

# COMBINING PRODUCTION AND TRANSMISSION SYSTEM USING RELAXED CONSTRAINTS

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**This paper presents a new method, based on relaxation techniques, for optimization of power production where power transfer capability can be included as transmission constraints in the optimization. The presented method combines different optimization techniques making the resulting technique useful for a widespread number of applications: planning of dispersed generation and renewable energy and how introduction of these might influence the existing system, optimal utilization of generation capacity and transmission grid, implementing strategies for congestion management and transmission grid planning. In the Nordic power market, as well as in other deregulated power markets, the application areas listed above is expected to become more important as focus has shifted towards environmental issues and new large power plants and large investments in the transmission grid become more unlikely. In the paper we present an example where the method is used to analyze the impact of introducing dispersed generation in a hydro power dominated system. The modeling includes uncertainty in inflow and prices where inflow is modeled on a daily basis.**

***Keywords: Power system operation, distributed generation, congestion management***

## 1 INTRODUCTION

The Nordic countries, Norway, Sweden, Finland and Denmark today share a common spot market for electrical power. Deregulation and market competition were introduced in the electricity market in Norway in 1991, in Sweden in 1996, while Finland and Denmark came later. The effect of the deregulation imposes new risk to the power sector but it also increases the effectiveness of the power sector in means of economy.

Increasing the effectiveness of the power sector calls for a new approach where production scheduling and expansion planning are seen in connection with the transmission capabilities rather than the traditional decoupled approach where these are treated separately. Power production companies are now facing challenges such as:

- It is quite common that the market is split up in different price areas with individual spot prices

due to transmission constraints. It is therefore important for the power producers to implement long-term strategies that account for reoccurring congestion of the transmission system due to bottlenecks.

- Joint optimization of the production and local transmission system with the objective to decrease the overall losses, and enhance market access.
- As wind power and other renewable energy sources become available, their interaction with the existing system becomes important. This type of production will often be placed locally at remote locations where the transmission grid is weak.

The variety of the challenges above calls for different approaches with regard to the choice of optimization method. But in cases where a linear optimization or dispatch model can be used, the transmission grid can be added to the model as suggested in this paper.

As wind power and distributed power production become focused in Norway we have developed a prototype for local expansion planning where the production and transmission systems are both included. Transmission constraints are tested separately in a security constrained optimal power flow, and transmission constraints are added to the optimization. The reasons why the transmission grid is indirectly included in the model are:

- A combined model of the production and transmission system could become impractically large and time-consuming.
- The decomposition approach applied enhances the flexibility of both the power scheduling model and the transmission model. An example of this is to allow the transmission model to include the security aspect for the transmission system.

The work is an enhancement of a model developed for hydropower expansion planning under price uncertainty described in [1]. The focus is on description of the mathematics and iteration techniques used in the current implementation.

The next chapter gives a brief overview of the expansion-planning problem in a hydropower dominated power market. Chapter 3 describes the iterative imple

mentation of the model, followed by a more detailed description of the mathematics of the inner loop in chapter 4. In chapter 5 we present the results from a case study.

## 2 PROBLEM OVERVIEW

This chapter gives a brief description of expansion planning in hydropower-dominated systems. Generation expansion or refurbishment planning should ideally solve a three-dimensional investment problem for a given planning period:

- Type and location of the new generation.
- Capacity expansion.
- Time of investment.

The planning period should at least be equal to the lifetime of the equipment. The generation expansion problem is affected by several external variables:

- Water inflow to the reservoirs.
- Future market prices.
- Investment costs.
- The properties of the new equipment.

These variables include degrees of uncertainty, the most uncertain being the future market price.

The values of the uncertain variables are coupled in time. For example, the probability of the market price for a given week is strongly dependent on the market price in the previous week. The operation of the production system can also depend on the planned investments. The generation expansion problem should therefore ideally be formulated as an integrated stochastic dynamic optimization problem, which includes both the investment costs and the operational scheduling.

For many practical purposes it is not possible to solve this optimization problem by current optimization methods. The most commonly used method by producers in Norway decomposes the generation expansion problem into two sub problems:

- Calculation of operational revenues for a given production system by simulating system operation. The time resolution used in these calculations is typically one week.
- Calculation of present value for a given system including the investment costs.

The uncertain variables, which are independent of the values the previous year, e.g. inflow and price, are modeled as stochastic variables in the system simulation model. The simulated expected operational revenues are used to calculate the present value. Normally the uncertainty in these variables is modeled by using statistics, based on historical observations, where each year is assumed to have the same probability.

The uncertain variables, whose values are strongly dependent on the value the previous year, are fixed at a constant level for a given period. The uncertainties in these variables are included manually by calculating operational revenues for different values, i.e. by assuming various scenarios.

The market price is usually modeled using a combination of stochastic variables and scenarios. On the one hand, market price is strongly dependent on precipitation and is therefore modeled by the same methods as inflow. On the other hand even more uncertainty exists when it comes to the long-term development in the market. A producer analyzing an investment in new power production in Norway would have to consider, in addition to the development of domestic demand and hydro capacity, the following factors when forecasting future prices:

- decommissioning of nuclear plants in Sweden
- increased exchange capacity (cables) to Europe
- Increasing thermal generation capacity in Norway.

When existing power plants are retired, new power production is needed in order to meet the power demand. As a result market prices will probably increase, as the substitutes tend to have higher marginal costs. Increased capacity of exchange will probably result in an increased demand for peak power, and thereby also introduce higher prices. The annual price variations will, however, probably be reduced.

Within the next few years, variations in price within the day and week are expected to increase. This is in partially due to the increasing peak load demand tied to new undersea cables, but increasing domestic demand combined with a lack of investment in new capacity the last few years also contributes to these expectations. This will of course affect which projects it will be profitable to invest in.

Forecasts for market prices are obtained by using a model such as the EFI Multi-area Production Scheduling (EMPS) model [2] for simulating optimal operation of the “global” market (Norway, Sweden, Finland and Denmark with their ties to continental Europe and Russia). The market price is coupled to external variables such as oil price or national and international policies, e.g. regarding environmental taxes. These uncertainties are typically modeled by calculating the operational revenues for different levels of these values, for example a high, medium and low level.

This type of investment analysis is often referred to as static analysis, where the results are valid for a given future.

## 3 A MODEL FOR COMBINING PRODUCTION SYSTEM AND TRANSMISSION SYSTEM

This chapter describes how to include the transmission system in a model for evaluating expected revenues from investments in a hydropower dominated system in a market regime by simulating (as closely as possible) optimal system operation. This is one of the possible application areas described in chapter 1. The new model is called Optimal Regional Planning (ORP), and is an extension of an existing long-term scheduling model, sometimes referred to as an Extended Single-Reservoir Model [3]. System operation is calculated in two stages:

1. An optimal strategy for an aggregated single reservoir equivalent is computed, using stochastic dynamic programming to calculate expected incremental water costs for an aggregate energy model of the hydro system [4], [5]. The time resolution is one week.
2. Hydro system operation is simulated week by week for a large number of price and inflow scenarios, implementing the calculated strategy from stage 1. Within each week a detailed hydro model is used to evaluate revenues. The time resolution is up to hourly, giving 168 intervals per week.

The model implies the following assumptions:

- The system owner is a price taker, i.e. the market price is not influenced by short-term variations in the owner's generation. Any effects an expansion might have on long-term price development could, however, be accounted for by recalculating the price forecast with the expanded system included in the price forecasting model.
- System operation reflects a risk neutral strategy. This can to a certain extent be obtained by introducing a penalty function [6].

Each inflow scenario corresponds to a market price scenario. Because market price for one week is very much dependent on the previous week's price, an autoregressive model is used to model future market prices for computing hydro system strategy. The refinement of adding the transmission system to the simulation takes place in stage 2.

### 3.1 Simulating system operation

In stage 2 the local power system is simulated for a number of price/inflow scenarios. This involves a weekly decision process in three stages:

1. Aggregate hydro generation is found in a market clearance process where hydro costs are represented by the incremental water costs, which are calculated in advance (stage one).
2. Aggregate generation is allotted to available power plants in a rule-based reservoir draw down model.
3. An optimal schedule within the area accounting for transmission constraints is calculated with a finer time resolution.

In the first two stages the time resolution is one week, but with a limited number of accumulated price intervals within the week (usually four). In stage three the time increment is up to 168 intervals (hours) per week. The decision process for stage 1 and 2 is illustrated as a flowchart in Fig. 1. and is explained below:

#### A Optimize aggregate generation

The optimum aggregate hydro generation is calculated, using the market price, system efficiency and water values in a linear model. The hydropower system is modeled as an aggregate reservoir, and an aggregate plant with a piecewise linear efficiency rate as well as a piecewise constant operational cost.

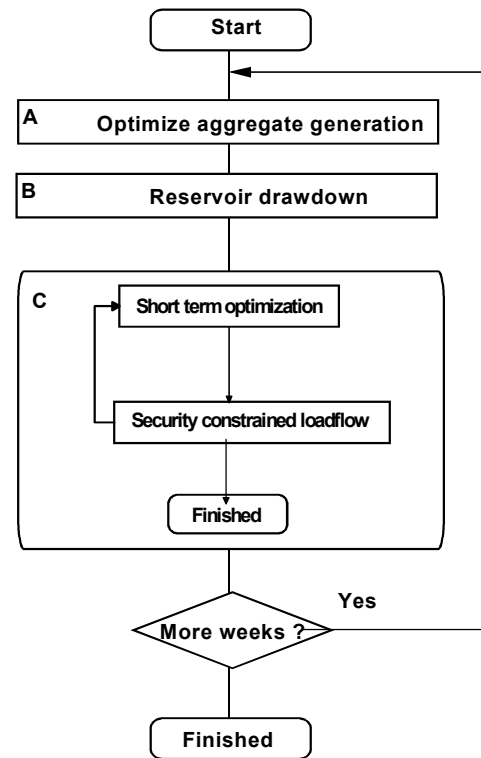


Fig. 1. Weekly simulation of hydro system operation

#### B Reservoir draw down

A detailed reservoir draw down model affords the distribution of aggregate generation among available plants, and thus the distribution of stored energy among available reservoirs, according to a rule-based strategy. When detecting an active constraint which increases costs of generation by more than 5%, the model temporarily exits B for an update of the aggregate hydro plant description in A, and recalculates the optimum generation, before continuing the reservoir draw down process in B. In [7] a more detailed description of the reservoir draw down model can be found.

#### C Scheduling including transmission grid

Using reservoir levels at the end of the week from B as target reservoirs levels for each reservoir, an optimal schedule of a detailed linear model is obtained using linear programming (LP). Given target values for end-of-week storage volumes for the reservoirs, and prices of electric power within the week, optimal plant discharges are found for the given week. In the first optimization of production schedules, transmission constraints are not included.

Then production schedules and consumption data, for each time interval, are used as input to a security constrained optimal power flow (SCOPF). In the SCOPF the objective is to minimize the required re-dispatch of power that is necessary in order to alleviate overloads. If no base-case or post-contingency (optional) overloads occur, the original power dis

patch is accepted as is. If, however, overloads exist, a linear constraint is added to the original LP-problem, which is then solved again resulting in a new power production schedule. Further details are described in chapter 4.

The new power dispatch can result in new overloads (base-case or post-contingency). The process is therefore repeated, checking for new overloads and adding new constraints if new overloads are found. The process is repeated until no more overloads occur. The process is illustrated in fig 2.

Then finally, from production and prices expected revenues can be calculated.

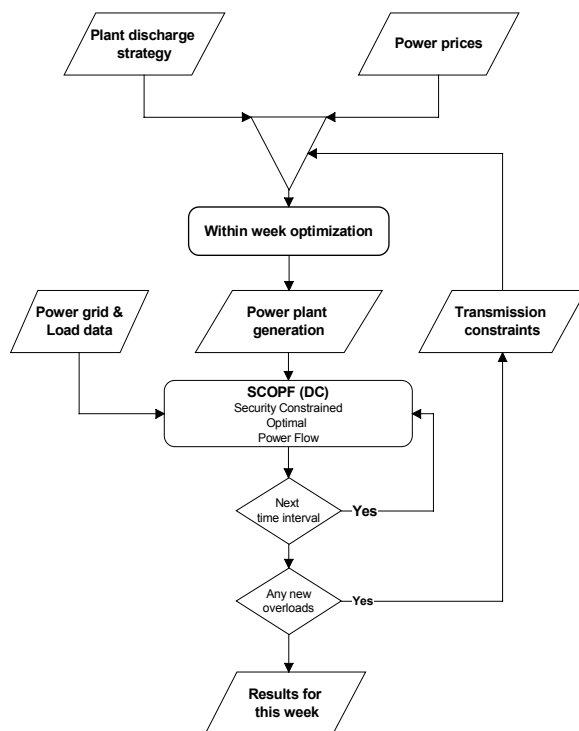


Fig. 2. Inclusion of transmission constraints.

This calculation is repeated for each week, for all the historical inflow values. Using a linear deterministic model results in a large degree of freedom with respect to the types of problems that can be modeled and analyzed within the week. This feature also makes it possible to include the modeling of the transmission system in the simulation.

In the next chapter we will describe in greater detail the optimization model we use for detailed scheduling within the week. We will also describe how we construct the hyper-plane constraints that represent the transmission system in the optimization model.

#### 4 THE SHORT-TERM OPTIMIZATION MODEL

The reservoir draw down model has already come up with a reasonably good schedule, but with a time resolution that does not satisfy our needs to account for hourly price variations and rapid changes in inflow. Neither does the reservoir drawn down model account

for possible problems in the transmission system. The only results that are kept from the draw down model are end-of-the-week stored volume for each reservoir.

In the short-term optimization the objective is to maximize revenues from sales of electrical energy for a week, assuming:

- A fixed market price, which may however vary from time interval to time interval within the week. All generated power is sold at market price.
- The target level for each reservoir is equal to the end-of-the-week storage from the reservoir draw down model. Deviation from the target reservoir level is allowed, but entails a penalty.
- Failure to meet minimum constraints on water flow is penalized.
- Daily values of inflow are used in the calculation instead of weekly inflow. This feature also contributes to better results, in terms of more realistic plant discharges and revenue potential, especially in poorly regulated hydro-systems.
- Impact from the transmission system can be accounted for in the short-term model using relaxed constraints.

The hydro system is based on the same modular description as in the reservoir draw down model, as shown in Fig. 3.

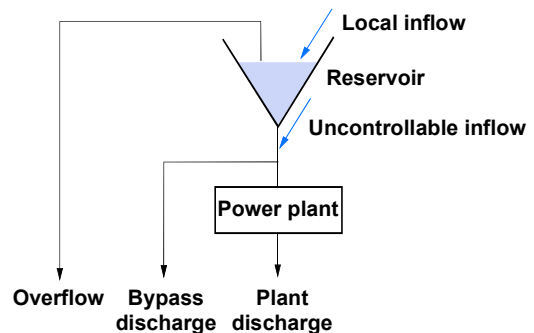


Fig. 3. A standard plant/reservoir module.

Properties, which may be attached to each plant/ reservoir module, include:

- A reservoir, defined by its volume and relationship between stored volume and elevation above sea level.
- A plant, defined by its discharge capacity and a piecewise linear relationship between discharge and generation. The relationship is put together from the data on each individual unit.
- Inflow on a daily basis, either to the reservoir or directly to the plant (implying it cannot be stored).
- Independent destinations for plant discharge, bypass discharge and reservoir overflow.
- Time variable constraints on reservoir contents and water flow (plant or bypass discharge).

- Plant discharge, which is limited by variations in head.

#### 4.1 Building the linear model

The objective is to maximize the income from operation of the local power system. Since equations are added for each individual bus bar, a traditional network formulation cannot be used.

$$MAX \left\{ \sum_{i=1}^M \sum_{t=1}^T \left( \sum_{k=1}^{N_k} (S_{k,t} - B_{k,t}) - Pm_{i,t} - Po_{i,t} \right) - Csp_i + Cst_i \right\} \quad (1)$$

Subject to:

Reservoir balances:

$$-X_{i,t-1} + X_{i,t} - \sum_{u=1}^{n_u} q_{u,t} + \sum_{d=1}^{n_d} q_{d,t} = 0 \quad (2)$$

Busbar power balances:

$$\sum_{i \in \{U_k\}} P_{i,t} + B_{t,k} - L_{t,k} - S_{t,k} = 0 \quad (3)$$

Constraints for target reservoir:

$$X_{i,T} - Target_i - ST_i + SP_i = 0 \quad (4)$$

Transmission cuts:

$$- \sum_{k=1}^K \lambda_k \cdot (P_{k,t} - P_{k,t-1}) - \omega_t \geq 0 \quad (5)$$

Minimum and maximum reservoir content:

$$X_{\min,i,t} < X_{i,t} < X_{\max,i,t} \quad (6)$$

Minimum and maximum discharge:

$$q_{\min,i,t} < q_{i,t} < q_{\max,i,t} \quad (7)$$

Constraints for target reservoir:

$$0 < ST_i < q_{\max,i,T} - Target_i \quad (8)$$

$$0 < SP_i < Target_i - q_{\min,i,T} \quad (9)$$

Explanation to notation:

- i = Module index {1, ..., M}
- t = Time index {1, ..., T}
- k = Busbar index {1, ..., K}
- u = Upstream index {set of upstream connections}
- d = Downstream index {set of downstream connections}
- S<sub>t,k</sub> = Sale at busbar k in interval t
- B<sub>t,k</sub> = Buy at busbar k in interval t
- Po = Penalty for overflow
- Pm = Penalty for not fulfilling minimum discharge
- C<sub>sp</sub> = Penalty for spending more water than Target
- C<sub>st</sub> = Revenue for storing extra water
- ST<sub>i</sub> = Penalty variable for ending over target
- SP<sub>i</sub> = Penalty variable for ending below target

X<sub>i,t</sub> = Reservoir volumes

λ<sub>i</sub> = Sensitivity for production changes in bus bar i

P<sub>0</sub> = Production before redistribution

P = Production after redistribution

ω = Lowest necessary redistribution of production

q<sub>i,t</sub> = Discharge at certain hours

P<sub>i,t</sub> = Production at module i in interval t

L<sub>t,k</sub> = Load at the busbar in interval t

Target<sub>i</sub> = Target reservoir module i

T = Last interval in the optimization period

Equation (2) gives the reservoir balances, while equation (3) gives the busbar power balances. The reservoirs must meet the target reservoirs. This is ensured by equation (4). The penalty for deviating from the target reservoir is calculated from:

$$Csp_i = Max_{price} \cdot (1 + \alpha_p) \quad (10)$$

$$Cst_i = Min_{price} \cdot (1 - \alpha_p) \quad (11)$$

Where:

Max<sub>price</sub> = Maximum price within the week

Min<sub>price</sub> = Minimum price within the week

α<sub>p</sub> = Form factor ∈ [0;1]

The operator chooses the form factor α<sub>p</sub>, so to a certain degree it is possible to choose how tightly the reservoir draw down model's reservoir operation is to be followed. It will usually not pay to deviate from target reservoir levels unless meeting them would entail a breach of constraints, running at excessively poor efficiency.

Equation (5), which adds the hyper plane "cuts", represents the transmission constraints. In this case, where each producer operates independently taking only fixed transmission tariffs into account, cuts are only added when base-case or post-contingency overloads occur, and not in order to obtain an optimal power flow. After rescheduling including transmission constraints, a new security constrained optimal load flow calculation is performed. If the load flow still results in an insecure or overloaded solution another scheduling in the linear model is needed and so fourth. It is important to keep all the calculated cuts otherwise the solution will not converge. Convergence is normally rapid as long as the transmission system is well dimensioned.

#### 4.2 Constructing the transmission constraint

From the hydro scheduling module an initial set of power injections are available. The purpose of the network module is then to form sensitivity information about how to make the network flow feasible.

An active power flow model is then used to represent the transmission system. In order to get a realistic load flow, losses are estimated for each line and distributed equally to the equivalent load vectors at the lines connection points. In an active power flow model, the rela

tion between voltage angles and power injection on nodes is given by:

$$B' \theta = P_{net} \quad (12)$$

The transmission line flow on line i-j can be expressed as:

$$f_{ij} = \frac{(\theta_i - \theta_j)}{X_{ij}} \quad (13)$$

Based on the relations given in equations (12-13), transmission line flows can be calculated as linear combinations of net power injection. It is then possible to apply a relaxation scheme and add transmission constraints iteratively when violations are detected. The optimisation problem is then formed as:

$$W = C_+^T \Delta P_+ + C_-^T \Delta P_- \quad (14)$$

Subject to:

$$\sum_k (P_{0k} + \Delta P_{+k} - \Delta P_{-k}) = 0 \quad (15)$$

$$-f_{ij} \leq A_{ij} (P_0 + \Delta P_+ - \Delta P_-) \leq f_{ij}$$

Explanation to notation:

- $P_{net}$  = Net node injection
- $\theta$  = Voltage angles
- $B'$  = Sub matrix for the load flow
- $f_{ij}$  = Transmission from i to j
- $X_{ij}$  = Line impedance
- $W$  = Minimum cost for obtaining a valid flow
- $C_+$  = Cost for increasing injection
- $C_-$  = Cost for decreasing injection
- $P_0$  = Initial injection  $P_{net}$
- $\Delta P_+$  = Needed increase in injection
- $\Delta P_-$  = Needed decrease in injection
- $A_{ij}$  = Sensitivity matrix

As seen from the objective function, the criterion is to minimize the deviation from the initial solution. At the optimal solution, the dual values (shadow prices) indi-

cate the impact on the objection function by an incremental change in power injection. When the cost coefficients in equation (14) are 1.0, the dual values can be interpreted as sensitivities and the value of the objective function as a measure of required rescheduling,  $\omega$ . After the optimal solution is found for a time step, a constraint can be formed as:

$$\omega + \lambda^T (P - P_0) \leq 0 \quad (16)$$

In principle the sensitivity rows ( $A_{ij}$ ) could have been used directly in the hydro optimisation problem. However, this would have given more constraints since the matrix  $A_{ij}$  may also include constraints related to outage conditions in the transmission system.

## 5 CASE-STUDY

In this chapter some results from a case study is presented. The purpose of the case is to demonstrate one of the appliances of the method. So far the method has been implemented as an early prototype in the ORP-model. Case data is constructed in order to demonstrate how the model can be used.

The target of the case is to analyze the effect of introducing a wind farm in a local area behind a bottleneck. A price forecast has been obtained from the EMPS model, with demand modeled in four intervals each week (peak load, mean load, off peak and weekend). The ORP-model is used to simulate before and after the introduction of a 50 MW wind farm at bus B110, see Fig. 4.

### 5.1 System data

The local power system has 10 reservoirs and 7 hydro power plants. The total installed capacity in the area is approximately 1150 MW and a total load is 875 MW (with a local load of 150 MW at bus B110). The local network is connected to the power market at bus B121.

To model the wind farm, a module as in Fig. 3. is used with the wind energy modeled as uncontrollable inflow based on daily average values.

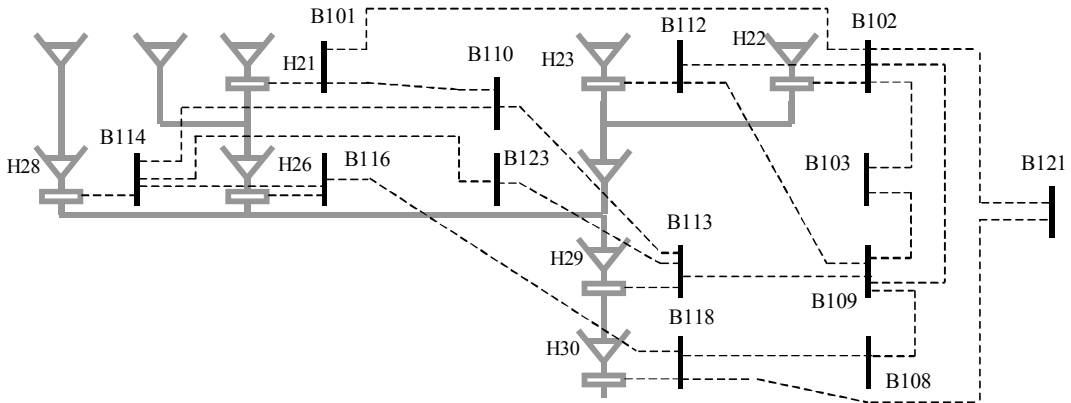


Fig. 4. Local energy system with production and transmission

## 5.2 Results

Prior to the introduction of a wind park, there is a local bottleneck. High load at bus B110, and limited transfer capacity on the transmission line B101-B110, due to security constraints (N-1-criterion), makes it necessary to redistribute power production.

Initially, the production on H21 must be reduced, and H26, H28, H29 and H30 are increased to deload the line B101-B110. The constrained operation of H21 will increase the spill of water in the cascaded system (H21, H26).

Introduction of local generation will improve utilization of the cascaded reservoir system and reduce the spill.

	Generation GWh	Bypass + overflow Mm <sup>3</sup>	Av. Hr. N-1	Economy MNOK <sup>1)</sup>
Before	2308	130.6	17.7	626
After	2457	129.9	16.0	667

**Table 1:** Results from the case in Fig. 4.

1) 9.30 NOK = 1US\$

The introduction of wind energy gives the results shown in table 1. Power generation in the local energy system increases by 149 GWh, of which the wind farm contributes with 145 GWh. The bypass and overflow are reduced by 0.7 GWh. This shows that by introducing wind power on the right side of a bottleneck, a more favorable dispatch can be found for the hydropower system. The table also shows that, even though security constraints still persist, the average number of hours per week has been slightly reduced, from 17.7 to 16 hours per week.

The economy values given in the table are not the value of the objective function, but production settled to market price. Economy shows improvement as new capacity has been added to the system. Income from wind energy counts for most of the improvement but a more flexible use of the hydro power also adds to the increased income.

The example suggests that an appropriate model of the network in the generation planning, may contribute to feasible production plans and improved overall strategy for utilization of limited hydro resources.

## 5.3 Experience

The experience from using the method so far is comforting. Convergence is swift and is normally obtained in 3-7 iterations, while the more complicated constraints can take more than 10 iterations. This depends, as always, on the problem size but in this case also on how

well dimensioned the energy system is. If the transmission system is strained, more overloads will be detected and more iterations are likely to be needed.

The model can give useful results such as shown in the example, but other results such as duration curves on transmission lines, losses and transmission tariffs can also be obtained from the model, yet this has not been implemented so far.

## 6 CONCLUSION

This paper presents a new method based on relaxation techniques for optimization of the production system with regard to the capabilities of the transmission grid. The paper describes the mathematics for calculation of the constraints from the results in the optimal power flow and how to add the transmissions constraints to the dispatch problem.

The usefulness of the method is demonstrated using a constructed case considering implementation of wind power in a local power system. Introduction of wind power in the local energy system together with other types of distributed power production makes this type of tool useful. A model of this type will be of great value both to potential investors and to authorities.

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