A NEW APPROACH FOR CIRCUIT BREAKER STATUS IDENTIFICATION IN GENERALIZED STATE ESTIMATION

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Abstract – This paper presents a method for detailed modeling and identification of breaker statuses in generalized state estimation. For each breaker its active and reactive power flows and its status (open or closed) are added as extra continuous state variables. Each breaker is characterized by its measured status as obtained by SCADA. Depending on this status, extra pseudomeasurements are introduced which define the corresponding breaker operating conditions. The method treats the analog and status measurements as "regular" measurements. Observability and bad data processing schemes are utilized both for analog and topological measurements. The $J(\hat{x})$–test and normalized residual techniques are applied for bad data detection and identification. The method is illustrated with a 20 node system.

Keywords: Generalized State Estimation, Circuit Breaker Status, Observability, Bad Data Analysis.

1 INTRODUCTION

In the conventional state estimator (SE), the bus/branch network model is determined by the topology processor, using switching device statuses, and the analog data is processed next by the state estimator. Generalized state estimation performs a detailed substation modeling (bus-section/switching device level), making it possible to detect and identify topology errors, which would appear as interacting bad data. In generalized state estimation, analog and switch data are processed as a single interacting information set.

Several methods for topology error identification can be found in the literature [1-5]. In [6] substation data is validated by a LP state estimation approach and power flows on circuit breakers (CB) are estimated. In [7-9] the WLS state estimator is presented, which models the circuit breakers as zero impedance branches and their power flows as variables. A production grade generalized state estimator is described in [10-11]. In [12] a LAV state estimation method is presented to detect and identify status errors of circuit breakers. Detailed physical level modeling of substations is done in [13] and a corresponding numerical observability scheme is presented in [14]. In [15] the well known $J(\hat{x})$-test is extended to validate hypothesis about switching device status.

In this paper a new approach is presented, for breaker status identification and bus voltage estimation. The conventional WLS state estimator is extended by incorporating the active and reactive power flows on circuit breakers and their statuses as state variables. The new feature of the algorithm is that breaker statuses are considered as unknown continuous variables. The method overcomes the drawbacks of the existing methods, which assume a certain status for each breaker and check a posteriori the status of doubtful switches by means of hypothesis testing [15]. Observability is determined by applying the reduced model based method of [16]. For the detection and identification of bad status and analog measurements, statistical tests on performance index $J(\hat{x})$ and normalized residuals are performed [18]. The proposed method is illustrated with a 20 node bus system.

2 STATE ESTIMATION BACKGROUND

For a N-bus system the state vector x of dimension $n = 2N - 1$ is defined as:

$$x^T = \left[x_1^T, x_2^T, \ldots, x_i^T, \ldots, x_N^T\right] \quad (1)$$

where $x_1 = V_1$ and $x_i^T = \left[\delta_i,\ V_i\right],\ i = 2,\ldots,N$. The measurement model is written as:

$$\begin{aligned} &z = h(x) + v \\ &E(v) = 0 \quad E(vv^T) = R \end{aligned} \quad (2)$$

where, z is a $mx1$ measurement vector, v is a $mx1$ noise vector, $h(.)$ is a $mx1$ vector of nonlinear functions and R is the diagonal measurement covariance matrix. The estimated state vector $\hat{x}$ is defined by:

$$min\ \ J(x) = \left[z - h(x)\right]^T R^{-1}\left[z - h(x)\right] \quad (3)$$

and satisfies the optimality condition:

$$\partial J(\hat{x}) / \partial x = 0 \quad (4)$$

3 CIRCUIT BREAKER MODELING

The *status* of a *telemetered* circuit breaker k - l can be defined by a discrete *(zero-one)* random variable X_{kl}, as follows:

X_{kl}*=1: closed breaker* X_{kl}*=0: open breaker*

Figure 1: Circuit breaker status modeling.

The probability function of this random variable is:

$$P(X_{kl} = 1) = s_{kl}\ ,\ \ P(X_{kl} = 0) = 1 - s_{kl}\ ,\ \ 0 \le s_{kl} \le 1 \quad (5)$$

where s_{kl} is a continuous *(non-integer)* variable. The expected value and variance of X_{kl} are defined as [19]:

$$E(X_{kl}) = \sum_i iP(X_{kl}=i) = 1P(X_{kl}=1) + 0P(X_{kl}=0) = s_{kl}$$

$$\begin{aligned} Var(X_{kl}) &= E[X_{kl} - E(X_{kl})]^2 = \sum_i (i - s_{kl})^2 P(X_{kl}=i) \\ &= (1-s_{kl})^2 P(X_{kl}=1) + (0-s_{kl})^2 P(X_{kl}=0) \\ &= (1-s_{kl})^2 s_{kl} + s_{kl}^2 (1-s_{kl}) = s_{kl}(1-s_{kl}) \end{aligned} \quad (6)$$

Let P_{kl} be the true active power flow on CB k - l. We define by Y_{kl} $\{\Theta_{kl}\}$ a zero-one random variable and by $E(Y_{kl})$ $\{E(\Theta_{kl})\}$ its expected value as follows:

$$Y_{kl} = X_{kl}P_{kl} \Rightarrow E(Y_{kl}) = E(X_{kl})P_{kl} = s_{kl}P_{kl} \quad (7)$$

$$\begin{aligned} &\Theta_{kl} = X_{kl}\Delta\delta_{kl} \equiv X_{kl}(\delta_k - \delta_l) \Rightarrow \\ &E(\Theta_{kl}) = E(X_{kl})\Delta\delta_{kl} = s_{kl}\Delta\delta_{kl} \end{aligned} \quad (8)$$

If $X_{kl} = 1$ (*closed*), P_{kl} may have any value. If $X_{kl} = 0$ (*open*) then $P_{kl} = 0$. Therefore, Y_{kl} always satisfies:

$$Y_{kl} = P_{kl} \Rightarrow E(Y_{kl}) = P_{kl} \quad (9)$$

If $X_{kl} = 0$ (open), $\Delta\delta_{kl}$ may have any value. If $X_{kl} = 1$ (*closed*) then $\Delta\delta_{kl} = 0$. Therefore, Θ_{kl} always satisfies:

$$\Theta_{kl} = 0 \Rightarrow E(\Theta_{kl}) = 0 \quad (10)$$

If we assume that status of breaker k - l is certain ($s_{kl} = 1$ for closed breaker, $s_{kl} = 0$ for open breaker), from (7), (9) and (8), (10) respectively we derive that:

$$(1 - s_{kl})P_{kl} = 0 \quad (11)$$

$$s_{kl}\Delta\delta_{kl} = s_{kl}(\delta_k - \delta_l) = 0 \quad (12)$$

Similarly, we derive that:

$$(1 - s_{kl})Q_{kl} = 0 \quad (13)$$

$$s_{kl}\Delta V_{kl} = s_{kl}(V_k - V_l) = 0 \quad (14)$$

where Q_{kl} is the true reactive power flow on CB k - l. Equations (11-14) are based on the hypothesis that status of breaker k - l is certain ($s_{kl} = 1$ or $s_{kl} = 0$). If (11-14) are not satisfied ($0 < s_{kl} < 1$) breaker status is uncertain.

4 THE EXTENDED MEASUREMENT MODEL

The measurement model (2) is extended, by taking into account the circuit breakers. For each circuit breaker k - l, *voltage angles* and *magnitudes* δ_k, V_k and δ_l, V_l of its terminal nodes, *active* and *reactive power flows* P_{kl}, Q_{kl} and *probability* s_{kl} of breaker being *closed,* are added as state variables (Fig. 2).

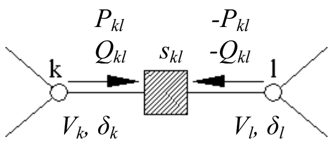

Figure 2: State variables related to circuit breaker k - l

For every circuit breaker k - l the following measurements are added:

- *pseudo measurements based on breaker status*

$$0 = \Delta\delta_{kl} \qquad 0 = \Delta V_{kl} \qquad (\text{if } X_{kl} = 1) \quad (15)$$

$$0 = P_{kl} \qquad 0 = Q_{kl} \qquad (\text{if } X_{kl} = 0) \quad (16)$$

- *pseudo measurements based on (11-14)*

$$0 = s_{kl}\Delta\delta_{kl} \qquad 0 = s_{kl}\Delta V_{kl} \quad (17)$$

$$0 = (1 - s_{kl})P_{kl} \qquad 0 = (1 - s_{kl})Q_{kl} \quad (18)$$

– *status measurements*

$$s_{kl}^m = s_{kl} + v_{s_{kl}} \quad (19)$$

where, s_{kl}^m is the *measured* status of *CB* k - l and $s_{kl}^m = 0$ ($s_{kl}^m = 1$) for *open (closed)* breaker.

For a flow measured circuit breaker k - l (Fig. 3) the following measurements are added:

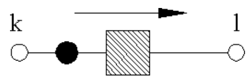

Figure 3: Flow measured circuit breaker k - l

$$P_{kl}^m = P_{kl} + v_{P_{kl}} \qquad Q_{kl}^m = Q_{kl} + v_{Q_{kl}} \quad (20)$$

For a flow measured regular branch k - l :

$$P_{kl}^m = h_{P,kl}(x) + v_{P_{kl}} \qquad Q_{kl}^m = h_{Q,kl}(x) + v_{Q_{kl}} \quad (21)$$

For an injection measured node k :

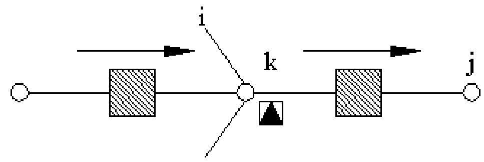

Figure 4: Injection measured node k

$$P_k^m = \sum_{i \in L_k} h_{P,ki}(x) + \sum_{j \in B_k} P_{kj} + v_{P_k} \quad (22)$$

$$Q_k^m = \sum_{i \in L_k} h_{Q,ki}(x) + \sum_{j \in B_k} Q_{kj} + v_{Q_k} \quad (23)$$

where, L_k (B_k) is the set of nodes connected to node k by *regular branches* (*circuit breakers*) and $h_{P,kl}(x)$, $h_{Q,kl}(x)$, $h_{P,ki}(x)$, $h_{Q,ki}(x)$ are the nonlinear power flow functions of x. Pseudomeasurements (15-18) are not treated as equality constraints, but as noisy measurements with proper weighting factors, as it will be explained in the next section.

5 STATE ESTIMATION ALGORITHM

The extended state vector can be defined as:

$$y = \left[x^T \; f^T \; s^T\right]^T \quad (24)$$

$$f = \left[P^T \; Q^T\right]^T \quad (25)$$

where P, Q are the $lx1$ vectors of the unknown P, Q

flows on CBs and s is the $lx1$ vector of CB statuses. The extended measurement model can be written as:

$$z = g(y) + v \equiv g(x,f,s) + v \tag{26}$$

or

$$\begin{bmatrix} z_A \\ z_B \\ z_I \\ z_S \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} h_A(x) \\ h_B(x) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ M_B \\ M_I \\ 0 \\ 0 \\ 0 \end{bmatrix} f + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_l s \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_C(x,s) \\ h_D(f,s) \end{bmatrix} + \begin{bmatrix} v_A \\ v_B \\ v_I \\ v_S \\ v_C \\ v_D \end{bmatrix} \tag{27}$$

where measurements are related to equations as follows

$z_A\ (m_A x1)$ to (15), (22) and (23) with $B_k = \emptyset$

$z_B\ (m_B x1)$ to (22), (23) with $L_k \neq \emptyset$, $B_k \neq \emptyset$

$z_I\ (m_I x1)$ to (16), (20), (22) and (23) with $L_k = \emptyset$

$z_S\ (lx1)$ to (19)

$h_C(x,s)\ (2lx1)$ to (17)

$h_D(f,s)\ (2lx1)$ to (18)

Matrices $M_B\ (m_B x2l)$, $M_I\ (m_I x2l)$ are measurement-to-breaker incident matrices and I_l is the lxl identity matrix. The state vector $\hat{y}$ minimizes the objective function:

$$J(y) = [z - g(y)]^T R^{-1} [z - g(y)] \tag{28}$$

The following iterative scheme is used to compute $\hat{y}$:

$$G(y^k)\Delta y^k = H^T(y^k)R^{-1}\Delta z^k \quad k{=}0,1,2,... \tag{29}$$

where $\Delta y^k = y^{k+1} - y^k$, $\Delta z^k = z - g(y^k)$, $H = \dfrac{\partial g}{\partial y}$ is the mxn Jacobian matrix of $g(x)$:

$$H(y) = \begin{array}{c} \begin{array}{ccc} 2N-1 & 2l & l \end{array} \\ \left[\begin{array}{ccc} H_A(x) & 0 & 0 \\ H_B(x) & M_B & 0 \\ 0 & M_I & 0 \\ 0 & 0 & I_l \\ H_{XS}(s) & 0 & H_{SX}(x) \\ 0 & H_{FS}(s) & H_{SF}(f) \end{array}\right] \end{array} \begin{array}{c} m_A \\ m_B \\ m_I \\ l \\ 2l \\ 2l \end{array} \tag{30}$$

and $G(y^k) = H^T(y^k)R^{-1}H(y^k)$ is the nxn gain matrix. The number of measurements is $m = m_A + m_B + m_I + 5l$ and the number of state variables is $n = 2N - 1 + 3l$. Pseudomeasurements (15-16) are given lower confidence and weights than those of the analog measurements. Breaker status measurements (19) and pseudomeasurements (17-18) are given weights comparable to those of analog measurements. Zero injection measurements of type (22-23) are given very large weights.

6 OBSERVABILITY ANALYSIS

The system is observable, if the available measurements are adequate to compute $\hat{y}$. Equations (29) are solvable if and only if H (or G) are of full column rank ($def[H] = def[G] = 0$). The deficiency t of matrix H (or G) is equal to the number of zero pivots in the diagonal matrix D of the $U^T DU$ decomposition of G. If $t = 0$ ($t \neq 0$), the system is *observable (unobservable)*. The $rank(H)$ is defined by Theorem 1.

Theorem 1 The column rank deficiency of the Jacobian matrix $H(y^0)$ at flat start is equal to:

$$t = def(H) = def(F) = def(F^T F) \tag{31}$$

where

$$F = \begin{bmatrix} H_A & 0 \\ H_B & M_B \\ 0 & M_I \end{bmatrix} \tag{32}$$

Proof: Without loss of generality the $P - \delta$ problem is examined. The rows of H corresponding to circuit breaker measurements are shown in Tables 1 and 2.

Equation	δ_k	δ_l	P_{kl}	s_{kl}
$\Delta\delta_{kl} = 0$	1	-1	0	0
$s_{kl} = s_{kl}^m$	0	0	0	1
$s_{kl}\Delta\delta_{kl} = 0$	s_{kl}	$-s_{kl}$	0	$\Delta\delta_{kl}$
$(1 - s_{kl})P_{kl} = 0$	0	0	$1 - s_{kl}$	$-P_{kl}$

Table 1: Part of Jacobian matrix related to closed CB k–l

Equation	δ_k	δ_l	P_{kl}	s_{kl}
$P_{kl} = 0$	0	0	1	0
$s_{kl} = s_{kl}^m$	0	0	0	1
$s_{kl}\Delta\delta_{kl} = 0$	s_{kl}	$-s_{kl}$	0	$\Delta\delta_{kl}$
$(1 - s_{kl})P_{kl} = 0$	0	0	$1 - s_{kl}$	$-P_{kl}$

Table 2: Part of Jacobian matrix related to open CB k–l

The first row in Table 1 (Table 2) corresponds to H_A (M_I) respectively. At flat start we consider:

- $\delta_k = \delta_l = 0$, $P_{kl} = 0$.
- $s_{kl} = 1$, if breaker k–l is closed (Table 1); the 4th row is zero and the 3rd row is equal to the 1st row.
- $s_{kl} = 0$, if breaker k–l is open (Table 2); the 4th row is equal to the 1st row and the 3rd row is zero.
- the 2nd row (Tables 1 and 2) is linearly independent on all other rows.

Hence $def(H) = def\begin{bmatrix} F & 0 \\ 0 & I_l \end{bmatrix} = def(F)$, which concludes the proof.

Lemma 1 $rank\, H(y^k) \geq rank\, F(y^0)$ at the $k-th$ iteration of (29).

As a consequence, if $def\ F(y^0) = 0$, then the system is observable at any iteration k ($def\ H(y^k) = 0$).

Observability determination and restoration is performed by extending the techniques of [16] on matrix F. If the system is *unobservable* (number of pivots

$t \neq 0$ during triangular factorization of $F^T F$), a minimal set of t additional *nonredundant (critical)* pseudo measurements should be selected to make the network barely observable, without affecting the estimated states of the already observable part. A measurement is said to be *critical* if its suppression from the measurement set makes the network unobservable. We form a list of candidate pseudo measurements, ordered by accuracy, which includes:

1. Measurements of z_I incident only to CBs (either breaker flow or injection at a breaker end).
2. Injection measurements of z_B incident to both CBs and regular branches. In most cases they are zero injections.
3. Flow or injection measurements of z_A incident only to regular branches.

We select a candidate pseudo measurement from the list and perform triangular factorization updating of matrix $F^T F$. If the deficiency t is decreased, this measurement is selected. This iterative procedure is repeated until matrix $F^T F$ becomes full rank.

7 BAD DATA ANALYSIS

Bad data in status and analog measurements are detected and identified by statistical tests on $J(\hat{y})$ and normalized residuals $\hat{r}_N$ respectively [18]. If $J(\hat{y}) > X^2_{k,p}$, bad data is detected; threshold $X^2_{k,p}$ is obtained from a X^2 distribution with $k = m - n$ degrees of freedom and $(1-p)$ false alarm probability. The normalized residuals are defined as:

$$\hat{r}_N = (diagP_r)^{1/2}\hat{r} \tag{33}$$

where,

$$\hat{r} = z - g(\hat{y}) \tag{34}$$

$$P_r = Cov(r) = R - H(HR^{-1}H)^{-1}H^T \tag{35}$$

Diagonal elements of P_r are derived by the sparse inverse matrix method [17]. Random vector $\hat{r}_N$ has unit normal distribution. We adopt a detection threshold of 3, corresponding to 0.003 false alarm probability. If $\left|\hat{r}_{N,i}\right|_{max} > 3$, the $i-th$ measurement is flagged as bad data. After a cycle of bad data removal, $\hat{y}$ and $\hat{r}_N$ reestimation, all bad data can be identified. At flat start, any circuit breaker $k-l$ has $\Delta\delta_{kl} = \Delta V_{kl} = 0$ and $P_{kl} = Q_{kl} = 0$. As a consequence, all status measurements s^m_{kl} of (19) become critical and if any of them is flagged as bad data, it cannot be removed from the measurement set. However, we can give very small weights to such a bad status measurement s^m_{kl} and during subsequent iterations it may become noncritical, due to values $\Delta\delta_{kl} \neq 0$ and $P_{kl} \neq 0$, and may be corrected.

The ability to detect and identify bad data, depends mainly on the measurement redundancy. We define the global measurement redundancy as:

$$r = m / n \tag{36}$$

where m, n is the total number of measurements and state variables respectively. More crucial to bad data detection and identification is the local redundancy, which is defined, for each node or substation k, as:

$$r_k = m_k / n_k \tag{37}$$

where m_k comprises the number of measurements at node or substation k and all its neighboring nodes or substations and n_k is the corresponding number of state variables. A local redundancy of $1.6 \div 2$ is considered to be optimal for a sufficient detection probability of bad data. The detailed representation of switching devices increases significantly the number of state variables and requires the availability of enough measurements at the substation level, so that the system remains observable and redundancy high. Without loss of generality, we assume that the measurement set consists of pairs of P, Q measurements. In general, for substation i with l_i CBs and N_i nodes (bus sections), the number of state variables is:

$$n_i = 2N_i + 3l_i \tag{38}$$

In practice, for a substation i we have:

- $t_i < N_i$ terminal nodes of transmission lines or transformers, taken as zero injections of z_B
- $b_i < N_i$ internal substation nodes, taken as zero injections of z_I
- $c_i \leq l_i$ flow measured circuit breakers
- l_i status measurements of type (19)
- $4l_i$ pseudomeasurements of type (17-18)
- $2l_i$ pseudomeasurements of type (15-16)

Hence, the number of measurements at substation k is:

$$m_i = 2(t_i + b_i + c_i) + 7l_i \tag{39}$$

As a consequence, the local redundancy at any substation i is significantly increased.

In the conventional state estimator, the bus/branch model is built, by merging bus sections connected by closed breakers into one or more network buses. Any substation i may be split into one or more buses and the corresponding measurement set will include t_i power flows on transmission lines or transformers. As an example, we consider the network of Fig. 5 and substation 4, for which $N_4 = 5$, $t_4 = 3$, $b_4 = 2$, $c_4 = 3$, $l_4 = 6$. According to (38) and (39), $m_4 = 58$ and $n_4 = 28$. For the computation of local redundancy r_4, nodes 1, 2, 14 *(generalized SE)* or buses 1, 2, 6 *(bus/branch model of the conventional SE)* and their corresponding injections, are taken into account. For the *conventional SE* the local redundancy r_4 of bus 4 is $r_4 = (2 \times 6) / 8 = 1.5$.

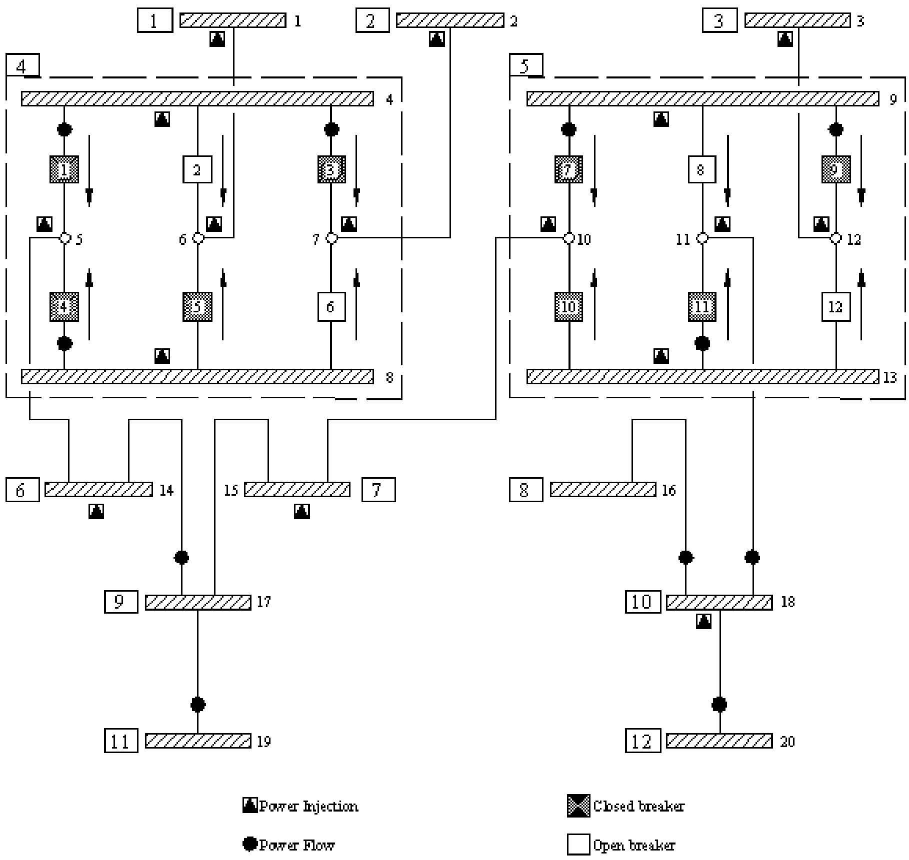

Figure 5: 20 node power system and associated measurement set.

For the *generalized SE* the local redundancy r_4 of substation 4 is $r_4 = (58+6)/(28+6) = 1.88$. It is obvious that the local redundancy in generalized SE is in general greater than that of the conventional SE due to the additional pseudomeasurements, without any addition of extra real-time analog measurements.

8 TEST SYSTEM

The method is illustrated by a 20-node system and an associated measurement set (Fig. 5). A direction is assigned arbitrarily to each breaker by an arrow. Substations 4 and 5, consisting of 6 breakers each, are modeled in detail. The remaining substations are modeled as single buses. In Fig. 5 the actual breaker statuses are shown. Without loss of generality the $P-\delta$ case is examined and branch impedances are set equal to $0+j1$. The Jacobian matrix H, as defined in (30), is of full column rank and the network is observable, since the number of pivots t in the triangular factorization of matrix $F^T F$, as defined by (32), is zero. Node 1 is the angle reference node and the first columns of Jacobian matrices H_A and H_B are omitted.

Breakers 1, 2, 3, 4, 7, 9, 10, 11 are *telemetered* as *closed*. Breakers 5, 6, 8, 12 are *telemetered* as *open*. Flow measurements P_{9-12} (on circuit breaker 9) and P_{18-11} (on regular branch) have gross errors. Breaker flows P_{8-6}, P_{8-7}, P_{9-11}, P_{13-12} are modeled as zero pseudo measurements of type (16) (open breakers). Breaker angle differences $\Delta\delta_{4-5}$, $\Delta\delta_{4-6}$, $\Delta\delta_{4-7}$, $\Delta\delta_{8-5}$, $\Delta\delta_{9-10}$, $\Delta\delta_{9-12}$, $\Delta\delta_{13-10}$, $\Delta\delta_{13-11}$ are zero pseudo measurements of type (15) (closed breakers). The values of analog measurements are shown in Tables 3, 4 and 5.

Power injection	P_1	P_2	P_3	P_{14}	P_{15}
Load flow value	1.0	1.0	2.0	-2.0	-3.0
Measured value	1.02	0.98	1.98	-2.03	-2.96

Table 3: Values of power injections (in p.u.)

Power flow	P_{4-5}	P_{4-7}	P_{8-5}	P_{9-10}	P_{9-12}	P_{13-11}
Load flow value	1.0	-1.0	1.0	2.0	-2.0	2.0
Measured value	1.02	-0.98	1.01	2.03	*-0.75*	1.97

Table 4: Values of power flows on CBs (in p.u.)

Power Flow	P_{17-14}	P_{18-16}	P_{18-11}	P_{19-17}	P_{20-18}
Measured value	0.02	3.95	*-1.0*	3.06	1.97
Load flow value	0.0	4.0	-2.0	3.0	2.0

Table 5: Values of power flows on regular branches (in p.u.)

Injection measurements at nodes 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 18 are zero injections. The estimated state vector $\hat{y} = \left[\hat{\delta}^T \ \hat{f}^T \ \hat{s}^T\right]^T$ is shown in Tables 6, 7 and 8.

Bus angles	Estimated values	Bus angles	Estimated values
δ_1	0.000	δ_{11}	-5.186
δ_2	0.105	δ_{12}	-5.186
δ_3	-3.501	δ_{13}	-5.186
δ_4	-0.924	δ_{14}	-2.878
δ_5	-0.924	δ_{15}	-5.487
δ_6	-0.924	δ_{16}	-10.220
δ_7	-0.924	δ_{17}	-2.829
δ_8	-0.924	δ_{18}	-6.569
δ_9	-5.186	δ_{19}	0.231
δ_{10}	-5.186	δ_{20}	-4.300

Table 6: Estimated vector $\hat{\delta}$

Breaker flows	Estimated values	Breaker flows	Estimate values
P_{4-5}	1.144	P_{9-10}	1.794
P_{4-6}	-0.309	P_{9-11}	-0.503
P_{4-7}	-0.835	P_{9-12}	-1.291
P_{8-5}	0.809	P_{13-10}	-1.492
P_{8-6}	-0.615	P_{13-11}	1.886
P_{8-7}	-0.194	P_{13-12}	-0.393

Table 7: Estimated vector $\hat{f}$

Breaker statuses	Estimated values	Breaker statuses	Estimated values
s_{4-5}	1.000	s_{9-10}	1.000
s_{4-6}	1.000	s_{9-11}	0.202
s_{4-7}	1.000	s_{9-12}	1.000
s_{8-5}	1.000	s_{13-10}	1.000
s_{8-6}	0.274	s_{13-11}	1.000
s_{8-7}	0.036	s_{13-12}	0.134

Table 8: Estimated vector $\hat{s}$

Bad analog measurements will be identified, based on normalized residuals. After the first estimate $\hat{y}$ is calculated, the five largest normalized residuals are:

Variable	P_{9-12}	s_{8-6}	P_{18-11}	s_{9-11}	P_3
$\lvert r_{N,i}\rvert$	76.759	63.276	53.869	53.714	52.760

Table 9: The five largest normalized residuals (1st SE run)

Flow measurement P_{9-12}, having the largest normalized residual, is flagged as bad data, since $\lvert r_{N,i}\rvert_{max} = 76.759 > 3$. After eliminating this measurement, the estimate $\hat{y}$ and the normalized residuals are reevaluated. The five largest normalized residuals are shown in Table 10.

Variable	P_{18-11}	s_{8-6}	P_{13-11}	P_{20-18}	P_{18-16}
$\lvert r_{N,i}\rvert$	69.461	63.276	40.311	39.320	39.320

Table 10: The five largest normalized residuals (2nd SE run)

Power flow P_{18-11} has the largest normalized residual and it is flagged as bad data, since $\lvert r_{N,i}\rvert_{max} = 69.461 > 3$. After removing this measurement, the estimate $\hat{y}$ is reevaluated and the five largest normalized residuals are shown in Table 11.

Variable	s_{8-6}	P_{8-5}	s_{8-7}	P_{4-7}	P_{4-5}
$\lvert r_{N,i}\rvert$	63.276	30.449	27.284	22.490	19.938

Table 11: The five largest normalized residuals (3rd SE run)

Status measurement s_{8-6} of circuit breaker 5 is flagged as bad data, with $\lvert r_{N,i}\rvert_{max} = 63.276 > 3$. After removing this measurement, all new normalized residuals have $\lvert \hat{r}_{N,i}\rvert < 3$, meaning that all bad data have already been eliminated. The new (correct) state vector $\hat{y}$ is computed and shown in Tables 12 and 13.

Bus angles $\hat{\delta}$	Estimated values	Bus angles $\hat{\delta}$	Estimated values
δ_1	0.000	δ_{11}	-5.929
δ_2	-0.042	δ_{12}	-5.929
δ_3	-3.935	δ_{13}	-5.929
δ_4	-1.027	δ_{14}	-3.040
δ_5	-1.027	δ_{15}	-5.955
δ_6	-1.027	δ_{16}	-11.841
δ_7	-1.027	δ_{17}	-3.021
δ_8	-1.027	δ_{18}	-7.897
δ_9	-5.929	δ_{19}	0.039
δ_{10}	-5.929	δ_{20}	-5.921

Table 12: Estimated vector $\hat{\delta}$

Breaker flows ($\hat{f}$)	Estimated values	Breaker statuses ($\hat{s}$)	Estimated values
P_{4-5}	1.011	s_{4-5}	1.000
P_{4-6}	-0.029	s_{4-6}	1.000
P_{4-7}	-0.982	s_{4-7}	1.000
P_{8-5}	1.001	s_{8-5}	1.000
P_{8-6}	-0.999	s_{8-6}	1.000
P_{8-7}	-2.6x10^{-3}	s_{8-7}	6.5x10^{-6}
P_{9-10}	2.016	s_{9-10}	1.000
P_{9-11}	-8.1x10^{-3}	s_{9-11}	6.5x10^{-5}
P_{9-12}	-2.008	s_{9-12}	1.000
P_{13-10}	-1.990	s_{13-10}	1.000
P_{13-11}	1.976	s_{13-11}	1.000
P_{13-12}	0.014	s_{13-12}	1.3x10^{-4}

Table 13: Estimated vectors $\hat{f}$ and $\hat{s}$

The erroreous telemetered status s_{4-6} of breaker *2* cannot be detected, because the estimated angle difference $\Delta\hat{\delta}_{4\text{-}6} = \hat{\delta}_4 - \hat{\delta}_6$ is zero. However the estimated power flow $\hat{P}_{4-6}$ is very small and is an indication that breaker *2* is actually open.

9 CONCLUSIONS

In this paper a new state estimation method was presented, for identification of breaker statuses in generalized state estimation. Any substation can be modeled in detail, by extending the conventional WLS state estimator and incorporating the active and reactive power flows on circuit breakers and their statuses as state variables. Depending on breaker statuses, additional pseudomeasurements are introduced. Analog and status measurements are treated as regular measurements. Observability is performed for analog and topological measurements. The $J(\hat{x})$-test and normalized residual techniques are used for bad data detection and identification. Depending on analog measurement redundancy, correct breaker statuses can be estimated. The method is illustrated by a 20 node system.

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