A STUDY OF THE EIGENVALUE ANALYSIS CAPABILITIES OF POWER SYSTEM DYNAMICS SIMULATION SOFTWARE

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Abstract - Increasingly, the investigation of the small-signal stability of power systems is considered an important task, complementary to the usual transient stability investigations. To study the small-signal stability of a power system, eigenvalue analysis is used. In the power systems area, studying eigenvalues does not have a history as long as that of transient stability analysis and the results are difficult to verify experimentally. It is therefore important to study the capabilities of a power system dynamics simulation software package used for small-signal stability analysis carefully, before applying it to practical problems. In this paper, the eigenvalue analysis capabilities of two power system dynamics simulation software packages are analysed and the results are compared.

Keywords: eigenvalue analysis, small-signal stability, power system dynamics, simulation software

1. INTRODUCTION

It is increasingly recognized that the investigation of the small-signal stability of power systems yields important results that are complementary to those yielded by the usual transient stability investigations [1-3]. The advantage of studying the small-signal stability by using eigenvalue analysis when compared to transient stability investigations, is that it gives a complete overview of the small-signal stability of the current system operating state, whereas in transient stability investigations only one event at a time can be simulated. The drawback of eigenvalue analysis is, that a linearized set of equations is used and that higher order terms are neglected, which may lead to erroneous results, particularly when a system is described by strongly non-linear equations.
Keeping in mind these limitations, eigenvalue analysis is nevertheless considered a powerful tool, which, however, does not have a history as long as transient stability analysis because of its computational requirements. Further, the results are often difficult to verify experimentally since this requires synchronous measurements at different locations. Therefore, it is important to assure that the results delivered by a software package are correct before applying it to practical problems.

In this paper, the eigenvalue analysis capabilities of two widely used power system dynamics simulation tools are investigated, namely ABB's Simpow and PTI's PSS/E. Descriptions of other software packages that can be used for eigenvalue analysis can be found in [4].
The paper is organized as follows. In section 2, the linearization of the non-linear equations describing an electrical power system is explained. Then, the algorithms to calculate the eigenvalues of a power system as implemented in Simpow and PSS/E are discussed. The modelling of the synchronous generator and of magnetic saturation in the two programs is commented upon in section 3. To investigate the capabilities of the programs, a number of cases are analysed in section 4:

- a synchronous generator, represented by the classical model and connected to an infinite bus
- a synchronous generator, represented by a sixth-order model and connected to an infinite bus both excluding and including magnetic saturation
- a test system described in [5]

The eigenvalues of some of these cases are also given in [5]. For the first case, they can be calculated analytically, whereas for the other cases, analytical calculation is difficult. The results of the eigenvalue analysis are evaluated in section 5, where the results from Simpow and PSS/E are compared mutually and to the eigenvalues given in [5], in so far as the latter are available. A comparison between the linear analysis capabilities of software packages has not been reported yet in the literature.

2. EIGENVALUE CALCULATION AND MEANING

2.1 Eigenvalues and small-signal stability

The behaviour of a dynamic system, of which an electrical power system is an example, can be described using the following general equation

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)\\ \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u})\end{aligned} \tag{1}$$

where

f is a vector containing n first-order non-linear differential equations

x is a vector containing n state variables

u is a vector containing *r* input variables
g is a vector containing *m* non-linear algebraic equations
y is a vector containing *m* output variables
and t is time. By assuming that the system in (1) is time-invariant, i.e. the time-derivatives of the state variables are not explicit functions of the time, t can be excluded from equation (1).

This system can be linearized and the resulting linearized description can be used to investigate the system's response to small variations in the input or state variables, starting at an equilibrium point [5-6]. To this end, equation (1) is expressed in terms of its Taylor's series expansion. With second and higher orders of the partial derivatives of **f** to the state variables omitted and only taking into account first-order terms, this gives the following for the *i*th component of vector **x**

$$\begin{aligned}\dot{x}_i = \dot{x}_{i0} + \Delta\dot{x}_i &= f_i[\boldsymbol{x_0} + \Delta\boldsymbol{x}, \boldsymbol{u_0} + \Delta\boldsymbol{u}] \\ &= f_i(\boldsymbol{x_0}, \boldsymbol{u_0}) + \frac{\partial f_i}{\partial x_1}\Delta x_1 + \ldots + \frac{\partial f_i}{\partial x_n}\Delta x_n \\ &\quad + \frac{\partial f_i}{\partial u_1}\Delta u_1 + \ldots + \frac{\partial f_i}{\partial u_r}\Delta u_r\end{aligned} \quad (2)$$

As can be seen from (1),

$$\dot{x}_{i0} = f_i(\boldsymbol{x_0}, \boldsymbol{u_0}) \quad (3)$$

and thus (2) can be written as

$$\Delta\dot{x}_i = \frac{\partial f_i}{\partial x_1}\Delta x_1 + \ldots + \frac{\partial f_i}{\partial x_n}\Delta x_n + \frac{\partial f_i}{\partial u_1}\Delta u_1 + \ldots + \frac{\partial f_i}{\partial u_r}\Delta u_r \quad (4)$$

The same can be done for the *j*th component of **y**

$$\Delta y_j = \frac{\partial g_j}{\partial x_1}\Delta x_1 + \ldots + \frac{\partial g_j}{\partial x_n}\Delta x_n + \frac{\partial g_j}{\partial u_1}\Delta u_1 + \ldots + \frac{\partial g_j}{\partial u_r}\Delta u_r \quad (5)$$

Doing this for all components of the vectors **x** and **y** gives the following linearized set of equations

$$\begin{aligned}\Delta\dot{\boldsymbol{x}} &= \boldsymbol{A}\Delta\boldsymbol{x} + \boldsymbol{B}\Delta\boldsymbol{u} \\ \Delta\boldsymbol{y} &= \boldsymbol{C}\Delta\boldsymbol{x} + \boldsymbol{D}\Delta\boldsymbol{u}\end{aligned} \quad (6)$$

with

$$\boldsymbol{A} = \begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \ldots & \frac{\partial f_1}{\partial x_n} \\ \ldots & \ldots & \ldots \\ \frac{\partial f_n}{\partial x_1} & \ldots & \frac{\partial f_n}{\partial x_n}\end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix}\frac{\partial f_1}{\partial u_1} & \ldots & \frac{\partial f_1}{\partial u_r} \\ \ldots & \ldots & \ldots \\ \frac{\partial f_n}{\partial u_1} & \ldots & \frac{\partial f_n}{\partial u_r}\end{bmatrix}$$
$$\boldsymbol{C} = \begin{bmatrix}\frac{\partial g_1}{\partial x_1} & \ldots & \frac{\partial g_1}{\partial x_n} \\ \ldots & \ldots & \ldots \\ \frac{\partial g_m}{\partial x_1} & \ldots & \frac{\partial g_m}{\partial x_n}\end{bmatrix} \quad \boldsymbol{D} = \begin{bmatrix}\frac{\partial g_1}{\partial u_1} & \ldots & \frac{\partial g_1}{\partial u_r} \\ \ldots & \ldots & \ldots \\ \frac{\partial g_m}{\partial u_1} & \ldots & \frac{\partial g_m}{\partial u_r}\end{bmatrix} \quad (7)$$

Thus, the matrices **A**, **B**, **C** and **D** contain the partial derivatives of the functions in **f** and **g** to the state variables **x** and the input variables **u**.

Equation (6) can be Laplace transformed. Thus, the state equations in the frequency domain are obtained

$$\begin{aligned}s\Delta\boldsymbol{x}(s) - \Delta\boldsymbol{x}(0) &= \boldsymbol{A}\Delta\boldsymbol{x}(s) + \boldsymbol{B}\Delta\boldsymbol{u}(s) \\ \Delta\boldsymbol{y}(s) &= \boldsymbol{C}\Delta\boldsymbol{x}(s) + \boldsymbol{D}\Delta\boldsymbol{u}(s)\end{aligned} \quad (8)$$

A solution to the state equations can be obtained by rearranging the upper equation of (8) as follows

$$(s\boldsymbol{I} - \boldsymbol{A})\Delta\boldsymbol{x}(s) = \Delta\boldsymbol{x}(0) + \boldsymbol{B}\Delta\boldsymbol{u}(s) \quad (9)$$

The values of s which satisfy

$$\det(s\boldsymbol{I} - \boldsymbol{A}) = 0 \quad (10)$$

are known as the eigenvalues of matrix **A** and equation (10) is defined as the characteristic equation of matrix **A**.

In practical situations it has to be kept in mind that the eigenvalues of a linearized set of equations have been calculated. In non-linear systems, the eigenvalues depend on the system state and the eigenvalues change when the system state evolves. Therefore, a set of eigenvalues only characterizes the actual state of the system and not necessarily other states.

2.2 Practical eigenvalue calculation in PSS/E

For most practical systems, equation (1), with which the eigenvalue calculation starts, is a small-signal equation. This also applies when an electrical power system is studied. It can be complicated to construct the matrix **A** in equation (7) analytically. Therefore, in many power systems simulation software capable of eigenvalue analysis, a method is implemented to construct the matrix **A** numerically, after which its eigenvalues can be calculated.

In PSS/E, this is done in the following way [7]: starting from a valid equilibrium condition $\mathbf{x}_0$, a second state vector is created, $\mathbf{x}_i$, in which the *i*th component of $\mathbf{x}_0$ is perturbed. The first-order differential equations in **f** are evaluated for $\mathbf{x}_i$ as well, resulting in d$\mathbf{x}_i$/dt. Now, it is possible to use the following equation

$$\frac{d\boldsymbol{x}_i}{dt} - \frac{d\boldsymbol{x}_0}{dt} = \frac{d\boldsymbol{x}_i}{dt} - \boldsymbol{0} = \boldsymbol{A}_i\Delta\boldsymbol{x}_i \quad (11)$$

to calculate the values of the *i*th column of the state matrix **A**. In equation (11), $\mathbf{A}_i$ is a matrix of the same dimensions of **A**, containing the *i*th column of the matrix **A** and zeros for the rest and $\Delta\mathbf{x}_i$ equals $\mathbf{x}_i$-$\mathbf{x}_0$. By sequentially perturbing all entries of the vector $\mathbf{x}_0$ to get different $\mathbf{x}_i$'s, all columns of the matrix **A** can be constructed. When the matrix **A** is available, its eigenvalues are calculated using numerical eigenvalue calculation routines [8].

The following remark must be made regarding this approach. The entries of the matrix **A** will partly depend upon the size of the applied perturbation. The larger the perturbation, the more inaccurate the resulting approximation of **A**. A perturbation as small as possible seems therefore preferable. However, when the perturbation is too small, numerical inaccuracies in calculating $\mathbf{x}_0$ and $\mathbf{x}_i$ will lead to inaccuracies in the approximation of **A** as well. Therefore, it must be investigated how robust the approximation of **A** is to changes in the perturbation size, and it is advised to treat the results carefully when they are very sensitive to such changes.

2.3 Practical eigenvalue calculation in Simpow

In Simpow, the matrix **A** is calculated in a different way. First, a vector **z** is prepared

$$z = \begin{bmatrix} v \\ x \end{bmatrix} \quad (12)$$

This vector consists of a vector with network quantities **v** and a vector with state variables **x**. The latter was already introduced in equation (1). The network quantities are calculated using algebraic equations, whereas the state variables are calculated using differential equations.

The value of the partial derivatives are computed using formulas that are derived analytically from the equations describing the system. With these, a Jacobian is constructed

$$J = \begin{bmatrix} J_{aa} & J_{as} \\ J_{sa} & J_{ss} \end{bmatrix} \quad (13)$$

This Jacobian consists of four parts. A row in the submatrix $\mathbf{J}_{aa}$ contains partial derivatives of an algebraic equation to network quantities, whereas a row in $\mathbf{J}_{as}$ contains partial derivatives of an algebraic equation to the state variables. Similarly, a row in $\mathbf{J}_{sa}$ contains partial derivatives of a differential equation to the network quantities and a row in $\mathbf{J}_{as}$ contains partial derivatives of a differential equation to the state variables.

The relation between (12) and (13) is as follows

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & I \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} J_{aa} & J_{as} \\ J_{sa} & J_{ss} \end{bmatrix} \begin{bmatrix} v \\ x \end{bmatrix} \quad (14)$$

Since there are no time differentials of the network quantities and no time differentials of the state variables in the algebraic equations, all elements of submatrices $\mathbf{D}_{11}$ and $\mathbf{D}_{12}$ equal zero. Further, since there are no time differentials of the network quantities in the differential equations, the elements of submatrix $\mathbf{D}_{21}$ also equal zero. **I** in (14) is the identity matrix. Thus,

$$0 = J_{aa}v + J_{as}x \quad (15)$$

and

$$v = -J_{aa}^{-1}J_{as}x \quad (16)$$

From (14) it can also be concluded that

$$\dot{x} = J_{sa}v + J_{ss}x \quad (17)$$

Using (16) in (17) gives

$$\dot{x} = -J_{sa}J_{aa}^{-1}J_{as}x + J_{ss}x = (-J_{sa}J_{aa}^{-1}J_{as} + J_{ss})x \quad (18)$$

so that, using (6), the matrix **A** can be identified as

$$A = -J_{sa}J_{aa}^{-1}J_{as} + J_{ss} \quad (19)$$

When the matrix $\mathbf{J}_{aa}$ is singular, which can be the case in the presence of network elements containing time delays such as HVDC models, more complex routines are necessary to arrive at equation (19).

When the matrix **A** has been constructed, the QR-algorithm is applied to it to find the eigenvalues of the linearized power system [9]. The QR-algorithm is fast and gives information on the number of well-isolated eigenvalues. The number of eigenvalues is limited to 700 by default but can be increased if desired. To improve the results of the eigenvalue calculation from the QR-algorithm, the Inverse Iteration method is applied to the eigenvalues of interest [10]. In case of closely positioned eigenvalues the Shifted Inverted Arnoldi method is applied.

2.4 Differences in eigenvalue calculation

From the above, it can be concluded that there exist differences between the eigenvalue calculation in PSS/E and Simpow. The first difference is that they do not use exactly the same numerical routines to calculate the eigenvalues of the matrix **A**, although both routines are based on the QR algorithm. This could theoretically lead to minor differences in the results.

A more important point is that they use different approaches to construct the state matrix **A** and that they take into account the interaction between network quantities and state variables in a different way.

In PSS/E, the matrix **A** is calculated using numerical differentiation, as described section 2.2. Further, when the eigenvalues are calculated in PSS/E, only the state variables are perturbed and the matrix **A** is calculated according to equation (11). The network quantities are, however, not perturbed and changes in the state variables that might result from the perturbation of the network quantities are not taken into account.

In Simpow, analytical expressions for all partial derivatives exist and are used, i.e. no numerical differentiation take place as in PSS/E. Besides, not only the partial derivatives of the state variables are included when deriving the matrix **A**, but also the partial derivatives of the network quantities are included, as was shown in (14).

3. SYNCHRONOUS GENERATOR MODELLING

3.1 Synchronous generator block diagrams

Apart from differences in the way the eigenvalues are calculated in Simpow and PSS/E, it is also important to investigate the synchronous generator models in both of the programs, because differences between these could also lead to differences in the results of the eigenvalue analysis.

In figure 1, the block diagram of the flux-current relationships implemented in the synchronous generator model in PSS/E is depicted [7]. Magnetic saturation is excluded for clarity. The block diagram of the mechanical part is also not given. It is important to note that in PSS/E, rotor speed variations are reflected in the internal voltage and thus in the generator terminal current [7]. In [5], it is advised not to take into account rotor speed variations in the internal voltage, because this counteracts the effects of neglecting the dψ/dt terms in the stator voltage equations, as is routinely done in power system simulations. This suggestion is not followed in PSS/E.

In figure 2, the block diagram of the synchronous generator model in Simpow is given, again with

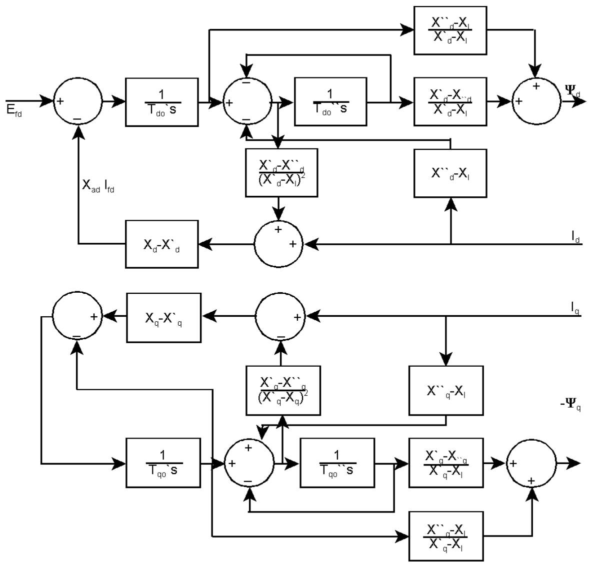

Figure 1: Block diagram of the flux-current relationships of the synchronous generator models GENROU/GENROE in PSS/E with magnetic saturation neglected [7].

magnetic saturation neglected [11]. The following differences between the PSS/E and the Simpow block diagram exist:

- In the PSS/E model, it is assumed that the stator resistance R_a equals zero, i.e. the stator resistance is neglected, whereas in Simpow, the stator resistance can be taken into account.
- In PSS/E, X_d'' must be equal to X_q'', whereas in Simpow different values can be used.
- In PSS/E, rotor speed ω_m is taken into account in the stator voltage equations, as already said above. In Simpow, it is possible to modify the synchronous generator model such that ω_m is not included in the stator voltage equations.
- In PSS/E, mechanical power is an input to the synchronous generator, in opposition to [5], where mechanical torque is the input. In Simpow, it is possible to select between mechanical torque and mechanical power as an input.
- The stator fluxes are calculated in a different way. In PSS/E, they are derived from the stator currents and the exciter voltage, whereas in Simpow the terminal voltage and the stator currents are used.

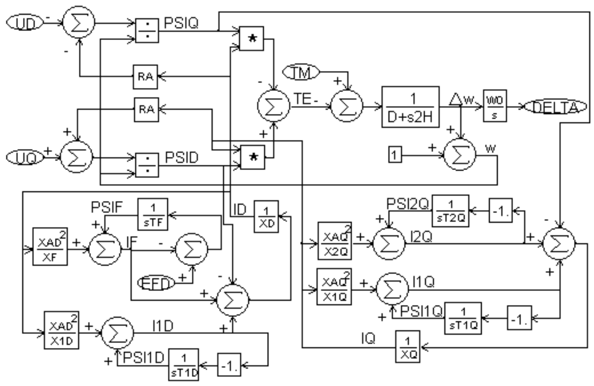

Figure 2: Block diagram of the synchronous generator model 1A in Simpow with magnetic saturation neglected [11].

Apart from the above, Simpow offers the possibility to take into account the $d\psi/dt$ terms in the stator voltage equations when simulating in the instantaneous value mode. This does not apply to PSS/E and this feature of Simpow is not used throughout the research presented here.

3.2 Modelling of magnetic saturation

When the flux in the iron of a synchronous machine becomes larger, magnetic saturation in the stator and rotor iron can no longer be neglected, i.e. it can no longer be assumed that the relative magnetic permeability μ_r of the iron equals infinity. Therefore, magnetizing current is no longer only needed to magnetize the air gap, but also to magnetize the iron. The magnetizing curve of saturated iron is non-linear. Therefore, the relation between magnetizing current and flux becomes non-linear as well.

The calculated eigenvalues partly depend on this non-linear relation, which is modelled in a different way in the software packages studied here. Therefore, some words are spent on the modelling of magnetic saturation in the two software packages studied, before turning to the actual calculations.

In figure 3, the non-linear relation between field current and flux, that results when magnetic saturation is included, is depicted. It is assumed that the stator terminals are not connected. This curve is known as the *open circuit characteristic* of the synchronous machine. The solid line depicts the air gap line, which coincides with the open circuit characteristic when magnetic saturation is neglected. The dotted line depicts the open circuit characteristic including magnetic saturation. It can be seen that the larger the flux, the more field current is required to magnetize the iron.

The non-linear magnetic saturation phenomenon can be approximated in different ways. All approximations are based on the following equation

$$I_{fd} = \frac{\Psi(1 + S(\Psi))}{L_{fd}} \qquad (20)$$

in which S gives the amount of saturation, Ψ is the flux, L_{fd} is the field inductance and I_{fd} is the field current.

In PSS/E, a quadratic or an exponential model of magnetic saturation $S(\Psi)$ can be chosen, which is characterized by the amount of saturation at nominal flux and at 1.2 times nominal flux: the quantities S(1.0)

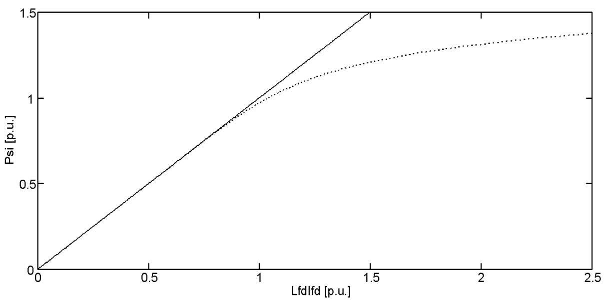

Figure 3: Open circuit characteristic of a synchronous generator with magnetic saturation neglected(solid) and magnetic saturation included (dotted).

and S(1.2). In Simpow, a linear saturation model to represent S(Ψ) is implemented and additionally any user-defined characteristic can be formulated in a table. In those computations below in which magnetic saturation is taken into account, the exponential model in PSS/E has been chosen and in Simpow, a user defined characteristic closely matching the PSS/E model has been used in order to arrive at set-ups with a high degree of similarity.

4. EIGENVALUE CALCULATION RESULTS

In this section, the eigenvalues of a number of set-ups are calculated. The calculation results are evaluated in section 5.

4.1 Case 1: synchronous generator, classical model

In this first case, a generator connected to an infinite bus and modelled with the classical model, i.e. a voltage source behind a transient reactance, is investigated. There are only two state variables, load angle and rotor speed deviation from 1 p.u.. The moment of inertia H equals 3.5 s and X_d' equals 0.3 p.u., both on a 2220 MVA base. The system is described in [5], p. 732 and depicted in figure 4. In case of this rather simple system, it is possible to verify the eigenvalue calculation analytically. The results are given in table 1.

PSS/E	Analytical calculation	Simpow	Ref. [4]
+6.603j	+6.387j	+6.385j	+6.39j
-6.603j	-6.387j	-6.385j	-6.39j

Table 1: Eigenvalues of a synchronous generator connected to an infinite bus when the generator is modelled with the classical model.

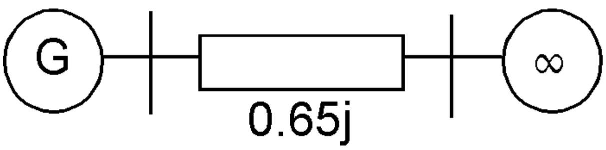

Figure 4: System used in cases 1, 2 and 3.

4.2 Case 2: synchronous generator, sixth-order model with magnetic saturation neglected

In the second case, the synchronous generator connected to the infinite bus is modelled with a sixth-order model, with the field winding flux and one damper winding flux in the d-axis, two damper winding fluxes in the q-axis, rotor speed speed deviation from 1 p.u. and load angle as the state variables. The generator's governor and exciter are not modelled, the stator resistance R_a equals 0 and the stator transients, represented by the dψ/dt terms in equation (21), are neglected. The latter is normal in power system dynamics simulation software [5,12]. These simplifications reduce the full form of the stator voltage equations in the d- and q-axis

$$e_d = -\omega_m \Psi_q - R_a i_d + \frac{d\Psi_d}{dt}$$
$$e_q = \omega_m \Psi_d - R_a i_q + \frac{d\Psi_q}{dt} \qquad (21)$$

to the following simplified form

$$e_d = -\omega_m \Psi_q$$
$$e_q = \omega_m \Psi_d \qquad (22)$$

The generator parameters are given in Appendix A.

The results are depicted in table 2. In the first column, the results for PSS/E are given, in which mechanical power is one of the model's inputs and rotor speed is taken into account in the stator voltage equations. In the last three columns, three results acquired with Simpow are given. In the column marked *Simpow 1*, mechanical power is the input signal and rotor speed is taken into account in the stator voltage equation (22) as in PSS/E. In the column marked *Simpow 2*, mechanical torque is the input signal and rotor speed is included in equation (22). In the column marked *Simpow 3*, mechanical torque is the input signal and rotor speed is not included in equation (22), making ω_m equal to 1 in (22). In the column marked *Simpow 3* the synchronous generator model is similar to that used in [5]. In tables 3 and 4, the definitions of columns *Simpow 1* and *Simpow 2* are the same as in table 2.

PSS/E	Simpow 1	Simpow 2	Simpow 3
-0.252 +6.355j	-0.238 +6.387j	-0.173 +6.386j	-0.191 +6.387j
-0.252 -6.355j	-0.238 -6.387j	-0.173 -6.386j	-0.191 -6.387j
-0.047	-0.051	-0.051	-0.051
-1.743	-2.145	-2.146	-2.146
-21.75	-22.07	-22.07	-22.07
-35.48	-35.49	-35.49	-35.49

Table 2: Eigenvalues of a synchronous generator connected to an infinite bus when the generator is modelled with a sixth-order model and magnetic saturation is neglected.

4.3 Case 3: synchronous generator, sixth-order model with magnetic saturation included

The set-up in this case is the same as in section 4.2, but with magnetic saturation included. The magnetization curve of the generator is depicted in figure 3. The resulting eigenvalues are given in table 3 for PSS/E and Simpow and now also from [5], p. 790. In the first column, the results for PSS/E are given. In the second and third column, results acquired with Simpow are given. The definition of the columns *Simpow 1* and *Simpow 2* is the same as in table 2. The model used in column *Simpow 3* in table 2 is hand-written and magnetic saturation can not be included. Therefore, it is omitted in the following sections. In [5], mechanical torque is the input and rotor speed is not included in the stator voltage equations.

There are some notable differences between [5] on one hand and Simpow and PSS/E on the other. In the calculations in Simpow and PSS/E, the following is assumed

$$R_a = 0 \quad X_d'' = X_q'' \qquad (23)$$

This assumption is due to PSS/E's model structure as mentioned in section 3.1, but has been used in Simpow as well, in order to make the set-ups as similar as possible to enable the results of the two programs to be compared. As a result, a direct comparison between the two software packages and [5] is invalid. From further simulations with Simpow, it could be concluded that assumption (23) reduces the damping of the complex eigenvalue pair. A careful examination of the effect of assumption (23) shows that the observed reduction of the damping is more specifically caused by assuming that X_d'' equals X_q''. The larger X_d'', the less the eigenvalue pair is damped.

PSS/E	Simpow 1	Simpow 2	Ref. [4]
-0.262 +6.34j	-0.225 +6.41j	-0.160 +6.41j	-0.171 +6.47j
-0.262 -6.34j	-0.225 -6.41j	-0.160 -6.41j	-0.171 -6.47j
-0.185	-0.114	-0.114	-0.2
-2.227	-2.148	-2.148	-2.045
-21.396	-22.08	-22.08	-25.01
-35.425	-35.88	-35.88	-37.85

Table 3: Eigenvalues of a synchronous generator connected to an infinite bus when the generator is modelled with a sixth-order model and magnetic saturation is included.

4.4 Case 4: 4-generator test system

To conclude, the case of a 4-generator test system described in [5], p. 813, is investigated. The system is depicted in figure 5. The synchronous generator parameters used are given in appendix B.

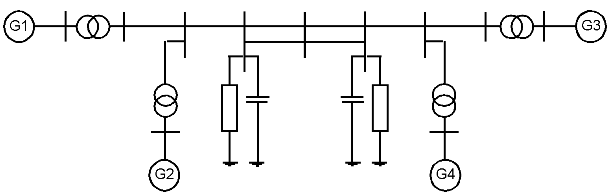

Figure 5: System used in case 4.

The results are given in table 4. The order of the columns is the same as in table 3. The methods used in [5] to calculate the eigenvalues are different from those in PSS/E and Simpow. For more information, [5] should be consulted.

PSS/E	Simpow 1	Simpow 2	Ref. [4]
0.32	0	0	-0.001 +0.002j
-0.106	0	0	-0.001 -0.002j
-0.151+3.41j	-0.152+3.42j	-0.121+3.42j	-0.111+3.43j
-0.151-3.41j	-0.152-3.42j	-0.121-3.42j	-0.111-3.43j
-0.606+ 6.74j	-0.661+6.64j	-0.628+6.64j	-0.492+6.82j
-0.606-6.74j	-0.661-6.64j	-0.628-6.64j	-0.492-6.82j
-0.619+6.94j	-0.678+6.84j	-0.644+6.84j	-0.506+7.02j
-0.619-6.94j	-0.678-6.84j	-0.644-6.84j	-0.506-7.02j
-0.217	-0.100 +0.069j	-0.072 +0.069j	-0.096
-0.225	-0.100 -0.069j	-0.072 -0.069j	-0.117
-0.250 + 0.210j	-0.191	-0.191	-0.265
-0.250 - 0.210j	-0.210	-0.211	-0.276
-2.93	-3.60	-3.60	-3.428
-3.85	-5.10	-5.10	-4.139
-5.19	-8.04	-8.05	-5.287
-5.21	-8.11	-8.12	-5.303
-28.43	-34.12	-34.12	-31.03
-29.92	-35.01	-35.01	-32.45
-33.32	-36.02	-36.02	-34.07
-34.71	-37.02	-37.02	-35.53
-35.67	-37.48	-37.48	-37.89 +0.142j
-35.85	-37.67	-37.67	-37.89 -0.142j
-37.08	-41.26	-41.26	-38.01 +0.038j
-37.15	-41.38	-41.38	-38.01 -0.038j

Table 4. Eigenvalues of the test system used in case 4 and described in [5].

5. EVALUATION OF EIGENVALUE CALCULATIONS

In this section, the results of the eigenvalue analysis carried out above will be evaluated. The following remarks can be made with respect to the differences observed in the results of case 1, as presented in table 1:

- The state matrix can be accessed in both programs. This makes it possible to calculate the eigenvalues of the state matrix using MATLAB, which resulted in exactly the same eigenvalues as given in the second and third column of table 1. Thus, the observed differences are not caused by differences in the routines used to calculate the eigenvalues numerically.
- It is not clear what causes the differences between Simpow and PSS/E when the classical generator model is used, as in case 1. According to the manuals, the model is exactly identical in both software packages and the different numerical routines that are used to calculate the eigenvalues once the matrix **A** has been constructed have only a minor impact. Therefore, it is most likely that the different ways in which the matrix **A** is constructed cause the differences between Simpow and PSS/E that can be seen in table 1.

From the second case, it can be concluded that the eigenvalues depend on the synchronous generator model used. The results are influenced by taking into account rotor speed in the stator voltage equations or not and by making mechanical power or mechanical torque the model's input. Only the real part of the eigenvalue pair is influenced by these changes as can be seen in columns 2, 3 and 4 of table 2.

With respect to the observed differences in the third and fourth case between [5] on one hand and Simpow and PSS/E on the other, the following remarks can be made:

- In [5], mechanical torque is an input to the synchronous generator model. In PSS/E, mechanical power is an input and in Simpow, this can be varied. It can be seen that when the set-up in Simpow is most similar to PSS/E, this also applies to the eigenvalues, whereas when the set-up is most similar to [5], the results are more similar to those given in [5].
- In [5], the stator resistance is not equal to zero and X_d'' is not equal to X_q'', whereas in the calculations with Simpow and PSS/E assumption (23) applies due to the structure of the synchronous generator model in PSS/E.

The observed differences between PSS/E and Simpow in cases 2, 3 and 4 can be caused by differences in the way the state matrix is constructed, by differences in the modelling of magnetic saturation and by differences in the synchronous generator model. It is difficult to point out which factors are most important.

An important general conclusion of the research presented in this paper, is that the results of the eigenvalue calculation depend on the synchronous generator model used. From tables 2 and 3 it can be concluded that the eigenvalues depend on whether the mechanical torque or power is chosen as an input and on whether rotor speed variations are included or not. Different choices with respect to these issues are made in prevailing software packages, which therefore yield different results.

The results obtained in case 4 lead to another general conclusion. As concluded above, differences in the construction of the state matrix and the generator model used can result in different eigenvalues. However, from table 4, it can be concluded that these differences in modelling and calculation can even result in a different judgement of the system's stability. An analysis of the four generator system of case 4 with Simpow and with the methods used in [5] would lead to the conclusion that it is stable, whereas an analysis with PSS/E would lead to the conclusion that it is unstable.

The latter conclusion was, however, denied by carrying out a time domain simulation with PSS/E. The dynamic simulation was initialized at an equilibrium, using a load flow solution. Then a simulation was run, in which the rotor speed of the synchronous generator at bus 1 was perturbed. In opposition to what would be expected based on the obtained eigenvalues, the system returned to its initial state and thus proved to be stable.

In this specific case, the eigenvalue with a positive real part corresponds to an eigenvalue that should be located at the origin of the complex plane. It is associated with state redundancies, as described in [5], and should therefore not be used to judge the system's stability. This also appeared from a linearization in which in equation (11) the state variables were perturbed with a perturbation of size 0.001 instead of 0.01. With a perturbation size of 0.001, the positive real part of the eigenvalue became slightly negative, corresponding to a stable system.

6. CONCLUSIONS

In this paper, the eigenvalue analysis capabilities of PTI's PSS/E and ABB's Simpow, two widely used power system dynamics simulation software packages, have been investigated. It was shown that differences between these two programs exist in the way the state matrix of the system is constructed, in the synchronous generator model and in the way magnetic saturation is modelled. It was shown that the different results that were obtained when calculating the eigenvalues of some test cases resulted from these differences.

The results were also compared with those presented in the literature. In this case, differences were present as well. These were influenced not only by differences in modelling the synchronous generator and constructing the state matrix, but also differences in the generator model parameters. It was not possible to exactly match the set-ups used in the reference, because in PSS/E, the synchronous generator model lacks some degrees of freedom: the stator resistance is neglected and X_d'' must be equal to X_q''.

An important overall conclusion of the research presented in this paper is that the different ways of modelling the synchronous generator and the different approaches to constructing the state matrix that are used in power system dynamics simulation software result in different eigenvalues. Even different conclusions with respect to a system's stability can be drawn depending on the software package used for the simulations. In the case analysed in section 4.4, the same set-up in each of the two investigated programs resulted in a stable system in one of them and an unstable system in the other. The assumed instability of the system was, however, denied by carrying out a time domain simulation. The corresponding eigenvalue is associated with state redundancies and should therefore equal zero. It is not possible to draw conclusions with respect to the topic which modelling approach yields results closest to reality from the results presented in this paper. To that end, the output of the programs would have to be compared with observations made in a real power system, which has not been done here.

REFERENCES

[1] T.-S. Luor, Y.-Y. Hsu, "Effect of non-utility generator siting on oscillatory stability of Taiwan power system", Electrical power and energy systems, v. 19, n. 2, p. 93-101, February 1997.

[2] H. Bruelmann, M. Lösing, et al., "Analysis and Damping of Inter-Area Oscillations in the UCTE/CENTREL Power System" Cigré Session 2000, August 27-September 1, 2000, Paris, F, paper 38-113.

[3] A. Fischer, I. Erlich, "Impact of Long-Distance Power Transits on the Dynamic Security of Large Interconnected Power Systems", IEEE Porto Power Tech Conference, September 10-13, 2001, Porto, PT, paper SSR2-074.

[4] J. Paserba (ed.), "Analysis and Control of Power System Oscillations", Paris: Cigré technical brochure, July 1996.

[5] P. Kundur, "Power system stability and control", New York, US, McGraw-Hill, Inc., 1994.

[6] M. Ilic, J. Zaborsky, "Dynamics and control of large electric power systems", New York, US, John Wiley & Sons, Inc., 2000.

[7] "PSS/E 25, On-line Documentation", Schenectady, US: Power Technologies, Inc., December 1997.

[8] B.T. Smith, "Matrix Eigensystem Routines - EISPACK guide", Berlin, Springer, 1974.

[9] David S. Watkins, "Understanding the QR Algorithm", SIAM Review, Vol. 24, No. 4, October 1982.

[10] G. Peters and J. H. Wilkinson, "Inverse Iteration, Ill-Conditioned Equations and Newton's Method", SIAM Review, v. 21, n. 3, July 1979.

[11] E. Johansson, "Detailed description of Synchronous Machine Models used in Simpow", Technical Report TR H 01-160, Västerås, Sweden, ABB Utilities, December 2001.

[12] P. Forsyth, R. Kuffel, R. Wierckx, J.-B. Choo, Y.-B. Yoon, T.-K. Kim, "Comparison of Transient Stability Analysis and Large-Scale Real Time Digital Simulation", IEEE Porto Power Tech Conference, September 10-13, 2001, Porto, PT, paper EMT-175.

APPENDIX A. SYNCHRONOUS GENERATOR DATA FOR CASES 2 AND 3

Quantity	Value	Quantity	Value
T'_{do}	8.0 s	X_q	1.76 p.u.
T''_{do}	0.03 s	X'd	0.3 p.u.
T'_{qo}	1.0 s	X'_q	0.65 p.u.
T''_{qo}	0.07 s	X''_d*	0.25 p.u.
H	3.5 s	X_l	0.16 p.u.
D	0	S(1.0)**	0.124 p.u.
X_d	1.81 p.u.	S(1.2)**	0.431 p.u.

Table A.1: Data for the synchronous generator used in cases 2 and 3.
* X''_d equals X''_q due to the PSS/E synchronous generator model structure
** Equals 0 when magnetic saturation is neglected

APPENDIX B. SYNCHRONOUS GENERATOR DATA FOR CASE 4

Quantity	Value	Quantity	Value
T'_{do}	8.0 s	X_q	1.7 p.u.
T''_{do}	0.03 s	X'd	0.3 p.u.
T'_{qo}	0.4 s	X'_q	0.55 p.u.
T''_{qo}	0.05 s	X''_d**	0.25 p.u.
H	6.5/6.175 s*	X_l	0.2 p.u.
D	0	S(1.0)	0.039 p.u.
X_d	1.8 p.u.	S(1.2)	0.223 p.u.

Table B.1: Data for the synchronous generator used in case 4.
* H equals 6.5 s for generators 1 and 2 and 6.175 s for generators 3 and 4
** X''_d equals X''_q due to the PSS/E synchronous generator model structure