CALCULATING THE START-UP COSTS OF HYDROPOWER GENERATORS

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Abstract - This paper presents several models for calculation of hydropower generator start-up costs. All start-up costs are assumed to be either directly proportional to the number of starts or increasing with the number of operating hours of the unit. Focusing on the latter, simplified models for both average and marginal start-up costs are developed and compared. Finally, an improved model is developed which takes into account both the direct start-up cost and the effect of the unit's refurbishment intervals. As for the simplified models, the start-up cost is assumed to increase with the number of operating hours (age) of the unit. As each start is assumed to cause abrasion equivalent to a certain number of operating hours, however, additional costs are incurred as each start will reduce the remaining service life of the unit and thus shift the refurbishment intervals closer in time.

Key words: Hydropower, Start-up costs, Service life, Maintenance, Refurbishment

1. INTRODUCTION

Traditionally, Norwegian hydropower plants have been designed and built for optimal energy generation with continuous operation. Lately, many power plants have experienced a change in operating pattern with more starting and stopping and larger and more frequent load changes than before. This development causes increased wear and tear on the unit, but few studies are made to examine how this might influence the service life of the equipment and thus the operating costs. An extensive literature search resulted in papers and reports mainly in three classes: 1) Technical aspects of abrasion and ageing, 2) Methods for maintenance scheduling, and 3) Methods to include (given) start-up costs in operation planning. Only one paper has been found actually trying to calculate specific start-up costs of hydropower units [1].

This paper treats the problem from a general point of view, and will not give a detailed technical examination of the different unit components and the respective abrasion mechanisms. In Section 2 two different cost models are defined for time-invariant and time-dependent costs, respectively. In Sections 3 and 4 simplified models for average and marginal start-up costs are developed and compared. In Section 5 a more formal model for the time-dependent costs is developed. Note that in this paper we use the terms "start-up" and "start" meaning one full start-stop cycle of the unit. Furthermore, the terms "refurbishment interval" and "service life" are used to denote the time period between two subsequent refurbishments.

2. START-UP COSTS AND COST MODELS

The start-up costs of hydropower units are related to the following aspects:

- A normal (successful) start will incur costs due to water losses and necessary job tasks.
- An unsuccessful start will occur with a certain probability, and increases both water losses and labor costs. Nilsson [1] indicates that up to 20% of all starts are unsuccessful due to control equipment failure.
- Any start induces abrasion of valves, mechanical equipment and generator, especially windings and insulation. This incurs losses due to efficiency degradation, and increased probability of failure or major breakdowns. Increased monitoring and maintenance are necessary to remedy these effects.
- Increased wear and tear of the unit may result in more frequent refurbishment. This decrease in the unit's service life will add costs.
- More frequent maintenance or refurbishment might or might not incur additional costs due to lost production and non-optimal water management, depending on the topology of the river system, the water storage capacity and the number of cascading plants in the river.

Hence, the total start-up cost is made up of several elements. Some of these elements are easy to identify, such as the water loss of a normal start or the extra labor cost of an unsuccessful start. Other elements are less obvious, like the effect on start-up costs due to reduced service life. The unit's service life is a result of some decision making in order to maximize profit or minimize generation cost. The start-up cost also depends on the residual service life as demonstrated in the next section.

Hydropower plants have very long service life, typically more that 50 years. It is not uncommon, however, that units are upgraded or retrofitted before they are worn out. This might be the case if new improved technology is introduced, e.g. new turbines with improved efficiency or upgrading of penstocks and tunnels. In this situation, the potential cost increase due to unit starting is irrelevant.

Similarly, if the plant owner has a higher level of maintenance than necessary, there are apparently no additional costs due to unit start-up, as the exaggerated maintenance schedule is "hiding" the real start-up costs of the unit.

In order to support intuition and analyses we now introduce some simple cost modeling: Refurbishment is assumed to incur a cost equal to *RC*. This figure includes all expenses and losses related to the refurbishment itself. All other costs and losses relevant to start-up and reduced service life can be assigned to one of the two following categories:

1. Costs directly related to the respective start-up sequence like labor and loss of water during a normal start, and expected additional costs due to control equipment failure. These costs are assumed to be a function of the number of starts within the relevant time period.
2. Costs dependent of accumulated number of hours of operation *d* for the actual unit. Furthermore, the unit abrasion caused by one start-up is assumed to be equivalent to Δd hours of "normal" operation. In this way, the abrasion of starting is allowed to accumulate over the unit's service life.

We capture these categories by two formal models:

- ***Model 1:*** The costs directly related to each start are assumed to be proportional only to the number of starts *n* in a given period:

$$c_1(n) = c_0 \cdot n \qquad \text{[USD]} \qquad (1)$$

- ***Model 2:*** Costs assumed to be increasing with the number of operating hours *d;* operating, monitoring and maintenance costs, costs due to (expected) breakdown, as well as losses due to unit efficiency degradation. One hour of operation adds one hour to *d* while one start is equivalent to a number of operating hours Δd. The quantity Δd is denoted "*equivalent operating time"* due to one single start, and can be constant or an increasing function of *d.* See also Section 5.1 for a more detailed formulation.

$$c_2 = c_2(d) \qquad \text{[USD]} \qquad (2)$$

Assuming that for a fixed start-up rate the income will be fixed, the optimal service life can be determined by minimizing the total cost comprising of refurbishment cost RC plus c_1 and c_2. Note that should the sum $c_1 + c_2$ *not* be increasing with operating hours, there will be no cost savings to obtain by refurbishment and the service life will be infinite and independent of the start-up rate.

Costs of the first kind (Model 1) are invariant over the service life of the unit and the number of start-up per period will influence the cost level only, and will not influence on optimal service life.

The second cost model implies increasing start-up cost over the unit's service life. This model thus establishes a link between the start-up rate and the unit's optimal service life as the start-up rate influence the shape of the cost function. It is evident that if the start-up cost is increasing over a time period, it will be advantageous to decrease the start-up rate over the same period, hence the time dependence should be considered when bidding in short term regulating markets and for short term planning.

Hara et.al. assume that one start-up decreases the service life of the generator by 10 hours [2]. Similarly, Nilsson assumes 15 hours [1], but when he calculates an average cost per start-up the time structure of the costs is lost.

3. AVERAGE START-UP COST

It is important to distinguish between *average* and *marginal* start-up cost, both when different methods of calculation are derived as well as when the results are to be employed. Average and marginal costs are relevant in different situations. Considering a long-term contract implying frequent unit start-up, it will be of interest to calculate the *average* start-up cost to ensure that it is covered over the contract period. In a short-term reserve market, however, it will be more interesting to set the price of one single start-up, thus the *marginal* cost is more relevant. Models for short-term operation planning also use marginal start-up cost as input.

In this section we focus on start-up costs as related to reduced service life of major components. Ignoring all other cost effects, we now assume that one start decreases the unit's *residual* operating time by a fixed number of hours ΔD. This is a simplification compared to the assumptions of Model 2 to enable an easier computation of start-up costs. In Section 5 the different cost effects will be formally assembled.

3.1 Undiscounted average cost

We assume the refurbishment cost *RC* to be known for the unit or component under consideration. Without discounting the amount, the *annual* refurbishment cost can be found by dividing *RC* by the service life (refurbishment interval). T_0 and $T_{\Delta n}$ indicate refurbishment intervals by *normal operation* and *intermittent operation* with Δn extra start/stop sequences, respectively. The transition from a traditional operating pattern to an intermittent pattern will cause the average annual refurbishment cost to increase from RC/T_0 to $RC/T_{\Delta n}$. The cost per start can then be calculated by dividing this *additional* operating cost with the number of *extra* starts per year Δn [3].

$$c_{st} = \frac{1}{\Delta n} \cdot \left(\frac{RC}{T_{\Delta n}} - \frac{RC}{T_0} \right) \qquad \text{[USD/start]} \qquad (3)$$

where
- Δn - No. of extra starts per year by intermittent operation
- RC - Refurbishment cost [USD]
- T_0 - Refurbishment interval at normal operation [year]
- $T_{\Delta n}$ - Refurbishment interval at intermittent operation [year]

Introducing *total annual Operating Time* by normal and intermittent operation yields:

$$OT_0 = (\alpha_0 \cdot 24 \cdot 365 + n_0 \cdot \Delta D) \qquad \text{[h/year]} \qquad (4)$$

$$OT_{\Delta n} = OT_0 + \Delta n \cdot \Delta D \qquad \text{[h/year]} \qquad (5)$$

where α_0 - Fraction of annual operation
n_0 - No. of starts per year in "normal" operation
ΔD - Equivalent operating time [h/start]

We now define *Designed Total Operating Time DTOT* [h] as the maximum operating time of a unit determined by its design. This total operating time may be "spent" in several ways. If the unit is in continuous operation, it will last exactly *DTOT* hours. Assuming that one start-up cycle is increasing the operating time of the unit by ΔD, the service life of the unit will be correspondingly shorter due to frequent starting. The following relations are valid:

$$DTOT = T_0 \cdot OT_0 = T_{\Delta n} \cdot OT_{\Delta n} \qquad \text{[h]} \qquad (6)$$

Inserting Eqs. (4)-(6) in Eq. (3) yields:

$$c_{st} = \frac{RC \cdot \Delta D}{T_0 \cdot OT_0} = RC \cdot \frac{\Delta D}{DTOT} \qquad \text{[USD/start]} \qquad (7)$$

c_{st} is an average cost calculated without considering any discounting. As can be seen from Eq. (7) the cost per start is independent of the total number of starts over the unit's lifetime. In other words, *the normal operating pattern does not influence the cost of starting the unit.* Only the Designed Total Operating Time is relevant for the undiscounted average cost calculated by this model.

The time aspect must also be considered, however. This is done in the following section by discounting.

3.2 Discounted average cost

Assuming the next refurbishment to be imminent (or just finished), the following refurbishment will occur in $T_{\Delta n}$ years when the unit is used for intermittent operation. Then a new refurbishment will follow indefinitely with $T_{\Delta n}$ years intervals. The present value of such an indefinite series is designated $PV_{\Delta n}$. Similarly, the present value of the refurbishment cycle with a "normal" operating pattern is denominated PV_0. The additional cost of an intermittent operating pattern is $(PV_{\Delta n}-PV_0)$.

Assuming the refurbishment cost by an intermittent operating pattern to occur at regular intervals of $T_{\Delta n}$ years, the present value of an indefinite series of refurbishment cycles is:

$$PV_{\Delta n} = \frac{RC}{(1-e^{-rT_{\Delta n}})} \qquad \text{(USD)} \qquad (8)$$

The additional cost of an intermittent pattern compared to a "normal" operating pattern with T_0 years between each refurbishment, is then:

$$\Delta PV = \frac{RC}{(1-e^{-rT_{\Delta n}})} - \frac{RC}{(1-e^{-rT_0})} = \frac{RC \cdot (e^{-rT_{\Delta n}} - e^{-rT_0})}{(1-e^{-rT_0}) \cdot (1-e^{-rT_{\Delta n}})} \qquad \text{(USD)} \qquad (9)$$

We now assume that this cost must be covered by a defined price for each extra start, designated ***p***. With Δn extra starts per year, the annual cash flow is $\Delta n \cdot p$. The present value of this indefinite series is $\Delta n \cdot p/r$. If this cash flow is to cover the additional refurbishment cost the present values must be equal, resulting in the following expression:

$$p = \frac{1}{\Delta n} \cdot r \cdot \frac{RC \cdot (e^{-rT_{\Delta n}} - e^{-rT_0})}{(1-e^{-rT_0}) \cdot (1-e^{-rT_{\Delta n}})} \qquad \text{[USD/start]} \qquad (10)$$

p thus indicate a balance price or an average cost when discounting is considered. Substituting for $T_{\Delta n}$ and calculating the limit when the interest rate *r* approaches zero, this average cost approaches the undiscounted average cost. Discounted average cost is always lower than undiscounted cost. The *undiscounted average cost thus represents an upper limit for the discounted average cost.* The importance of the discounting is shown in the following examples.

Example 1:

Refurbishment cost: $RC = 2.5$ *mill. USD*
Designed ref. interval: $T_0 = 40$ *years*
Operating fraction: $\alpha_0 = 0.8$
Designed no. of start: $n_0 = 5\ day^{-1} = 1825\ year^{-1}$
Equivalent operating time: $\Delta D = 15$ *h/start*

=> Designed total operating time: DTOT = 157 yrs

It is assumed that "normal" operation corresponds to the design criteria such that normal lifetime is T_0 = 40 years. It is further assumed that one single start is equivalent to 15 hours of operation. This yields the following correspondence between the no. of extra start, the refurbishment interval (lifetime) and the cost:

No. of extra starts per day	Refurb. interval T_n [years]	Undisc. cost (Eq. 7) [USD/start]	Disc. cost r = 5% (Eq. 10) [USD/start]
1	34.5	27.3	20.6
2	30.3		21.3
3	27.1		21.9
4	24.4		22.4
5	22.3		22.8

Table 1 Comparison of discounted and undiscounted start cost

When calculating the discounted average cost, the unit or component is assumed to be in "new" condition. If this assumption is incorrect, the costs will increase as the refurbishment occurs sooner than assumed. This effect is marginal, however, and will only influence the cost with a small amount.

The costs calculated in Example 1 might seem low. This is mostly due to the assumption of the high Designed Total Operating Time of 157 years continuous operation. The importance of this assumption is illustrated in Example 2 where designed operating time is reduced to 57 years of continuous operation.

Example 2:

Refurbishment cost: $RC = 2.5$ *mill. USD*
Designed refurb. interval: $T_0 = 40$ *years*
Operating fraction: $\alpha_o = 0.8$
Designed no. of start: $n_o = 1\ day^{-1} = 365\ year^{-1}$
Equivalent operating time: $\Delta D = 15$ *h/start*

=> Designed total operating time: DTOT = 57 years

The correspondence between the number of extra starts, the refurbishment interval (lifetime) and the costs are:

No. of extra start per day	Refurb. interval T_n [years]	Undisc. cost (Eq. 7) [USD/start]	Disc. cost r = 5% (Eq. 10) [USD/start]
1	27.8	75.1	60.0
2	21.3		63.2

Table 2 Comparison of discounted and undiscounted start cost

This method is sensitive to the assumption that the design criteria are taking into account a certain number of "normal" starts per day. Comparing Example 1 with Example 2, both assume the *normal* refurbishment interval to be 40 years, and both examples yield refurbishment intervals of approximately 27 years. In Example 1 this is caused by *3 extra starts per day* at an undiscounted average cost of 27.3 USD. In Example 2, however, only *1 extra start per day* causes an undiscounted cost of 75.1 USD.

The relation between the costs is *1:3* since the first example allows three times as many starts per day as the other example for the same refurbishment interval (approx. 27 years). In other words, the average cost is an expression for the cost of making *one extra start each day infinitely*, but is not the cost for *one single extra start*. The cost for a single start has to be calculated by its marginal value, which is done in the following section.

4. MARGINAL COST

Assuming that the unit or component has a certain residual service life, the next refurbishment will occur in T_F years. With a defined operating pattern, the following refurbishments will then occur at constant intervals T_R, where $T_F \leq T_R$. At each refurbishment a cost of *RC* is incurred. This is illustrated in Fig. 1.

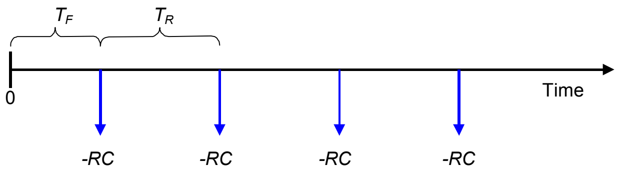

Figure 1 Cash flow due to refurbishment intervals

Assuming that each start will reduce the residual operating time of the unit with ΔD, one start will cause each following refurbishment to move closer to present time, as illustrated in Fig. 2. Note that only *one single extra start* is made, not one extra start per day over one week, one year or infinitely.

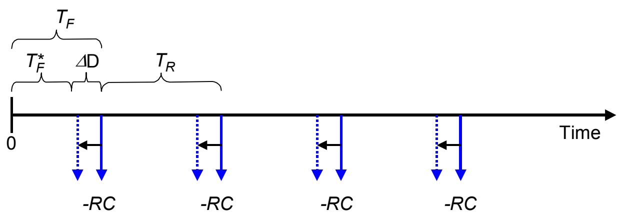

Figure 2 Cash flow time shift

We have assumed that one start is equivalent to a number of operating hours ΔD. In the following we are considering only the service life of the unit, and it is beneficial to substitute ΔD with ΔLT. While ΔD denotes equivalent *operating time*, ΔLT denotes *lost service life*.

If the unit is operated continuously towards the end of the refurbishment interval, ΔLT is equal to ΔD. If there is frequent start-up also towards the end of the refurbishment interval, this is not valid. The operating time of a unit is given in Eq. (4):

$$OT_0 = n_0 \cdot \Delta D + \alpha_0 \cdot 24 \cdot 365 \qquad \text{(h/year)} \qquad (4)$$

n_0 denotes the number of *planned* starts per year towards the end of the refurbishment interval, and α_0 denotes the fraction of the day when the unit is planned to be in operation. When the unit is operating continuously ($n_0 = 0$, $\alpha_0 = 1.0$), ΔLT measured in hours of the day is equal to *equivalent operating time* ΔD measured in hours of operation. The relation between ΔLT and ΔD for one single start is thus:

$$\Delta LT = \frac{24 \cdot 365 \cdot \Delta D}{n_0 \cdot \Delta D + \alpha_0 \cdot 24 \cdot 365} \qquad \text{(h)} \qquad (11)$$

The relation between operating time and service life is further explained in Fig. 4.

The value of *all* future refurbishment costs without any extra start at the time for the *first* refurbishment is:

$$V_0 = \frac{RC}{(1 - e^{-rT_R})} \qquad \text{(USD)} \qquad (12)$$

The present value is found by discounting this value with the remaining time to the first refurbishment T_F:

$$PV_0 = \frac{RC}{(1 - e^{-rT_R})} \cdot e^{-rT_F} \qquad \text{(USD)} \qquad (13)$$

Making one extra start today will reduce the remaining service life with ΔLT, and the remaining time to the next refurbishment is then (T_F -ΔLT). This is the only effect of the start. The present value of all future refurbishment under the assumption of one extra start today is thus:

$$PV_{st} = \frac{RC}{(1-e^{-rT_R})} \cdot e^{-r(T_F - \Delta LT)} \qquad \text{[USD/strt]} \qquad (14)$$

The additional cost, or marginal cost MC, for the extra start is then:

$$mc_{st} = PV_{st} - PV_0$$
$$= \left(\frac{RC}{(1-e^{-rT_R})} \right) \cdot (e^{r\Delta LT} - 1) \cdot e^{-rT_F} \qquad \text{[USD/strt]} \qquad (15)$$

This expression has a simple interpretation. The first parenthesis indicates the present value of all future refurbishment at the time of the first refurbishment. The denominator inside the parenthesis scales the cost to account for all future refurbishment. The second parenthesis is a factor that indicates the cost of shifting all future refurbishment a certain time ΔLT. The last term is the discounting from the time of the first refurbishment to the present time.

ΔLT is typically in the range of a few hours, such that we without loss of accuracy can set $(e^{r\Delta LT} - 1)$ equal to $r\Delta LT$.

Eq. (15) shows that the marginal cost is influenced both by the refurbishment intervals and the remaining time to the next refurbishment. It is worth emphasizing that it is the refurbishment intervals that decide the marginal cost, not the number of start per year. In other words, it is sufficient for the user to define the refurbishment interval. Eq. (15) then yields:

$$mc_{st} = \frac{RC}{(1-e^{-rT_R})} \cdot r\Delta LT \cdot e^{-rT_F} \qquad \text{[USD/start]} \qquad (16)$$

Example 3

Refurbishment cost:	*RC = 2.5 mill. USD*
Designed refurb. interval:	T_n *= 40 years*
Lost service life:	ΔLT *= 15 h/start*
Interest rate:	*r = 5% pa*

One single start is assumed to shift the next refurbishment with ΔLT *= 15 hours. With 5% annual interest rate the cost of this shift is 214 USD.*

Note that all future refurbishment must also be considered. With 5% interest rate and 40 years refurbishment intervals the total cost of all future refurbishment is 247.5 USD calculated at the time of the first refurbishment. However, the present value is dependent on how far into the future this cost will occur. If the next refurbishment is due in 40 years, (T_F *= 40) the present value will of course be considerably reduced. With an interest rate of 5% the present value of the total cost of 247.5 is reduced to 33.5 USD.*

If the next refurbishment is close, however, (T_F *= 0), there is no effect of the discounting, and the marginal cost will reach its maximum value of 247.5 USD. If the first refurbishment is due in one year (*T_F *= 1), the discounted cost will be 235.5 USD.*

The marginal cost of one single start is thus strongly dependent on when the next refurbishment is due. In this example, the marginal cost vary between 33.5 USD and 247.5 USD. Both these numbers are present values that indicate the cost of *one single start*.

5. IMPROVED MODELS FOR START-UP COST

In section 2 we established two cost models, or cost functions denoted $c_1(n)$ and $c_2(d)$. The cost function $c_1(n)$ is uncomplicated, especially if a unit cost is applicable. This function is independent of time, hence it is not influenced by refurbishment intervals or service life. Of course $c_1(n)$ adds to the total start-up cost, but focusing on the unit service life, we ignore this cost.

In this section we elaborate on the second cost model, taking into account the unit's service life (refurbishment intervals) and refurbishment cost RC. In the end we relate our results to the simplified calculations of Section 3.

5.1 Time dependent start-up cost

The cost $c_2(d)$ comprises all operating costs including monitoring and maintenance between refurbishments etc. We also assume that $c_2(d)$ comprises expected breakdown cost and relevant losses.

A start contributes to increasing cost as a start-up add ΔD hours to the accumulated operating time d of $c_2(d)$. There is some discussion, however, whether the equivalent operating time ΔD actually *increases* over the service life of the unit [4]. To include such an effect in our models, the equivalent operating time $\Delta d(d)$ is set equal to a fixed number of hours ΔD plus a small portion δ of total operating time:

$$\Delta d(d) = \Delta D + \delta \cdot d \qquad (17)$$

where δ - proportionality factor

Setting $\delta = 0$ yields the original assumption of constant increase ΔD in operating hours. Note that with frequent start-up the total operating time within one day can be more than 24 hours.

Fig. 3 illustrates the direct effect of a start. $c_2(d)$ is assumed to be increasing and convex. Two starts are shown, and both add ΔD to total operating time. A start implies that ΔD hours of operating time elapse instantaneously. Costs associated with the two start-ups are equal to the areas S_1 and S_2.

In the figure there is one "early" start and one "late" start. It is evident that with an increasing $c_2(d)$, a "late" start will induce greater costs than an "early" start.

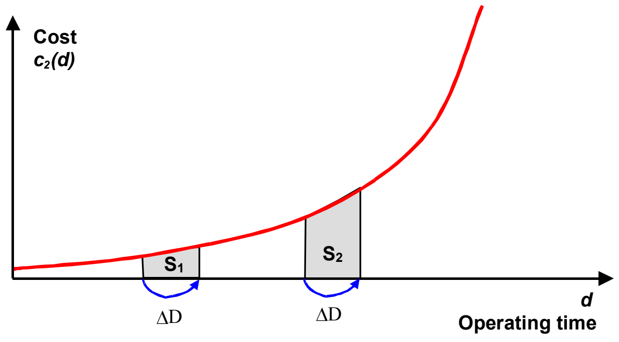

Figure 3 Start costs depending on operation time [4]

Note that even though a start-up instantaneously adds ΔD to total operating time, there is no change in elapsed (calendar) time. Assuming a situation with no start-ups and optimal total running time equal to D^*, optimal service life is found to be T_0^* - see Fig. 4.

Introducing one single start-up, either "early" or "late", the same amount ΔD of total operating time is spent, but elapsed (calendar) time is not changed. If one sticks to the optimal total operation time D^*, the service life, or refurbishment interval is reduced from T_0^* to T^*.

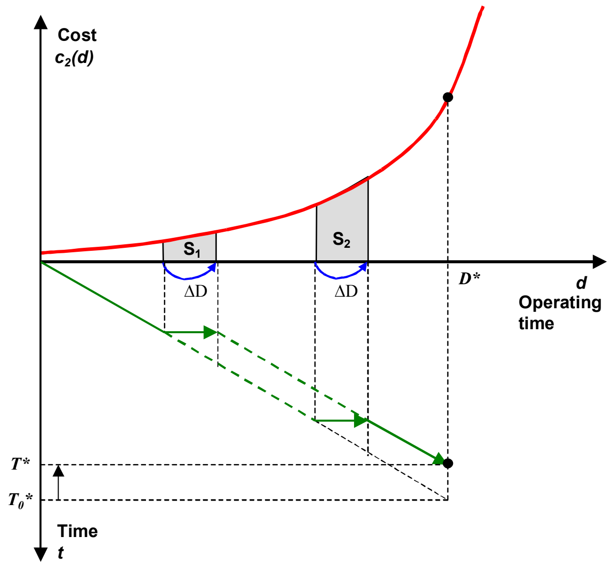

Figure 4 Relation between calendar and operating time [4]

Although the direct costs (shaded areas) associated by the starts differ, the service life reduction is equal for an "early" and a "late" start.

Furthermore, if the equivalent operating time ΔD is increasing with total operating time as in Eq. (17), there will be a further cost difference between the "early" and the "late" start-up. Thus, comparing a "late" start-up and an "early" start-up, the "late" start-up is more costly because of *i)* Greater direct costs (shaded areas) and *ii)* Greater indirect costs due to reduced service life.

5.2 Overall start-up cost model

In order to determine the optimal service life and the relation to start-up cost, we use a standard economic replacement methodology, assuming an indefinite chain of identical refurbishment intervals, each starting with a new refurbishment.

The first step is to establish the cost minimising strategy for future refurbishment. For our purpose it is sufficient to characterize the strategy by the present value or by the average discounted cost, usually denoted *Equivalent Annuity – EA*. Assuming that the minimal present value of all future costs related to refurbishment and operating costs are designated PV_{RC} and PV_{c2}, respectively, the *Equivalent Annuity EA* is calculated as $r\cdot(PV_{RC} + PV_{c2})$. These present values are valid at the moment of refurbishment. Hence, instead of modelling future costs by repeated refurbishment and operating cost cycles, the optimal future cost level is set equal to this *Equivalent Annuity*.

Refurbishment should thus take place when the cost $c_2(d)$ equals the *Equivalent Annuity.* This optimal moment of refurbishment corresponds to **G** in Fig. 5.

This standard result is used in Fig. 5. Without any new start-up, the cost $c_2(d)$ is increasing until T_F* when it equals *EA* and refurbishment is effected (at point **G**). Making an extra start-up as indicated in the figure, the direct cost given by the shaded area below the cost function is incurred. Note, however, that both the future operating costs and optimal moment of refurbishment shift closer to present time (to point **F**). Thus, total start-up cost is equal to the sum of the shaded and the hatched areas of Fig. 5.

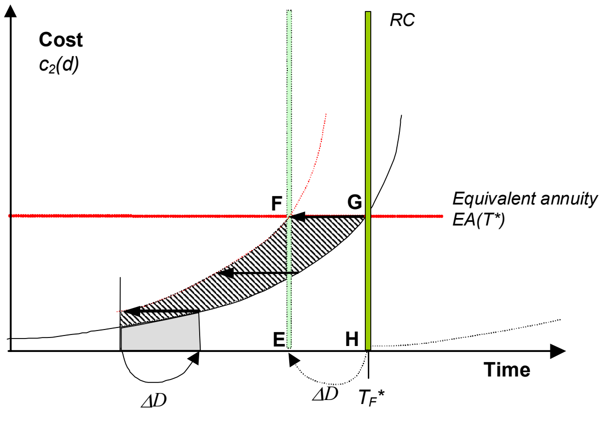

Figure 5 Cost increase due to one start [4]

By geometric observation, the size of the shaded and hatched areas in Fig. 5 is equal to the area of the rectangle *EFGH*. The exact shape of the function $c_2(d)$ is not known, thus we calculate the present value of the area *EFGH* instead. By doing so, we underestimate the start-up cost, as the present value of the start-up cost will be greater than the present value of the area *EFGH*.

The area of *EFGH* is $EA\cdot\Delta D$ as the height is *EA* and the width is ΔD. The calendar time to refurbishment is assumed to be $(T_F^* - \Delta D)$. Note that Fig. 5 is exaggerated to emphasize the different cost effects. ΔD is in reality very small compared to T_F*.

The following expression yields the present value of *EFGH*, and is then a lower limit for the marginal cost of one single start-up:

$$mc_{\min} = EA \cdot \Delta D \cdot e^{-rT_F^*}$$
$$= \left(PV_{RC} + PV_{C2}\right) \cdot r \cdot \Delta D \cdot e^{-rT_F^*} \text{ [USD/start]} \quad (18)$$

where $e^{-rT_F^*}$ - Discounting factor
PV_{RC} - Present value of all future refurbishments at the time for the first refurbishment
PV_{C2} - Present value of all future cost due to c_2 , at the time for the first refurbishment

So far we have assumed just one start-up. Under this assumption the shifting of the refurbishment timing was particularly easy to calculate. The equation also applies in a more general setting with a number of start-ups. However, the influence on the refurbishment timing will be less clear and will depend on start-up rate at the end of the unit service life (close to T_F^*).

Thus, three cost effects must be considered when calculating hydropower unit start-up costs:

1. The direct cost related to the start-up itself, equal to the shaded area under the cost function $c_2(d)$.
2. The increased future costs related to the cost function $c_2(d)$ as this shifts closer to present time.
3. The refurbishment cost *RC* as this shifts closer to present time.

Setting PV_{C2} equal to *0*, Eq. (18) is equivalent to Eq. (16) when ΔD is substituted by ΔLT. This implies, however, that both cost effects related to operating time $c_2(d)$ are neglected.

6. CONCLUSIONS

In this paper several models for calculation of hydropower generator start-up costs have been presented. All start-up costs are assumed to be either directly proportional to the number of starts or increasing with the number of operating hours of the unit. The former is independent of time and is not influenced by refurbishment or service life. Focusing on the latter, simplified models for both average and marginal start-up costs are developed and compared. As average or marginal start-up costs have relevance in different situations, both must be considered.

An improved model is also developed which takes into account both the direct start-up cost and the effect on the unit's refurbishment intervals. As for the simplified models, the start-up cost is assumed to increase with the number of operating hours of the unit. As each start is assumed to cause abrasion equivalent to a certain number of operating hours, however, additional costs are incurred as each start will reduce the remaining service life of the unit and thus shift the refurbishment intervals closer in time.

The cost function $c_2(d)$ is meant to represent all costs that are functions of operating time *d*. This includes also expected breakdown cost and loss of income due to more frequent failures as these increase with *d*. Other costs that are independent of operating time (*Model 1*) are not considered in this paper.

A hydropower unit start-up will influence the unit's service life and future refurbishment intervals. The exact formulation of this influence, however, is so far based on rather simplified assumptions. To enable a further improvement of these calculations, more complicated models must be developed which include several different assumptions and cost/time effects.

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