (L in Hinry, C in Farad, R in Ohm, G in Semens, Voltage source (E) in volts, Current Source (J) in Amperes)

SC: Sensitivity analysis control (If SA=1 the program will perform sensitivity with respect to the parameter, SA=0

the program will not carry sensitivity calculation with respect to this parameter)

B. Output Results

The MNFSA program calculates the sensitivity of any system variable (voltage or current), with respect to any circuit element parameter or the sensitivity of any system variable with respect to system frequency.

Table 4.2 shows the MNFSA output results that correspond to the input data file described in Table 4.1. In addition to calculating the sensitivity of any node voltage to any system parameter, the program can also calculate similar sensitivities to branch current variables. To calculate the sensitivities of load current (I_1), output variable in Table 4.1 is changed to (OUTNOD=5), incorporating current variables in the MNF equations as described previously. Table 4.3 shows the sensitivity of load current with respect to parameters L_2 , C_1 and R_2 .

Table: 4.2 Sample Output For Parametric Sensitivity of Node-2 Voltage at Different Frequencies

Frequency (Hz)	SV2L2	SV2R2	SV2C1	SV2L2
0	0.00E+00	0.00E+00	0.00E+00	0.00E+00
50	4.10E-02	1.88E-02	2.77E-02	4.10E-02
100	5.21E-02	1.19E-02	9.22E-02	5.21E-02
150	5.83E-02	8.90E-03	1.60E-01	5.83E-02
200	6.06E-02	6.95E-03	2.08E-01	6.06E-02
250	5.95E-02	5.46E-03	2.32E-01	5.95E-02
300	5.61E-02	4.29E-03	2.36E-01	5.61E-02
350	5.15E-02	3.37E-03	2.27E-01	5.15E-02

Multiplying the actual sensitivity with the parameter actual value divided by the base value normalizes the sensitivities shown in Tables 4.2 and 4.3. Fore example the sensitivity of node 2 with respect to L_2 (SV2L2) is calculated based on the following equation:

$$SV_{2L2} = \frac{\partial V_2}{\partial L_2} \cdot \frac{L_2}{BV}$$

Where SV2L2 is the sensitivity of node-2 voltage to system parameter L_2 .

To calculate the sensitivity of node-2 voltage (V_2) with respect to system frequency, the only change to the input data is to change Type of analysis from (TYP=3) to (TYP=4). The results of the MNFSA are shown in Table 4.4. The results showed both the magnitude and the phase angle of the network sensitivity.

C. Applications and Capabilities of the Algorithm

The MNFSA program is designed to perform a wide range of sensitivity analysis; more specifically the code is designed to compute the sensitivity of all system states variables (voltages and currents) with respect to any network parameters. The program will be utilized to perform the following analysis:

- Quantify the significance of each parameter on system response relative to other parameters
 - Analyze the effect of changes in system parameter on sensitivities.
 - Perform a quick assessment to system frequency response at different harmonic frequencies.
 - The above analysis can be used in power system design optimization.
- MNFSA has Computational Advantages due to the following:

- The algorithm is generic for current and voltage variables, which means that the same equations are used for calculation of current and voltage sensitivities.
- Inversion of T is needed only once.
- Although the size of system matrices has increased due to the use of MNF, the structure of system equations has been greatly simplified by modeling the system as a set of linear and first order differential equations.

Frequency (Hz)	SISL2	SISR2	SISC1
0	0.00E+00	2.85E-03	0.00E+00
50	1.05E-03	4.81E-04	7.09E-04
100	6.67E-04	1.53E-04	1.18E-03
150	4.97E-04	7.60E-05	1.37E-03
200	3.88E-04	4.45E-05	1.33E-03
250	3.05E-04	2.80E-05	1.19E-03
300	2.39E-04	1.83E-05	1.01E-03
350	1.88E-04	1.23E-05	8.31E-04

Frequency (Hz)	SV2F (Mag)	SV1F (Phase)
0	3.07E+01	180.0001
50	1.05E+01	-115.95
100	2.53E+01	-148.211
150	3.24E+01	-173.659
200	3.46E+01	164.626
250	3.37E+01	146.2357
300	3.12E+01	130.7622
350	2.82E+01	117.7515

V. PRACTICAL APPLICATION OF MNFSA

In the following case, the MNFSA is applied to study the effect of installing a Power Factor Correction Capacitor (PFC) at the load bus. Introducing a capacitor in power system can lead to resonance phenomenon [15, 16]. Resonance is a condition whereby the capacitive reactance of a system, offsets its inductive reactance leaving the resistive elements in the network as the only impedance. The frequency at which this offsetting effect takes place is called the resonant frequency of the system. Depending on how the reactive elements are arranged throughout the system, the resonance can be of a series or a parallel type.

A. Plain Power Factor Correction Capacitor

A radial system feeding a group of induction motors load (20MVA) operating at 60% lagging power factor is evaluated. The load is fed from a 13.8kV utility company via a 15kV cable. Power Factor Correction compensation (PFC) of 15MVar is connected at the load end. The single line diagram of the system is shown in Figure 5.1 and the Circuit diagram is shown in Figure 5.2. The system equipment data are as follows:

Utility data

Short Circuit Level (MVA_{sc})= 350 MVA

X/R ratio =15

Voltage level = 13.8 kV

Cable data

Cable size 350 MCM, 3-Coconductors/Phase (3-1/350MCM)

Cable length = 3 km

Cable Impedance = 0.0427+j0.0386 ohm/km, $Y_c=0.2 \mu F/km$

Load data

20 MVA Induction motors load

Power factor = 60% lagging

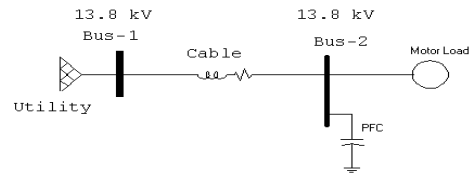


Figure 5.1: Single Line Diagram for Case Study (Plain PFC)

Utility Source Equivalent Impedance

$$Z_s = \frac{KV^2}{MVA_{sc}}$$

$$Z_s = \frac{13.8^2}{350} = 0.38088 \text{ Ohm}$$

$$Z_s = \sqrt{R_s^2 + X_s^2}$$

$$Z_s = X_s \sqrt{(R_s / X_s)^2 + 1}$$

$$X_s = \frac{Z_s}{\sqrt{(R_s / X_s)^2 + 1}} = 0.38 \text{ Ohm}$$

$$L_s = L_1 = \frac{X_s}{2pf} = 1.0078 \text{ mH}$$

$$R_s = R_1 = X_s (R_s / X_s) = 0.0253 \text{ Ohm}$$

Cable impedance

$$R_c = 0.0427 \times 3 = 0.1281 \text{ Ohm}$$

$$L_c = L_2 = \frac{X_c}{2pf} = \frac{0.0386 \times 3}{377} 1.0078 \text{ mH}$$

$$C_c = C_2 = C_3 = 0.2 \times 3 = 0.6 \mu F$$

Load equivalent impedance

$$Z_L = \frac{KV^2}{MVA_L}$$

$$Z_L = \frac{13 \cdot 8^2}{20} = 9.522 \text{ Ohm}$$

$$R_L = R_3 = p \cdot f \times Z_L = 5.713 \text{ Ohm}$$

$$X_L = MVA_L \times \sqrt{1 - p \cdot f^2} = 7.617 \text{ Ohm}$$

$$L_L = L_3 = \frac{X_L}{2\pi f} = 20.20 \text{ mH}$$

Power Factor Correction Capacitor

$$X_{C1} = \frac{KV^2}{MVA_{PFC}}$$

$$Z_L = \frac{13 \cdot 8^2}{15} = 12.696 \text{ Ohm}$$

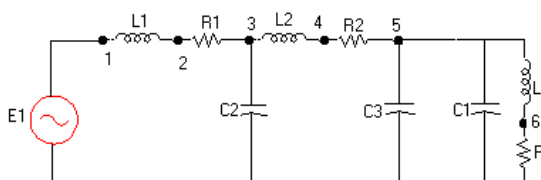
$$C_{PFC} = C_1 = \frac{1}{2\pi f X_C} = 208.9 \text{ uF}$$


Figure 5.2: Circuit Diagram for Case Study (Plain PFC)

B. Sensitivity Analysis and Results:

Impedance frequency scan and voltage response of node 5 (load bus), shown in Figures 5.3 and 5.4 indicate that the system has a parallel resonance near the 5th harmonic (315Hz). The circuit has an amplification factor of 14.02 p.u at the resonance frequency. Due to this amplification, the presence of a 5th harmonic current source near this bus will lead to over voltages and possible damage to the PFC capacitor and other equipment. Nevertheless, this resonance frequency is very close to one of the commonly encountered characteristic harmonics that can be produced from nonlinear loads such as 6-pulse converter circuit. The program is applied to calculate parametric sensitivities for node 5 at the 300 Hz and 60 Hz, to be used to analyze the causes of the parallel resonance near the 300 Hz. Results shown in Table 5.1, shows a significant increase in the sensitivity of node-5 voltage to all system parameters at 300 Hz as compared to the 60 Hz case. The increase in the sensitivity is consistent with the frequency scan results and can be used as another means to locate the system resonance frequencies. In fact the sensitivity curves maintain the same profile as the frequency scan curve. This can be demonstrated by comparing node 5 the frequency scan curve shown in Figure 5.3 with node-5 parametric sensitivity versus frequency curves shown in 5.4 From these curves it is clear that the sensitivity curve can be used instead of normal frequency scan to calculate series and parallel resonance frequencies.

Moreover, results in Table 5.1 shows that V_5 sensitivities with respect to certain parameter are substantially higher than others. For example, in the case where the PFC is connected to the load bus, the sensitivities for the parameters C_1 , L_1 and L_3 are 81.04, 58.33 and 5.232 respectively. Therefore parameters L_1 , C_1 and L_2 are the main cause of the 300 Hz resonance. When the PFC removed from the system, the sensitivities shown in Table 5.1 are comparatively smaller at both the 60 Hz and the 300 Hz, which indicate that the 300 Hz is no longer a system resonance frequency.

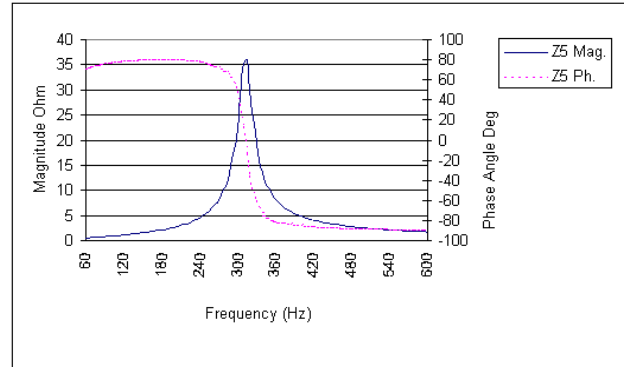


Figure 5.3: Impedance Frequency Scan at Node 5 (Radial System With a Plain PFC Capacitor)

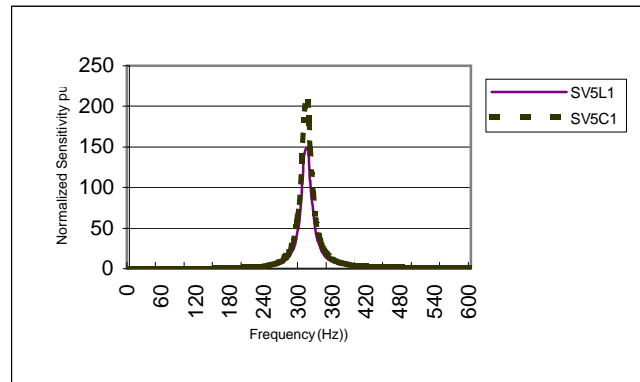


Figure 5.4: Parametric Sensitivities of Node 5 Voltage versus Frequency (Radial System With a Plain PFC Capacitor)

VI. CONCLUSIONS

A Parametric Sensitivity Analysis approach based on the Modified Nodal Formulation for power system analysis (MNFSFA) has been proposed. The MNFSFA approach provides a good help in understanding how parametric variation of power system equipment such as motor, generators, transformer, cable and PFC capacitors influences the power system response.

Table 5.1: Sensitivity of load bus voltage (Node 5) to System Parameters at Different Frequencies with and without PFC

Freq	With PFC		Without PFC	
	60	300	60	300
SV5L1	0.02483	58.33	0.03317	0.03993
SV5R1	0.001657	0.7784	0.002213	0.000533
SV5L2	0.007569	17.71	0.01013	0.01273
SV5R2	0.008374	3.918	0.01121	0.002817
SV5C1	0.04027	81.04	N/A	N/A

SV5C2	8.57E-05	0.1408	8.02E-05	0.001944
SV5C3	0.000116	0.2333	0.000108	0.002503
SV5R3	0.03358	0.9838	0.03115	0.01056
SV5L3	0.03572	5.232	0.03313	0.05615

It also provides a tool to assist system designers and operation engineers in comparing the quality of alternative power system design having the same purpose. The tool can also provide an assessment mechanism to changes in the power system due to addition of new component such as power factor correction and harmonic filters.

The proposed MNFSA formulation has a modular and general mathematical algorithm that can incorporate the parameters of all power system elements. The MNFSA combines the MNF superior modeling capabilities and the computational efficiency of the sensitivity analyses.

VII. ACKNOWLEDGMENT

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