

# Methodologies for Loss and Line Flow Allocation under a Pool-Bilateral Market

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**Abstract:** This work proposes methodologies to allocate losses and line flows to the participants of a Pool-Bilateral energy market. The allocation schemes are based on the results provided by an optimal power dispatch model that, using the Superposition Principle (SP), represents each transaction and the power traded with the Pool (spot market) by an individual set of network equations. The model provides the optimal values of the voltages and line currents associated with every system user. These quantities are used to allocate line flows and losses to the transactions and the Pool. The results obtained for loss allocation are translated into bus allocation factors and compared with the results provided by two previous works.

**Keywords:** Line Flow Allocation, Loss Allocation, Optimal Power Dispatch, Pool-Bilateral Markets.

## 1. INTRODUCTION

The restructuring of the electric power systems has brought about a redefinition of the obligations, roles and practices of the traditional industry, also creating different energy market structures. Under Pool-Bilateral markets, among the independent system variables are the power transactions. They influence the behavior of the power system, the transmission losses and the line flows. Transmission losses form a significant component of power that has to be generated in order to meet the demand, whereas transmission congestion affects the bus incremental costs, thus influencing the price of energy. Since the incurred additional costs must be divided among the network users, there is a need to allocate losses and line flows to those users in a systematic manner and several allocation schemes have been proposed. These methodologies can be divided into two groups: those that allocate the transmission losses to the buses (generators and loads) of the system and those that allocate them among the transactions.

A number of proposals for allocating transmission losses among the system buses are based on approximations of the load flow equations. Kirschen *et al.* [1] and Bialeck [2] express the power flows in terms of the generation levels based on the supposition that the power injections are proportionally shared among the outflows of each network bus. The methodology proposed by the Brazilian System Operator (ONS) [3] uses a linear approximation of the line losses. Loss factors are calculated for every bus by dividing the line losses between generator and load in the ratio of 50%.

Researchers have also developed loss allocation methods without recourse to approximations. One of such methods is presented in [4]. It is based on the Bus Impedance Matrix and on the current injections.

The first transaction loss allocation scheme was proposed in 1995 [5] and is based on quadratic approximations of the networks equations. Two years later, [6] presented a methodology for line flow allocation based on formulae for computing the contribution of each transaction to the line flows. Shortly after, Gross and Tao [7] proposed a methodology for loss allocation where the losses are expressed analytically in terms of the transactions in a quadratic function. Although this approach yields negative transaction losses, only the absolute values of the transactions losses are used and the allocations are normalized. Recently, [8] presented a loss allocation method based on the resolution of differential equations stated in terms of the transactions. The method allocates negative losses and also considers the case where the losses allocated to a particular transaction are supplied by a generator not participating in such transaction.

The consequences of different loss allocation criteria were also analyzed. In the work by Exposito *et al.* [9], this analysis was made using the DC network model.

Methodologies for allocating general transmission services based on the load flow equations were also proposed [10,11,12]. The first work starts the allocation procedure by running two load flows: one with no transaction and the other with all transactions in the system. Next, to allocate services to transaction  $t$ , two more load flows are run: one with only such transaction in the network and other with all transactions except  $t$ . The incremental components of line flows, power losses, reactive support, etc., are obtained from these load flow cases. The second approach allocates transmission services, among a number of transactions, according to the average sensitivities of the service functions obtained considering one and all transactions in the system. Finally, in the last approach, the allocation is performed through a process of integration of the first derivative of the function representing the transmission service considered.

This work aims to contribute to the loss and line flow allocation studies. It proposes allocation schemes based on the results of an optimization model for the power dispatch under a Pool-Bilateral market [13]. In this model, using the Superposition Principle (SP), each transaction, and also the power traded in the spot market, is represented by an individual set of network equations. The model provides the optimal values of the voltages and line currents associated with every system user (that is, a transaction or the Pool). These are used for allocating line flows and transmission losses to the

users. The results obtained by the loss allocation scheme are translated into bus loss allocation factors which are compared with those provided by [3] and [4] for a 5-bus and the IEEE test systems with 14 and 118 buses.

The paper is organized as follows. First of all, the network equations associated with the transactions and to the Pool (spot market) are derived. Next, the optimal power dispatch model is described. Subsequently, the expressions of the line flows and transmission losses allocated to every transaction and to the Pool are derived. Tests results are then presented and, finally, some conclusions are put forward.

## 2. POWER BALANCE EQUATIONS [13]

The power transactions and also the power injections associated with the Pool define a set of current injections in the system. Thus, a particular bilateral or multilateral transaction, or even the Pool, can be regarded as *an individual participant* of the transmission system, which has a set of current sources (generators) and a set of current sinks (loads). According to the SP: "The current or the voltage across the terminals of a linear circuit's component is equal to the algebraic addition of the currents and voltages independently produced by each one of the sources". Using this principle, the total current injections can be separated into a set of individual circuits, one for each participant, where, with the exception of its own current sources and sinks, all the others are represented by open circuits. Thus, for a system with  $nb$  buses, to each participant  $k$  is assigned a vector of current injections,  $\dot{\mathbf{I}}^k$ , and a vector of bus voltages,  $\dot{\mathbf{V}}^k$ . The total voltages and current injections of any bus are obtained by adding the corresponding values of the individual voltages and current injections. The vector of total power injections,  $\dot{\mathbf{S}}^{total}$ , is then

$$\dot{\mathbf{S}}^{total} = \text{diag}(\dot{\mathbf{V}}^{total}) \dot{\mathbf{I}}^{total*} \quad (1)$$

where,  $\dot{\mathbf{V}}^{total}$  and  $\dot{\mathbf{I}}^{total}$  are the  $(nb \times 1)$  vectors of total voltages and current injections, respectively.

Supposing that there are  $ntr+1$  participants in the power market ( $ntr$  transactions and the spot market) equation (1) can be rewritten as:

$$\dot{\mathbf{S}}^{total} = \text{diag}(\dot{\mathbf{V}}^1 + \dots + \dot{\mathbf{V}}^{ntr} + \dot{\mathbf{V}}^{Pool}) (\dot{\mathbf{I}}^1 + \dots + \dot{\mathbf{I}}^{ntr} + \dot{\mathbf{I}}^{Pool})^* \quad (2)$$

Equation (2) expresses the total bus power injections in terms of the voltages and current injections associated with each network user. Due to its non-linearity, it cannot be decomposed into a unique sum of individual power injections, each one associated with a particular user. However, if the decomposition is made based on the current injection, we can express the power injection due to participant  $k$  as

$$\dot{\mathbf{S}}^k = \text{diag}(\dot{\mathbf{V}}^{total}) (\dot{\mathbf{I}}^k)^* = \text{diag}(\dot{\mathbf{V}}^{total}) (\dot{\mathbf{Y}} \dot{\mathbf{V}}^k)^* \quad (3)$$

where  $\dot{\mathbf{Y}}$  is the Bus Admittance Matrix  $(nb \times nb)$  and  $\text{diag}(\dot{\mathbf{V}}^{total})$  is a diagonal matrix composed of  $\dot{\mathbf{V}}^{total}$ .

A bilateral or multilateral transaction is defined by the power injected into a set of buses  $A = \{i, j, \dots, nb\}$ . Thus, for such participant, equation (3) can be written as

$$\begin{bmatrix} 0 \\ \vdots \\ \dot{S}_i^A - \dot{S}_i^A \\ \vdots \\ \dot{S}_j^A \\ \vdots \\ -\dot{S}_{nb}^A \end{bmatrix} = \text{diag} \begin{bmatrix} \dot{V}_1^{total} \\ \vdots \\ \dot{V}_i^{total} \\ \vdots \\ \dot{V}_j^{total} \\ \vdots \\ \dot{V}_{nb}^{total} \end{bmatrix} \dot{\mathbf{Y}}^* \begin{bmatrix} \dot{V}_1^A \\ \vdots \\ \dot{V}_i^A \\ \vdots \\ \dot{V}_j^A \\ \vdots \\ \dot{V}_{nb}^A \end{bmatrix} \quad (4)$$

where  $\dot{V}_n^A$  is the voltage of bus  $n$ , due to transaction  $A$ .

The current injections associated with the Pool are due to the power traded in the spot market. The Pool can be regarded as a multilateral transaction comprising the entire system, which supplies the loads at the spot market price and, in case all transactions are firm, also the losses. Its power balance equations are:

$$\begin{bmatrix} -\dot{S}_1^{Pool} \\ \vdots \\ \dot{S}_i^{Pool} - \dot{S}_i^{Pool} \\ \vdots \\ \dot{S}_j^{Pool} - \dot{S}_j^{Pool} \\ \vdots \\ -\dot{S}_{nb}^{Pool} \end{bmatrix} = \text{diag} \begin{bmatrix} \dot{V}_1^{total} \\ \vdots \\ \dot{V}_i^{total} \\ \vdots \\ \dot{V}_j^{total} \\ \vdots \\ \dot{V}_{nb}^{total} \end{bmatrix} \dot{\mathbf{Y}}^* \begin{bmatrix} \dot{V}_1^{Pool} \\ \vdots \\ \dot{V}_i^{Pool} \\ \vdots \\ \dot{V}_j^{Pool} \\ \vdots \\ \dot{V}_{nb}^{Pool} \end{bmatrix} \quad (5)$$

Thus, the complete set of power balance equations is:

$$\dot{\mathbf{S}} = \text{diag} \begin{bmatrix} \dot{V}_1^{total} \\ \vdots \\ \dot{V}_{nb}^{total} \\ \vdots \\ \dot{V}_1^{total} \\ \vdots \\ \dot{V}_{nb}^{total} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{Y}}^* & & & \\ & \ddots & & \\ & & \dot{\mathbf{Y}}^* & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \dot{V}_1^1 \\ \vdots \\ \dot{V}_{nb}^1 \\ \vdots \\ \dot{V}_1^{Pool} \\ \vdots \\ \dot{V}_{nb}^{Pool} \end{bmatrix} \quad (6)$$

where

$$\dot{\mathbf{S}} = \mathbf{P} + j\mathbf{Q} = \begin{bmatrix} \dot{S}_1^1 & \dots & \dot{S}_1^{nb} & \dots & \dot{S}_1^{ntr} \\ \dots & \dot{S}_{nb}^{ntr} & \dot{S}_1^{Pool} & \dots & \dot{S}_{nb}^{Pool} \end{bmatrix}^T$$

with  $\dot{\mathbf{S}}$ ,  $\mathbf{P}$  and  $\mathbf{Q}$  being the  $[nb(ntr+1) \times 1]$  vectors of complex, active and reactive power injections, respectively.

Using rectangular coordinates, let vector  $\mathbf{x}$  be

$$\mathbf{x} = \left[ (\mathbf{e}^1)^T \dots (\mathbf{e}^{ntr})^T (\mathbf{e}^{Pool})^T (\mathbf{f}^1)^T \dots (\mathbf{f}^{ntr})^T (\mathbf{f}^{Pool})^T \right]^T$$

where  $\mathbf{e}^k = [e_1^k, e_2^k, \dots, e_{nb}^k]^T$ ,  $\mathbf{f}^k = [f_1^k, f_2^k, \dots, f_{nb}^k]^T$  for  $e_i^k$  and  $f_i^k$  being the real and imaginary components of  $\dot{V}_i^k$ . If all the transformer tap settings and phase shifter angles are fixed, equation (6) can be rewritten as two real quadratic equations in  $\mathbf{x}$ :

$$\mathbf{Pg} - \mathbf{Pd} = \begin{bmatrix} \mathbf{Pg}_1^1 - \mathbf{Pd}_1^1 \\ \vdots \\ \mathbf{Pg}_{nb}^1 - \mathbf{Pd}_{nb}^1 \\ \vdots \\ \mathbf{Pg}_1^{Pool} - \mathbf{Pd}_1^{Pool} \\ \vdots \\ \mathbf{Pg}_{nb}^{Pool} - \mathbf{Pd}_{nb}^{Pool} \end{bmatrix} = \begin{bmatrix} \mathbf{x}^T \mathbf{Hp}_1^1 \\ \vdots \\ \mathbf{x}^T \mathbf{Hp}_{nb}^1 \\ \vdots \\ \mathbf{x}^T \mathbf{Hp}_1^{Pool} \\ \vdots \\ \mathbf{x}^T \mathbf{Hp}_{nb}^{Pool} \end{bmatrix} \mathbf{x} = \mathbf{P}(\mathbf{x})\mathbf{x} \quad (7)$$

$$\mathbf{Qg} - \mathbf{Qd} = \begin{bmatrix} \mathbf{Qg}_1^1 - \mathbf{Qd}_1^1 \\ \vdots \\ \mathbf{Qg}_{nb}^1 - \mathbf{Qd}_{nb}^1 \\ \vdots \\ \mathbf{Qg}_1^{Pool} - \mathbf{Qd}_1^{Pool} \\ \vdots \\ \mathbf{Qg}_{nb}^{Pool} - \mathbf{Qd}_{nb}^{Pool} \end{bmatrix} = \begin{bmatrix} \mathbf{x}^T \mathbf{Hq}_1^1 \\ \vdots \\ \mathbf{x}^T \mathbf{Hq}_{nb}^1 \\ \vdots \\ \mathbf{x}^T \mathbf{Hq}_1^{Pool} \\ \vdots \\ \mathbf{x}^T \mathbf{Hq}_{nb}^{Pool} \end{bmatrix} \mathbf{x} = \mathbf{Q}(\mathbf{x})\mathbf{x} \quad (8)$$

where  $\mathbf{Pg}$ ,  $\mathbf{Qg}$ , are vectors composed of the real and reactive generations of all participants,  $\mathbf{Pd}$  and  $\mathbf{Qd}$  are vectors composed of all the loads, and  $\mathbf{Hp}_i^k$  and  $\mathbf{Hq}_i^k$  are symmetric constant matrices defined by the network structure, bus  $i$  and participant  $k$ .

### 3. OPTIMAL POWER DISPATCH MODEL [13]

Using equations (7) and (8), an optimization problem can be formulated to dispatch the power associated with the transactions and the spot market. The optimal power dispatch (OPD) problem is written

$$\min F = w_1 c(\mathbf{U}_g \mathbf{Pg}) + w_2 \mathbf{u}^T \mathbf{Pg} + 0.5 w_3 \left\{ \left\| \mathbf{U}_t \mathbf{Pg} - \mathbf{Pg}^p \right\|^2 + \left\| \mathbf{U}_t \mathbf{Qg} - \mathbf{Qg}^p \right\|^2 \right\} \quad (9)$$

subject to

$$\mathbf{Pg} - \mathbf{Pd} = \mathbf{P}(\mathbf{x})\mathbf{x} \quad (10)$$

$$\mathbf{Qg} - \mathbf{Qd} = \mathbf{Q}(\mathbf{x})\mathbf{x} \quad (11)$$

$$\mathbf{u}_{ref}^T \mathbf{x} = 0 \quad (12)$$

$$\mathbf{Gp}^{\min} \leq \mathbf{U}_g \mathbf{Pg} \leq \mathbf{Gp}^{\max} \quad (13)$$

$$\mathbf{Gq}^{\min} \leq \mathbf{U}_g \mathbf{Qg} \leq \mathbf{Gq}^{\max} \quad (14)$$

$$\mathbf{0} \leq \mathbf{Pg} \quad (15)$$

$$\mathbf{Qg}^{\min} \leq \mathbf{Qg} \quad (16)$$

$$\mathbf{V}^{\min} \leq \mathbf{V}(\mathbf{x})\mathbf{x} \leq \mathbf{V}^{\max} \quad (17)$$

where

$\mathbf{x}$	vector of voltage components;
$c(\cdot)$ :	total generation cost;
$w_1, w_2, w_3$ :	weighting factors associated with the generation cost, transmission losses and the deviation from proposed values of the transactions, respectively;
$\mathbf{u}$ :	unit vector;
$\mathbf{U}_t$ :	incidence matrix that extracts from $\mathbf{Pg}$ and $\mathbf{Qg}$ the power transactions' components;
$\mathbf{U}_g$ :	incidence matrix that associates the $\mathbf{Pg}$ and $\mathbf{Qg}$ components to the generator buses;
$\mathbf{Pg}^p, \mathbf{Qg}^p$ :	proposed values of the transactions;
$\mathbf{P}(\mathbf{x}), \mathbf{Q}(\mathbf{x})$ :	matrix functions used to represent the active and reactive power injections;

$\mathbf{u}_{ref}$ :	vector which has ones at the positions of $\mathbf{x}$ corresponding to the imaginary components of the voltage at the reference bus and zeros everywhere;
$\mathbf{Gp}^{\min}, \mathbf{Gp}^{\max}$ :	minimum and maximum active generation limits;
$\mathbf{Gq}^{\min}, \mathbf{Gq}^{\max}$ :	minimum and maximum reactive generation limits;
$\mathbf{Qg}^{\min}$ :	minimum limits on the values of the reactive power contracts or Pool's generations;
$\mathbf{V}(\mathbf{x})$ :	matrix function; $\mathbf{V}_i(\mathbf{x}) = [\mathbf{x}^T \mathbf{M}_i]$ , where $\mathbf{M}_i$ is a matrix composed of zeros and ones defined by the bus index $i$ ;
$\mathbf{V}^{\min}, \mathbf{V}^{\max}$ :	vectors composed of the square of the minimum and maximum bus voltage limits.

In the OPD model, the first term of  $F$  is the total generation cost, the second is the total transmission losses and the third represents the deviation of the power contracts from the proposed values. Equations (10) and (11) are the power balance equations. Equation (12) fixes at zero the sum of the imaginary components of the voltage at the reference bus. Equations (13)-(16) ensure that the limits of the generators and the transactions are satisfied. Finally, equation (17) imposes limits on the square of the bus voltage magnitudes.

The OPD problem is solved by a Nonlinear Primal-Dual Interior Point Method. It provides the voltages and current injections associated with each participant. These are used to obtain the line flows and transmission losses allocated to the transactions and the Pool.

### 4. ALLOCATION METHODOLOGIES

Transmission services allocation methodologies should: (i) reflect the actual use of the network by each participant and (ii) be consistent, in the sense that the sum of the components associated with the users is equal to the actual amount of the service provided by the network. The schemes proposed here meet these points and are a by-product of the OPD model. It is important to notice that the methodologies described bellow treat all generators and loads that participate in the spot market as *one* network user: the Pool. Thus, the line and loss components allocated to the Pool represent the sum of the losses and line flows due to all the entities trading in the spot market and an appropriate strategy must be used to allocate those quantities among these entities.

#### 4.1 Line Flow Allocation Methodology

Using the AC network model, the complex flows through line  $l$ , from bus  $i$  to bus  $m$ ,  $\dot{S}_{im}^l$ , and from bus  $m$  to bus  $i$ ,  $\dot{S}_{mi}^l$ , can be written:

$$\dot{S}_{im}^l = Pl_{im} + jQl_{im} = \dot{V}_i [\dot{y}_{im} (\dot{V}_i - \dot{V}_m) + jb_{im}^s \dot{V}_i]^* \quad (18)$$

$$\dot{S}_{mi}^l = Pl_{mi} + jQl_{mi} = \dot{V}_m [\dot{y}_{mi} (\dot{V}_m - \dot{V}_i) + jb_{mi}^s \dot{V}_m]^* \quad (19)$$

where  $Pl_{im}$  and  $Ql_{im}$  are the real and reactive power flows,  $\dot{V}_i$  and  $\dot{V}_m$  are the voltages at buses  $i$  and  $m$ , and

$\dot{y}_{im}$ , and  $b_{im}^s$  are the series admittance and shunt susceptance of this line.

The above expressions represent the total line flows, which are caused by all network users. To allocate  $\dot{S}l_{im}$  and  $\dot{S}l_{mi}$  to the participants, first of all, we rewrite expressions (18) and (19) using the notation introduced in equation (1):

$$\dot{S}l_{im}^{total} = \dot{V}_i^{total} [\dot{y}_{im} (\dot{V}_i^{total} - \dot{V}_m^{total}) + j b_{im}^s \dot{V}_i^{total}]^* \quad (20)$$

$$\dot{S}l_{mi}^{total} = \dot{V}_m^{total} [\dot{y}_{im} (\dot{V}_m^{total} - \dot{V}_i^{total}) + j b_{im}^s \dot{V}_m^{total}]^* \quad (21)$$

As it occurs with the total injection of complex power,  $\dot{S}l_{im}^{total}$  can be divided among the network users using different criteria. If the division is based on the line current associated with each participant, we have:

$$\dot{S}l_{im}^{total} = \dot{V}_i^{total} (\dot{I}_{im}^1)^* + \dots + \dot{V}_i^{total} (\dot{I}_{im}^{Pool})^* \quad (22)$$

where  $\dot{I}_{im}^k$  is the line current, from bus  $i$  to bus  $m$ , caused by participant  $k$ . By doing so, the sum of the line flow components is equal to the total flow.

The expression of the line flow, from bus  $i$  to bus  $m$ , allocated to participant  $k$  can then be written:

$$\dot{S}l_{im}^k = \dot{V}_i^{total} (\dot{I}_{im}^k)^* = \dot{V}_i^{total} [\dot{y}_{im} (\dot{V}_i^k - \dot{V}_m^k) + j b_{im}^s \dot{V}_i^k]^* \quad (23)$$

A similar expression can be derived for the flow component from bus  $m$  to bus  $i$ .

#### 4.2 Transmission Losses Allocation Methodology

The total losses of line  $l$ , connecting bus  $i$  to  $m$ , are:

$$\dot{S}loss_l^{total} = Ploss_l^{total} + jQloss_l^{total} = \dot{S}l_{im}^{total} + \dot{S}l_{mi}^{total} \quad (24)$$

The total active losses ( $Ploss\_total$ ) are obtained by adding the active losses of all the  $nl$  lines of the network:

$$Ploss\_total = \sum_{l=1}^{nl} Ploss_l^{total} \quad (25)$$

Using the line flow components,  $\dot{S}l_{im}^k$  and  $\dot{S}l_{mi}^k$ , obtained from equation (23), it is possible to calculate the active losses of line  $l$  due to participant  $k$ :

$$Ploss_l^k = \text{real}(\dot{S}l_{im}^k + \dot{S}l_{mi}^k) = Pl_{im}^k + Pl_{mi}^k \quad (26)$$

where  $Pl_{im}^k$  e  $Pl_{mi}^k$  are the active flows through line  $l$  due to participant  $k$ .

The total active losses of line  $l$  is therefore the sum of (26) over all participants, that is:

$$Ploss_l^{total} = Ploss_l^1 + \dots + Ploss_l^{ntr} + Ploss_l^{Pool} \quad (27)$$

Using equations (23) and (26), we can express the active losses of line  $l$  caused by participant  $k$  as

$$Ploss_l^k = \text{real} \left\{ \dot{V}_i^{total} [\dot{y}_{im} (\dot{V}_i^k - \dot{V}_m^k)]^* + \dot{V}_m^{total} [\dot{y}_{im} (\dot{V}_m^k - \dot{V}_i^k)]^* - j b_{im}^s [\dot{V}_i^{total} (\dot{V}_i^k)^* + \dot{V}_m^{total} (\dot{V}_m^k)^*] \right\} \quad (28)$$

Equation (28) can be simplified if we consider the total losses of line  $l$ . Let the third term of this equation be

$$(Ploss_l^k)_{shunt} = \text{real} \{ -j b_{im}^s [\dot{V}_i^{total} (\dot{V}_i^k)^* + \dot{V}_m^{total} (\dot{V}_m^k)^*] \} \quad (29)$$

Adding  $(Ploss_l^k)_{shunt}$  for  $k=1, \dots, ntr+1$ , we have

$$(Ploss_l^{total})_{shunt} = \text{real} \left\{ -j b_{im}^s [\dot{V}_i^{total} (\dot{V}_i^1 + \dot{V}_i^2 + \dots + \dot{V}_i^{Pool})^* + (\dot{V}_m^{total}) (\dot{V}_m^1 + \dot{V}_m^2 + \dots + \dot{V}_m^{Pool})^*] \right\} = 0 \quad (30)$$

Therefore, although the shunt susceptances influence the values of  $Ploss_l^k$ , they do not influence the total losses since there is no active flow through  $b_{im}^s$ . Using this fact, we can express the active loss component allocated to participant  $k$  as:

$$Ploss_l^k = \sum_{i,m} \text{real} \left\{ \dot{V}_i^{total} [\dot{y}_{im} (\dot{V}_i^k - \dot{V}_m^k)]^* + \dot{V}_m^{total} [\dot{y}_{im} (\dot{V}_m^k - \dot{V}_i^k)]^* \right\} \quad (31)$$

Equation (31) can be used to allocate the transmission losses among the participants in the power market. By doing so, equations (25) and (27) ensure that the total losses are recovered.

## 5. NUMERICAL RESULTS

Three systems were used to test the proposed allocation methodologies: a 5-bus system, whose data is in the Appendix, the 14-bus and the 118-bus IEEE test systems. To compare the results obtained by the loss allocation methodology with those provided by the schemes described in [3] and [4], first of all, the losses allocated to every participant  $k$  are proportionally divided between its generators and loads, thus obtaining bus loss components associated with such participant. Subsequently, the bus loss components of the  $ntr+1$  participants are added to obtain the total bus loss factors.

The values presented below are in pu on a 100MVA basis.

### 5.1 5-Bus System

This system was tested supposing 2 multilateral transactions: the first one between the generators at buses 1 and 3 and the loads at buses 1 (0.25+j0.1 pu), 2 (1.125+j0.2 pu) and 3 (0.3+j0.2 pu) and; the second one between the generator at bus 4 and the loads at buses 4 (0.3+j0.2 pu) and 5 (0.4+j0.2 pu). The Pool's loads are, at bus 1: 0.2+j0.05 pu; 2: 0.5+j0.0 pu; 3: 0.5+j0.0 pu; 4: 0.2+j0.0 pu and; 5: 0.5+j0.05 pu. The solution for the minimization of the transactions' deviations from the accorded values and the Pool's generation cost is shown in Table 1. In it,  $Pg^p$ ,  $Qg^p$ , are the real and reactive power generated for the Pool;  $Pg^m$  and  $Qg^m$ , are the real and reactive power generated for the transactions and;  $Pg^{total}$  and  $Qg^{total}$  are the total real and reactive generations.

Bus	Volt.	$Pg^{pool}$	$Pg^m$	$Pg^{total}$	$Qg^{pool}$	$Qg^m$	$Qg^{total}$
1	1.0500	0.8894	1.3750	2.2644	0.1637	0.3000	0.4637
2	0.9897	0	0	0	0	0	0
3	1.0326	0.4495	0.3000	0.7495	0.4220	0.2000	0.6220
4	1.0500	0.6631	0.7000	1.3631	-0.1789	0.4000	0.5789
5	1.0154	0	0	0	0	0	0

Table 1: Optimal dispatch- 5-bus system.

Table 2 presents the total and the allocated line active flows of the system. The line flows allocated to transaction 1 and the Pool are, in most cases, larger than

those allocated to transaction 2 thus reflecting the fact that the load supplied by transaction 2 is the smallest one. In addition, the flow components of transaction 2 are larger for the lines connecting buses 4 and 5, that is, lines 3-5, 3-4 and 5-4. Therefore, the allocation scheme reflects the network use.

Line i-j	Active Flows ij			
	Total	Trans.1	Trans.2	Pool
1-2	1.1392	0.7985	-0.0442	0.3849
2-3	-0.5366	-0.3394	-0.0471	-0.1502
3-5	-0.1731	-0.2483	0.0959	-0.0207
3-4	-0.4242	-0.0918	-0.1444	-0.1880
5-4	-0.4137	0.0821	-0.2634	-0.2324
5-1	-0.6618	-0.3196	-0.0448	-0.2974

Table 2: Active line flows - 5-bus system.

Table 3 presents the losses associated with the multilateral transactions and the Pool. The sum of the losses allocated to the participants is equal to the total losses. It can be noticed that the losses allocated to the Pool, which has the largest amount of generation and load, are the highest ones. Also, transaction 2, which is the smallest one, gets allocated the lowest losses. Considering the total values of generation and load of transaction 1 and the Pool, the losses allocated to transaction 1 are lower than expected. However, this can be explained by the fact that two of the loads supplied by such transaction are located at the generators' buses. Thus, the losses allocated to transaction 1 are basically caused by the currents through the lines connecting buses 1 and 3 to bus 2.

Trans.	Losses	Load	Generation
1	0.0296	1.675	1.675
2	-0.0034	0.700	0.700
Pool	0.0758	1.900	2.002
Total	0.1020	4.275	4.377

Table 3: Participants' losses - 5-bus system

The results presented in Table 3 can be translated into bus loss components. By doing so, they can be compared with those obtained by the methods described in [3] (ONS) and [4] (Z-bus). This translation was made using the criterion explained in the beginning of this section. The results provided by the three methods are presented in Table 4. It can be noticed that, although all the methods recover the total losses, they provide different loss components. This is due to the nonlinear nature of the loss allocation problem, which imposes some kind of arbitrariness to the three allocation procedures. The allocations provided by the OPD approach are based on the line currents created by each participant, and on the proportional division of the participant losses among its generators and loads. The ONS method, on the other hand, calculates the losses using the DC load flow equations and expresses them as a linear function of the power injections. Finally, the Z-bus method is based on the intuitive division of the total losses among the buses of the system. Despite the differences, all methodologies provide allocations that are consistent with the bus power injections.

Bus	Power Injection	Methods		
		OPD	ONS	Z-bus
1	1.8144	0.0360	0.0272	0.0388
2	-1.6250	0.0245	0.0514	0.0399
3	-0.0505	0.0184	0.0009	0.0024
4	0.8631	0.0151	0.0182	0.0226
5	-0.9000	0.0080	0.0050	-0.0010
Total		0.1020	0.1020	0.1020

Table 4: Comparative results - 5-bus system.

## 5.2 IEEE-14 Bus System

The studies with the IEEE-14 bus system were performed considering the Pool and two multilateral transactions, the first contracted by the generator at bus 1 and the second contracted by the generator at bus 2. The optimization criterion is the minimization of the total transmission losses. Table 5 presents the values of the transactions' and Pool's active loads and generations, whereas Table 6 presents the optimal dispatch.

Bus	Transaction 1		Transaction 2		Pool	
	Gen.	Load	Gen.	Load	Gen.	Load
1	0.2932				0.2932	
2		0.2147	1.2234		1.2234	0.0170
3				0.9420		0.0420
4				0.4780		0.0780
5				0.0760		
6		0.1120				0.0120
9				0.2950		0.0950
10		0.0900				
11				0.0350		0.0050
12		0.0610				
13				0.0800		0.0350
14		0.1490				0.0490

Table 5: Transactions and Pool's load - 14-bus system.

Bus	Angle	Voltage	Pg[pu]	Qg[pu]
1	-0.0000	1.0500	0.5863	-0.0507
2	-0.0035	1.0500	2.4467	0.0740
3	-0.1616	1.0178	0	0.3903
4	-0.1323	1.0197	0	0
5	-0.1086	1.0236	0	0
6	-0.2272	1.0320	0	0.2400
7	-0.2063	1.0337	0	0
8	-0.2063	1.0500	0	0.0969
9	-0.2447	1.0332	0	0
10	-0.2471	1.0250	0	0
11	-0.2399	1.0239	0	0
12	-0.2457	1.0164	0	0
13	-0.2497	1.0102	0	0
14	-0.2725	1.0001	0	0

Table 6: Optimal Dispatch - 14-bus system.

The line flow components obtained from equation (23) are presented in Table 7. From Tables 5 and 7, it can be observed that the allocations depend on the values of the participant's generations and loads. For example, the load at bus 3 has a contract of 0.9420 pu with generator 2. This is a large contract and, therefore, the flows through lines 2-3 and 3-4 allocated to transaction 2 are considerable and define the direction of the total flow. In addition, the flows allocated to the Pool are considerable. This due to the fact that the Pool's generation is large (Table 5).

Element i-j	Active Flows ij			
	Total	Trans.1	Trans.2	Pool
1-2	0.0583	0.2206	-0.1487	-0.0136
1-5	0.5280	0.0726	0.1487	0.3068
2-3	0.8583	0.0039	0.5667	0.2876
2-4	0.7722	0.0145	0.3217	0.4359
2-5	0.6428	0.0190	0.2429	0.3809
3-4	-0.1572	0.0175	-0.3358	0.1611
5-4	0.5638	-0.0231	0.3391	0.2478
4-7	0.3728	0.0345	-0.0496	0.3879
4-9	0.2126	0.0193	-0.0289	0.2221
5-6	0.4960	0.1614	0.0528	0.2819
6-11	0.0716	-0.0561	-0.0031	0.1308
6-12	0.0867	0.0483	0.0122	0.0263
6-13	0.2137	0.0662	0.0426	0.1049
7-9	0.3728	0.0337	-0.0521	0.3912
9-10	0.0592	0.1467	0.0380	-0.1255
9-14	0.1362	0.1549	0.0262	-0.0450
10-11	-0.0310	0.0557	0.0377	-0.1245
12-13	0.0248	-0.0131	0.0121	0.0258
13-14	0.0652	-0.0024	-0.0256	0.0932

Table 7: Active line flows - 14-bus system.

Table 8 presents the transmission losses and also the total active loads and generations of each participant. Since the dispatch was performed without considering the values of the contracts, the optimal generations associated with the transactions and the Pool are quite different from the respective loads. Each generator injects an equal amount of power to supply its transaction and the Pool (Table 5). Thus, the total Pool generation is equal to the sum of the generations of transactions 1 and 2. Consequently the value of  $Ploss^{Pool}$  is approximately equal to  $Ploss^1$  plus  $Ploss^2$ .

Trans.	Losses	Load	Generation
1	0.0198	0.6817	0.2932
2	0.0393	1.9060	1.2234
Pool	0.0533	0.3330	1.5165
Total	0.1124	2.9207	3.0331

Table 8: Participants' losses - 14-bus system.

Bus	Injection.	Methods		
		OPD	ONS	Z-bus
1	0.5863	0.0139	0.0108	0.0173
2	2.2150	0.0584	0.0411	0.0652
3	-0.9894	0.0185	0.0235	0.0131
4	-0.5560	0.0089	0.0106	0.0042
5	-0.0760	0.0022	0.0009	0.0000
6	-0.1240	0.0006	0.0014	-0.0001
9	-0.3900	0.0000	0.0078	0.0026
10	-0.0900	0.0005	0.0020	0.0009
11	-0.0400	0.0010	0.0008	0.0003
12	-0.0610	0.0019	0.0013	0.0006
13	-0.1700	0.0038	0.0047	0.0028
14	-0.1980	0.0028	0.0077	0.0053
Total		0.1124	0.1124	0.1124

Table 9: Comparative results - 14-bus system.

The bus loss factors obtained by the OPD, ONS and Z-Bus approaches are presented in Table 9. The methods yield different allocations to most of the buses but the values of the allocated losses are closely related with the bus power injections. For example, generator 2, with about 80% of the total generation, gets allocated the highest losses according to all methods. In addition, the load at bus 3 and the generator at bus 1, both buses with

large power injections, also receive high allocations. In spite of this, only the OPD and the ONS approaches allocate their second highest loss components to the load at bus 3, with the second largest power injection; the Z-bus method allocates its second largest loss factor to generator 1. The remaining buses also receive varying allocations, depending on the method.

### 5.3 IEEE-118-Bus System

Three multilateral transactions, located in separate portions of the system, were considered. The optimization criterion was the total generation cost. The first multilateral transaction is accorded by the generators and loads of subsystem A, which comprises buses 1-23, 25-32, 113-115 and 117; the second is accorded by the generators and loads of subsystem B, which comprises buses 24, 33-76 and 116, and; the third is accorded by the generators and loads of subsystem C, comprising buses 77-112 and 118. The total transactions' loads and generations are shown in Table 11.

Table 10 presents the total active flows and flow components of the lines interconnecting the physical regions of the transactions. The first five circuits connect subsystem B and C and the last four connect subsystems A and B (Figure 1).

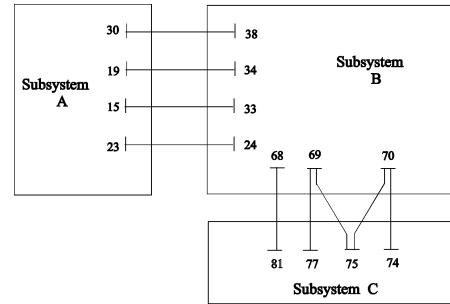


Figure 1: Interconnections - 118-bus system.

Element i-j	Active Flows (OPD)			
	Total	Trans. A	Trans. B	Trans. C
81-68	1.0040	-2.3978	1.0743	2.3275
77-69	0.0919	-0.7465	0.2399	0.5984
75-70	0.2059	-0.5442	0.1846	0.5654
75-69	0.4065	0.0744	-0.0360	0.3681
74-70	0.4986	-0.4878	0.1663	0.8201
38-30	-0.2272	-4.4833	1.0383	3.2178
34-19	-0.0654	-0.1734	-0.2402	0.3482
33-15	-0.1920	-0.2201	-0.3248	0.3529
24-23	1.3775	-1.4417	1.3804	1.4389

Table 10: Active flows and line flow components.

Table 10 shows that the net power flows from subsystems B and C to subsystem A. This is expected since the total generation of subsystem A is smaller than the total load (Table 11). However, the line flow components of transaction A have mostly the direction from subsystem A to B and from B to C. On the other hand, most of the line flow components of transactions B and C have the same direction of the total flows. Thus transaction A creates counter flows in the interconnection lines and, consequently, in some of the internal lines of the subsystems. These counter flows

help to reduce the total losses, which is consistent with the negative losses allocated to transaction A (Table 11). Note also that the flow components of transaction B are lower than those of transaction C. Accordingly, B gets allocated lower losses.

Table 12 presents the power injections and also the losses allocated to some buses by the Z-bus, ONS and OPD methods. The buses with the largest power injections (74, 90 and 103) get allocated the highest losses. However, the allocations do not always conform to the values of the injections. For example, although the power injection at bus 103 is higher than that of bus 74, the OPD method allocates higher losses to the last one. Similarly, the ONS and Z-bus methods allocate higher losses to bus 90 than to bus 103, despite of the lower power injection of bus 90. Besides some similarities, the allocations vary significantly, reinforcing the arbitrariness involved in the three methodologies.

Trans.	Losses	Load (pu)	Gen. (pu)
A	-1.8911	8.020	7.2893
B	0.3246	14.495	13.4664
C	2.6225	13.490	16.3053
Total	1.0560	36.005	37.061

Table 11: Participant's losses - 118-bus system.

Bus	Injection	Methods		
		OPD	ONS	Z-bus
10	0	-0.0238	-0.0140	-0.0539
26	-0.5289	-0.0148	-0.0077	-0.0364
33	-0.2300	0.0027	0.0153	0.0068
61	0.4824	0.0125	-0.0146	0.0132
74	1.1046	0.2429	0.0417	0.0792
90	0.8631	0.0770	0.1004	0.1276
103	1.3456	0.1860	0.0969	0.1236

Table 12: Active Losses in the interconnection buses.

Tables 3, 8, and 11 show that the OPD method allocated negative losses to some transactions. When interacting with the other transactions in the systems, such transactions help to reduce the total losses.

## 6. CONCLUSIONS

New line flow and loss allocation methodologies for use in the context of Pool-Bilateral markets were proposed and tested. They are based on the voltage and line current components provided by an optimal power dispatch model and have the following principal characteristics: (i) the methods are based on the exact network equations as defined by the SP; (ii) in allocating line flows and losses, the methods emphasize the interactions among the complex currents associated with every network user and; (iii) in a similar way to incremental loss and line flow allocation approaches, the proposed methods can yield negative allocations.

Simulation studies show that the loss allocation method yields results generally consistent with expectations and alternative methods. The line flow allocations were also consistent, and will be particularly useful in transmission pricing mechanisms that take into consideration transmission congestion. However, in order to use the OPD model to calculate transmission tariffs, it must consider line flow limits. This is the next

step of our study regarding the optimal power dispatch under Pool-Bilateral markets.

## 7. ACKNOWLEDMENT

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## APPENDIX

Element	r (pu)	x (pu)	bsh (pu)
1-2	0.042	0.168	0.030
2-3	0.031	0.126	0.020
3-5	0.053	0.210	0.015
3-4	0.084	0.336	0.012
5-4	0.063	0.252	0.011
5-1	0.013	0.126	0.010

Table A1: Branch data - 5-bus system

Bus	Gq <sup>max</sup>	Gp <sup>min</sup>	Gq <sup>max</sup>	Gq <sup>min</sup>	V <sup>max</sup>	V <sup>min</sup>	a (\$/pu)	b (\$/pu <sup>2</sup> )
1	3.5	0.0	2.05	-1.05	1.05	0.95	2	1
2	0.0	0.0	0.00	0.0	1.05	0.95	0	0
3	2.0	0.0	2.00	-1.00	1.05	0.95	4	2
4	2.0	0.0	1.00	-1.00	1.05	0.95	1	2
5	0.0	0.0	0.00	0.00	1.05	0.95	0	0

Table A2: Bus data - 5-bus system.