ADDRESSING NONLINEAR OBSERVABILITY ISSUES IN POWER SYSTEMS

Chris J. Dafis and C. O. Nwankpa
Center for Electric Power Engineering (CEPE)
Drexel University
Philadelphia, USA
cdafis@cbis.ece.drexel.edu, chika@nwankpa.ece.drexel.edu

Abstract – In recent years the power system industry has witnessed an increase of automation techniques in system operation. This development is especially evident with smaller systems, such as naval shipboard power systems. Traditionally, a nonlinear algebraic model of the power system is used to determine the system observability. In particular, the sensitivity of the system measurements (real and reactive power for example) to the change in the system states (bus voltages and angles) is used as a measure of observability, derived from the power system state-estimation problem. It ignores, however, the non-linear dynamics of the system related to generator performance, non-linear components, etc. The proposed observability formulation accounts for these nonlinearities providing a more comprehensive observability determination with the ability to track the trajectories of the dynamic variables over time and is derived from a DAE model of the power system. An example 3-bus power system is presented to illustrate the proposed method.

Keywords: Observability, DAE, Nonlinear Dynamics

1 INTRODUCTION

Traditionally, power system observability requires that enough measurements exist and they are distributed throughout the network, in such a way that the solution to the state estimation problem is possible. This focuses on the nonlinear algebraic model of the system, namely the power flow equations that describe the system interactions. These approaches are divided into two major areas: topological and numerical based observability formulations. These techniques have been successfully implemented in large systems to evaluate the observability of the system.

The topological approach, based on graph theory, focuses on determining the maximum spanning tree (a tree that contains every node in the network), of full rank, where each branch of the tree is assigned to a different measurement. With this configuration, all branch flows can be determined from the measurements, making the network observable (assuming the entire network is incorporated in the spanning tree). A more detailed description of topological observability formulations can be found in [1-4].

Numerical approaches center upon the evaluation of the Jacobian matrix. When the Jacobian matrix is of full-rank, the state estimation problem is solvable, and therefore the network is said to be observable. The Jacobian matrix represents the sensitivity of the available measurements to the states of the system. . A more detailed description of numerical observability formulations can be found in [5-9].

In summary, the existing observability formulations for power systems either evaluate the topology of the system using graph theoretic approaches, or focus on the determination of the state-estimation problem and are thus formulated with a non-linear algebraic equation description of the system. Therefore the non-linear dynamics of the system, related to the performance of the power generation source, for example, are neglected. With increases in system automation, it is desired to have a more detailed evaluation of the system observability as dynamic changes in the system occur. Currently the observability evaluations focus on the system being at an operating point, and evaluating whether this operating point is observable. To extend this evaluation, it would be useful to be able to track the system observability function for trajectories around these operating points, caused by small perturbations in the system. Therefore it is desired to incorporate these dynamics into an observability formulation that will enable us to track both the algebraic variables of the system, as well as the dynamic variables of the system. By including the non-linear dynamics of the system into our model, we form a Differential Algebraic Equation (DAE) model of the power system, which has a general form:

$$x' = f(x,u,Y) \tag{1.1}$$

$$0 = g(x,u,Y) \tag{1.2}$$

$$p = H(x,Y) \tag{1.3}$$

where *f(.)* is a set of non-linear differential equations (dependent on the generator model), *g(.)* is a set of non-linear algebraic equations (load flow mismatch equations), *H(.)* is the set of nonlinear algebraic equations related to the available measurements, p is the measurement vector, x is the set of dynamic state variables (generator phase angle δ, angular speed ω), and the set of static state variables (bus voltages V_i and phases θ_i), u is a set of independent control parameters and generator parameters, and Y represents the network parameters (bus admittance matrix - Y_{bus}). The observability formulation is derived by reforming Equations 1.a-1.c as:

$$F(x', x, Y) = u \tag{2.1}$$

$$p = H(x, N) \tag{2.2}$$

where *F(.)* is the set of functions relating to both the nonlinear dynamics and the non-linear algebraic equations of the system model (previously *f(.)* and *g(.)* in Equation 1.) In the power system application, the DAE system described by Equation 2 is of index 1 (the system can be decomposed to an equivalent ODE system with a single differentiation of the algebraic equations). The observability formulation, derived from Equation 2, in the special case of index 1 DAEs, is given in terms of the following Jacobian (J_O) [12]:

$$J_O = \begin{bmatrix} G_x & G_{x'} & G_{x''} \\ K_x & K_{x'} & K_{x''} \end{bmatrix} \tag{3.1}$$

where

$$G = \begin{bmatrix} F(x', x, u, Y) \\ F_x(x', x, u, Y)x' + F_{x'}(x', x, u, Y)x'' \end{bmatrix} \tag{3.2}$$

$$K = \begin{bmatrix} H(x, Y) \\ H_x(x, Y)x' \end{bmatrix} \tag{3.3}$$

and where F_x is the sensitivity of the nonlinear dynamic equations and nonlinear algebraic equations to the dynamic variables and algebraic variables of the system *x*, $F_{x'}$ is the sensitivity of *F(.)* to the derivatives of the dynamic and algebraic variables *x*, and H_x is the sensitivity of the measurement equations to the dynamic and algebraic variables *x*.

The system is considered observable, as long as the Jacobian in Equation 3, (J_O), has full rank. The benefit of this approach is the ability to evaluate an observability determination along the trajectory of the dynamic system states.

This paper is organized as follows: the problem formulation of the observability determination for a general *N*-bus system is developed in the Section 2; the formulation is further illustrated with a 3-bus power system example in Section 3, followed by the Conclusions section which includes the direction of future work on the topic.

2 PROBLEM FORMULATION

As mentioned, it is desired to derive an observability formulation including the non-linear dynamics of power systems. Overall, each observability determination follows the flow diagram of Figure 1, having four main elements:

- Network Processor – responsible for providing the power system network information to the observability formulation
- Meter Placement – responsible for placing meters in appropriate locations for observability
- Observability Formulation – the specific algorithm that determines the system observability
- Observability Determination – a qualitative analysis of the system observability – if the system is not observable what corrective actions can be taken

In the case of traditional topological observability, the network processor consists of a hierarchical graph of the power system, and the observability formulation is focused on determining the maximum spanning tree of full-rank. In the case of traditional numerical observability, the network information is in the form of the bus-admittance matrix, Ybus, and the observability formulation is based on the rank of the Jacobian matrix, defined by the state-estimation process.

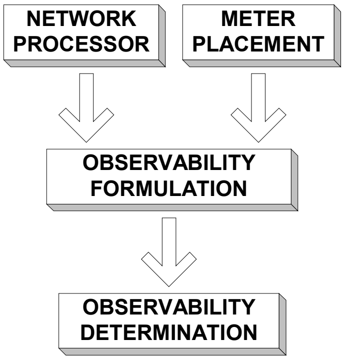

Figure 1: Outline of a Generic Power System Observability Formulation

In the proposed formulation, the network processor again is in the form of the bus admittance matrix, Ybus, however now the observability formulation includes the nonlinear dynamics of the power system, and is focused on the evaluation of the Jacobian, J_O, outlined in Equation 3. This paper focuses on the development of the observability formulation portion of the power system observability problem. The issues arising from meter placement have in the past been based on the particular observability formulations used (increase the measurement set to ensure that the Jacobian is of full-rank in the traditional numerical observability methods). Clearly the set of initial measurements affects the observability formulation. However, the problem of meter number and type or optimal meter placement is not addressed within this observability formulation. Therefore it is necessary to now examine our power system model.

Starting with the nonlinear system dynamics, the behavior of the generation equipment, in an electromechanical sense can be modeled by the swing equation:

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i + P_{G_i}(\delta_i) = P_{M_i}(\dot{\delta}_i) \tag{4}$$

where δ_i is the mechanical rotor angle of the rotating machine, M_i is the inertia constant of the generator, D_i is the damping constant of the generator, *i* is the system bus index, ranging from 1…*n* for the *n* generator buses in the system, and P_{Gi} is the per phase electrical power supplied by the generator given by:

$$P_{G_i}(\theta_i)=\sum_{j=1}^{m}|V_i||V_j|\left[B_{ij}\sin(\theta_i-\theta_j)+G_{ij}\cos(\theta_i-\theta_j)\right] \tag{5.1}$$

where G_{ij} and B_{ij} are the conductance and the susceptance between buses i and j respectively, θ_i is the phase of voltage V_i at bus i, m is the total number of buses in the system, and P_{mi} is the mechanical power for the i^{th} generator, given as:

$$P_{M_i}(\dot{\theta}_i)=P^0_{M_i}+\omega_i k_i \tag{5.2}$$

where k_i is a constant and P^0 is an initial value of the mechanical input. This expression for the mechanical input incorporates the speed control of the machine, and is assumed to be the same for each generator, therefore k_i=k for all i=1…n generators in the system. This assumption is made to simplify the analysis but can be removed in the future without any change in the proposed observability formulation.

Simplifying to a first order system, and using generator bus n as the reference bus we have:

$$\tilde{f}_1=\omega_i \tag{6.1}$$

$$\tilde{f}_2=\dot{\omega}_i=-\frac{D_i}{M_i}(\omega_i)+\frac{1}{M_i}\left(P_{M_i}(\omega_i)-P_{G_i}(\theta)\right) \tag{6.2}$$

where the assumption was made that all generators in the system are identical, and therefore the inertia constant M_i along with the damping constant, D_i of each generator is the same value. Again, as in the case of the identical input mechanical power model for all generators, this assumption, is made to simplify the initial analysis or the system, but can be relaxed in the future without compromising the formulation of the problem.

Next, considering the nonlinear algebraic equations of the power system model, we have the traditional power flow equations based on the real and reactive power calculations at each load bus of the network, namely:

$$\begin{aligned}\tilde{f}_3&=0=P_i-P_i^k(\theta,V)\\&=P_i-\sum_{j=1}^{m}|V_i^k||V_j^k|\left[B_{ij}\sin(\theta_i^k-\theta_j^k)+G_{ij}\cos(\theta_i^k-\theta_j^k)\right]\\\tilde{f}_4&=0=Q_i-Q_i^k(\theta,V)\\&=Q_i-\sum_{j=1}^{m}|V_i^k||V_j^k|\left[-B_{ij}\cos(\theta_i^k-\theta_j^k)-G_{ij}\sin(\theta_i^k-\theta_j^k)\right]\end{aligned} \tag{7}$$

where i=n+1…m load buses, P_i and Q_i are constants representing the numerical solution of for the real and reactive power when the power flow solution is satisfied, and k is the iterate of the power flow solution. Notice that the structure preserving power system model is selected [11].

Suppose now we introduce the following notations:

$$x=[\overbrace{\underbrace{(\theta_1-\theta_n)\cdots(\theta_{n-1}-\theta_n)}_{generator\ angles}}^{1\cdots n-1},\ \overbrace{\underbrace{\omega_1\ldots\omega_{n-1}}_{generator\ speeds}}^{n\cdots 2(n-1)}, \tag{8}$$

$$\overbrace{\underbrace{V_{n+1}\cdots V_{n+m}}_{\substack{load-bus\\ voltages}}}^{2n-1\cdots 2n+m-2},\overbrace{\underbrace{(\theta_{n+1}-\theta_n)\cdots(\theta_{n+m}-\theta_n)}_{load-bus\ angles}}^{2n-m-1\cdots 2(m+n-1)}]^T$$

$$u=\frac{P^0_{M_i}}{M_i}\quad for\quad i=1...n\ \ generator\ \ buses$$

$$Y=G_{ij},B_{ij}\quad for\ i,j=1...m+n\ \ buses$$

we will have a system of equations in the form of $F(.)$ in Equation 2.1, namely:

$$\begin{aligned}&f_1=\dot{x}_i-x_{n+i-1}=0\quad for\ i=1\cdots n\text{-}1\\&f_2=\dot{x}_i+\frac{D}{M}x_i+\frac{1}{M}P_{G_i}(x)-kx_i=\frac{P^0_{M_i}}{M}\\&\qquad\qquad for\ i=n\cdots 2(n\text{-}1)\\&f_3=P_i-P_i(x)=0\quad for\ i=n+1\cdots m\ \ load\ buses\\&f_4=Q_i-Q_i(x)=0\quad for\ i=n+1\cdots m\ \ load\ buses\end{aligned} \tag{9}$$

The last element of the model corresponds to the available measurements of the system. These equations, like the power flow equations, are nonlinear algebraic relationships between the voltage and phase variables (algebraic variables) of the system. For this formulation, we assume that the available measurements are either:

- real and reactive power measurements at load buses
- real power branch measurements

The measurement equations have the form:

$$\begin{gathered}p=\left[z_1\ \vdots\ z_2\right]^T\\z_1=\begin{bmatrix}P_1 & P_2 & \cdots & P_p & Q_1 & Q_2 & \cdots & Q_p\end{bmatrix}^{\mathrm{T}}\\z_2=\begin{bmatrix}P_1 & P_2 & \cdots & P_r\end{bmatrix}^T\end{gathered} \tag{10}$$

where z_1 corresponds to the vector of load buses with available P-Q measurements (2p such measurements), and z_2 corresponds to the vector of branch power flow measurements (r such measurements). The measurement vector p corresponds to the measurement vector in Equation 2.2. Therefore the model of the system, as described in Equation 2 is complete.

The observability determination follows from this model, with the evaluation of the Jacobian (J_O), as outlined in Equation 3.1. The main elements of Equation 3.2 and 3.3 consist of F_x, $F_{x'}$, and H_x. Starting with F_x, which is basically the Jacobian of F with respect to x:

$$F_x=\left[F_1\,\vdots\,F_2\,\vdots\,F_3\,\vdots\,F_4\right]^T \tag{11.1}$$

where

$$F_1=\left[\,0_{(n-1)x(n-1)}\ \vdots\ -I_{(n-1)x(n-1)}\ \vdots\ 0_{(n-1)x2m}\,\right] \tag{11.2}$$

$$F_2=\left[\begin{array}{c:c:c}\underbrace{\frac{\partial f_2}{\partial x_1}\cdots\frac{\partial f_2}{\partial x_{n-1}}}_{(n-1)x(n-1)} & \beta I_{(n-1)x(n-1)} & \underbrace{\frac{\partial f_2}{\partial x_{2n-1}}\cdots\frac{\partial f_2}{\partial x_{2(n+m-1)}}}_{(n-1)x2m}\end{array}\right]$$

$$F_3=\left[\begin{array}{c:c:c}\underbrace{\frac{\partial f_3}{\partial x_1}\cdots\frac{\partial f_3}{\partial x_{n-1}}}_{mx(n-1)} & 0_{(n-1)x(n-1)} & \underbrace{\frac{\partial f_3}{\partial x_{2n-1}}\cdots\frac{\partial f_3}{\partial x_{2(n+m-1)}}}_{mx2m}\end{array}\right]$$

$$F_3 = \left[\underbrace{\frac{\partial f_4}{\partial x_1} \cdots \frac{\partial f_4}{\partial x_{n-1}}}_{mx(n-1)} \;\vdots\; 0_{(n-1)x(n-1)} \;\vdots\; \underbrace{\frac{\partial f_4}{\partial x_{2n-1}} \cdots \frac{\partial f_4}{\partial x_{2(n+m-1)}}}_{mx2m} \right]$$

$$\beta = \frac{D}{M} - \frac{1}{M} k$$

where I is the identity matrix, and β is a constant. Next, we examine the evaluation of $F_{x'}$, which is expressed as:

$$F_{x'} = \left[\hat{F}_1 \;\vdots\; \hat{F}_2 \;\vdots\; \hat{F}_3 \;\vdots\; \hat{F}_4 \right]^T \tag{11.1}$$

$$\hat{F}_1 = \left[I_{(n-1)x(n-1)} \;\vdots\; 0_{(n-1)x(n-1)} \;\vdots\; 0_{(n-1)x2m} \right] \tag{11.2}$$

$$\hat{F}_2 = \left[I_{(n-1)x(n-1)} \;\vdots\; 0_{(n-1)x(n-1)} \;\vdots\; 0_{(n-1)x2m} \right]$$

$$\hat{F}_3 = \left[0_{(n-1)x2(m+n-1)} \right]$$

$$\hat{F}_4 = \left[0_{(n-1)x2(m+n-1)} \right]$$

The remaining element for the Jacobian J_o evaluation is the H_x element, which is expressed as:

$$H_x = \left[H_1 \;\vdots\; H_2 \right]^T \tag{12.1}$$

$$H_1 = \left[\underbrace{\frac{\partial z_1}{\partial x_1} \cdots \frac{\partial z_1}{\partial x_{n-1}}}_{(2p)x(n-1)} \;\vdots\; 0_{(2p)x(n-1)} \;\vdots\; \underbrace{\frac{\partial z_1}{\partial x_{2(n-1)}} \cdots \frac{\partial z_1}{\partial x_{2(n+m-1)}}}_{(2p)x(n-1)} \right] \tag{12.2}$$

$$H_2 = \left[\underbrace{\frac{\partial z_2}{\partial x_1} \cdots \frac{\partial z_2}{\partial x_{n-1}}}_{r\ x\ (n-1)} \;\vdots\; 0_{(r)x(n-1)} \;\vdots\; \underbrace{\frac{\partial z_2}{\partial x_{2(n-1)}} \cdots \frac{\partial z_2}{\partial x_{2(n+m+1)}}}_{r\ x\ 2m} \right]$$

To better illustrate the development of the observability Jacobian, J_o, a generic 3-bus power system is evaluated in the following section. This illustration also focuses on the development of Equations 3.2-3.3. Because in this system we have two generators, we have one degree of freedom in the model, in terms of dynamic variables. Therefore, the applied perturbations are on a single variable. Future cases, where systems are larger, will provide more degrees of freedom, with multiple perturbations of the dynamic states and illustrate the ability to track all the dynamic state trajectories.

3 ILLUSTRATIVE EXAMPLE

To illustrate the observability formulation of a DAE model, a 3-bus power system depicted in Figure 2 was selected, where GS represents a generator, R and L represent the resistance and inductance of the lines respectively, and P and Q represent the real and reactive power respectively. The equations of the system can be expressed as :

$$f_1 = \dot{x}_1 - x = 0 \tag{13}$$

$$f_2 = \dot{x}_2 + \frac{D_1}{M_1} x_2 + \frac{1}{M_1} P_{G_1}(x) - \frac{1}{M_1} k x_2 = \frac{1}{M_1} P_{M_1}^0 \tag{14}$$

where

$$x_1 = \theta_1, x_2 = \omega_1, x_3 = V_3, x_4 = \theta_3$$

and

$$\begin{aligned} f_3 &= P_3 - P_3(x) = 0 \\ &= P_3 - |x_3||V_1|\left[B_{31}\sin(x_4 - x_1) + G_{31}\cos(x_4 - x_1)\right] + \\ &\quad |x_3||V_2|\left[B_{32}\sin(x_4) + G_{32}\cos(x_4)\right] + |x_3|^2 G_{33} \end{aligned} \tag{15}$$

$$\begin{aligned} f_4 &= Q_3 - Q_3(x) = 0 \\ &= Q_3 - |x_3||V_1|\left[-B_{31}\cos(x_4 - x_1) + G_{31}\sin(x_4 - x_1)\right] + \\ &\quad |x_3||V_2|\left[-B_{32}\cos(x_4) + G_{32}\sin(x_4)\right] - |x_3|^2 B_{33} \end{aligned} \tag{16}$$

where bus-2 is taken as the reference bus.

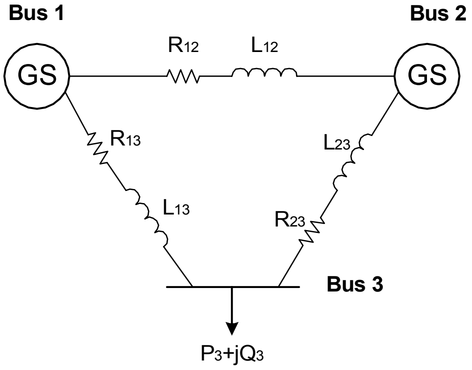

Figure 2: 3-Bus Power System Example

For this particular example, the system parameters used are given in Table 1.

Table 1: 3-bus System Parameter Data

	Line 1-2	Line 1-3	Line 2-3
R	0.0194	0.054	0.047
X	0.0592	0.223	0.198
	Bus 1	**Bus 2**	**Bus 3**
V	1.06	1.045	1.0099
θ	-0.00114	0	-0.14431
P	0.183	2.324	-1.442
Q	0.297	-0.169	0.044
	GS 1	**GS 2**	
M	1	1	
D	0.3	0.3	
k	0.005	0.005	

To check if it is possible to observe the dynamic state trajectories, we examine the upper half of the Jacobian matrix J_o in Equation 3.1, and from the associated deficiency, add the appropriate number of measurements to make the Jacobian J_o full-rank. The dynamic trajectories in this case are the trajectories for x_1 and x_2. We arbitrarily perturb the dynamic state variable x_1 from an initial operating point (given in Table 1), by 10% to obtain a trajectory of the state as it settles back to the initial operating point. A sample trajectory of x_1 is provided in Figure 3.

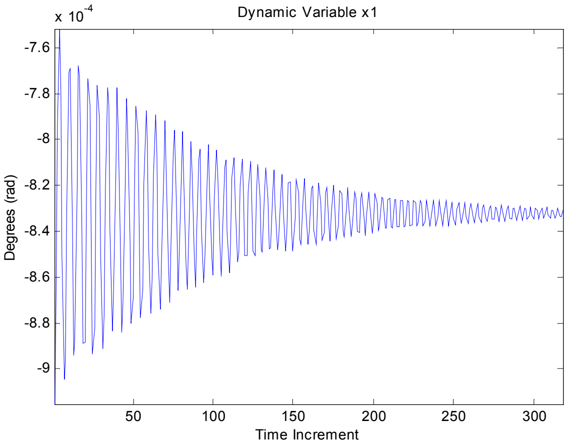

Figure 3: Sample Trajectory of a System Dynamic Variable

Evaluating the upper half of the Jacobian matrix J_o in Equation 3.1, namely the G entries, we notice that when the dynamic variable x_2 is exactly zero, there is a deficiency in the rank, namely rank=7 (whereas full-rank in this case would be rank=8). An example of which is illustrated below. Given x_2=0, the evaluation of the G entries of the Jacobian J_o, provide a rank-deficient matrix such as:

$$
\begin{bmatrix} \mathrm{G}_{\mathrm{x}} & \mathrm{G}_{\mathrm{x}'} & \mathrm{G}_{\mathrm{x}''} \end{bmatrix} =
\begin{bmatrix}
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
-12.6279 & .2950 & 5.7965 & -4.2524 & 0 & 1 & 0 & 0 \\
5.7970 & 0 & 10.1767 & -10.8637 & 0 & 0 & 0 & 0 \\
-4.338 & 0 & 7.9698 & 8.1145 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -12.6279 & .2950 & 0 & 1 \\
0 & 0 & 0 & 0 & 5.7970 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -4.3338 & 0 & 0 & 0
\end{bmatrix}
$$

In order to maintain full-rank in the evaluation of J_o, it is necessary to add a measurement that will allow you to track the trajectory of x_2. Therefore the branch flow measurement P_{23} is added, providing a full Jacobian J_o, which is full-rank (rank=8):

$$
\begin{bmatrix} G_x & G_{x'} & G_{x''} \\ K_x & K_{x'} & K_{x''} \end{bmatrix} =
\begin{bmatrix}
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
-12.6279 & .2950 & 5.7965 & -4.2524 & 0 & 1 & 0 & 0 \\
5.7970 & 0 & 10.1767 & -10.8637 & 0 & 0 & 0 & 0 \\
-4.3338 & 0 & 7.9698 & 8.1145 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -12.6279 & .2950 & 0 & 1 \\
0 & 0 & 0 & 0 & 5.790 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -4.3338 & 0 & 0 & 0 \\
4.3135 & 0 & 5.8798 & -4.3135 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4.3135 & 0 & 0 & 0
\end{bmatrix}
$$

Therefore, in order to track the dynamic state trajectory x_2 it is necessary to have the branch flow measurement present. Notice that the rank of the G entries for the Jacobian J_o are full-rank when the dynamic variable x_2 is not zero, which indicates that any measurement set would allow you to track the trajectory of the state. However, when the trajectory passes zero, it is necessary to have the branch flow measurement present in order to observe the state trajectory.

This example illustrates the ability to track the observability of the system, about the system state trajectories, and illustrates the significance of the required measurements in the system to successfully accomplish this task.

4 CONCLUSIONS

This paper focused on the formulation of an observability Jacobian that includes the nonlinear behavior of the system and the ability to track the trajectories of the dynamic states during nonlinear behavior. The formulation is illustrated with a 3-bus system example. Future results will focus on systems of larger size, and incorporate load dynamics.

5 ACKNOWLEDGEMENTS

The authors would like to thank the In-House Laboratory Independent Research Program (ILIR) for their funding and support, thru the Office of Naval Research (ONR).

6 REFERENCES

[1] K. A. Clements and B. F. Wollenberg, "An Algorithm for Observability Determination in Power System State Estimation", IEEE PES Summer Meeting, Paper A75447-3, San Francisco, July 1975.

[2] K. A. Clements, et al., "State Estimator Measurement System Reliability Evaluation – an Efficient Algorithm Based on Topological Observability Theory,", IEEE Transactions on Power App. and Systems, Vol. PAS-101, No. 4, pp.997-1004, April 1982.

[3] K. A. Clements, et al., "Power System State Estimation Residual Analysis: an Algorithm Using Network Topology", IEEE Transactions on Power App. and Systems, Vol. PAS-100, No. 4, pp.1779-1787, April 1987.

[4] H. Mori and Y. Sone, "Tabu Search Based Meter Placement for Topological Observability in Power System State Estimation", IEEE Document # 0-7803-5515-6, 1999.

[5] A. Monticelli and F. F. Wu, "Network Observability: Theory", IEEE Transactions on Power App. and Sys., Vol. PAS-104, No. 5, pp. 1042-1048, May 1985

[6] A. Abur and A. Exposito, "Bad Data Identification when Using Ampere Measurements", IEEE Transactions on Power Systems, Vol.12, No.2, pp. 491-500, May 1997

[7] F. F. Wu et al., "Observability Analysis and Bad Data Processing for State Estimation with Equality Constraints", IEEE Transactions on Power Systems, Vol. 3, No. 2, pp. 541-548, May 1988

[8] A. Exposito and A. Abur, , "Generalized Observability Analysis and Measurement Classification", IEEE Transactions on Power Systems, Vol.13, No.3, pp. 1090-1095, August 1998.

[9] O. Alsac, N. Vempati, B. Scott, A. Monticelli (1998): "Generalized State Estimation", IEEE Transactions on Power Systems, Vol. 13, No.3, pp.1069-1075

[10] G. Chartrand, Introductory Graph Theory, Dover Publications, New York, 1985.

[11] A. R. Bergen, D. J. Hill, “Structure Preserving Model for Power System Stability Analysis”, IEEE Transactions on Power Appar. and Syst., PAS-100, 1, January 1981

[12] W. Terrell, “Observability of Nonlinear Differential Algebraic Systems”, Circuits, Systems and Signal Processing, Vol. 16, No. 2, pp. 271-285, 1997