AN ESTIMATION OF LOAD POWER MARGINS FOR BRANCH OUTAGE CONTINGENCIES

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Abstract – This paper proposes a fast computation method to evaluate the voltage stability of the power system for branch outage contingencies. The load power margins with respect to saddle node bifurcations under post-contingency conditions are efficiently evaluated. The accuracy is considerably improved based on sensitivity analysis taking into account nonlinearities of the power flow equations and using a fast line search technique. The effectiveness of the proposed method is demonstrated through IEEE 14, 30, 57 bus systems. The maximum error in load power margins is not more than 15% for all possible branch outage contingencies including route open outages.

Keywords: voltage stability, contingency selection, load power margin, saddle node bifurcation.

1 INTRODUCTION

In recent years, much more attention has been paid to voltage security analyses of electric power systems. This issue is thought to be an inevitable and emergent task in the power system as well as steady state and dynamic security assessment. A faster and more accurate method is required in general in order to perform the analyses more efficiently for a numerous number of contingencies on-line.

Various methods have been proposed so far for voltage stability evaluation. For example, a series of methods [1~4] have been proposed to predict the load power margins for post-contingency systems based on the sensitivity analysis of the pre-contingency system. A look-ahead method is another approach to predict post-contingency conditions based on a quadratic curve fit [5]. The use of the reactive power reserve has been proposed as an index to evaluate the voltage stabilities of post-contingency systems [6~7], which may also be able to treat the Q-limit instability but the accuracy is unknown. Other stability indices have been proposed based on the second order information derived from the singular value analysis of the Jacobian in [8]. Furthermore, the relations between voltages and local reactive supports have been utilized to propose a stability index for branch contingencies in [9]. Those methods are fast enough for on-line applications but a common problem in general is an accuracy of the estimations. Specially, the accuracy in estimating load power margins is usually a critical factor.

The load power margin is widely accepted as a most informative index representing directly the degree of voltage stability as shown in figure 1. Let λ be a parameter varying in proportion to the total load power and $\lambda=0$ be considered as the base loading at an operating point. Then, the critical value of λ directly represents the load power margin. Generally, when a contingency such as branch outage contingency occurs, the load power margin changes. The post-contingency system is stable when λ is positive, while voltage collapse occurs when λ is negative.

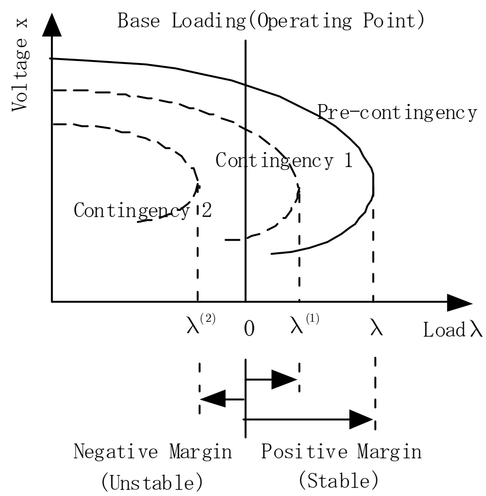

Figure 1: Changes in load power margins due to contingencies

This paper proposes a fast and more accurate computation method to estimate the load power margins for branch outage contingencies by extending the method in reference [10], where the saddle node bifurcations are the targets. Based on the linear sensitivity approaches [1~4], the accuracy of estimations will be considerably improved through taking into account the nonlinearities of the power flow equations and through the use of a fast line search technique. The effectiveness of the proposed method is demonstrated through the IEEE 14, 30, 57 bus systems.

The proposed method first computes the exact nose point for the pre-contingency condition in figure 1 and then linearly estimates nose points for post contingency systems, which is the same manner as the conventional sensitivity method. A major difference lies in the latter step in that the linear method directly estimates the post-fault loading margin, while the proposed method first linearly estimates the nose point voltages for the post-fault system and then the loading margin is evaluated as a function of the estimated voltage deviations by solving a scalar nonlinear equation.

2 MATHEMATIC FORMULATION

2.1 Contingency Parameter

Branch outage contingencies may be expressed with contingency parameter $p^{(k)}$

$$(1 - p^{(k)})y_k^{(k)} \tag{1}$$

where $y_k^{(k)}$ represents the admittance of branch (k) to be faulted. $p^{(k)}=0$ represents the pre-fault conditions and $p^{(k)}=1$ represents the post-fault conditions.

2.2 Power Flow Equations Using Loading parameter λ

The power flow equations are expressed as follows.

$$f^{(k)}(x,\lambda,p^{(k)}) = y_0 + \lambda y_d - g(x,p^{(k)}) = 0 \tag{2}$$

where y_0 represents real and reactive power node injections at an operating point. y_d is a change direction of y_0. $g(x,p^{(k)})$ is the power flow function. Vector x stands for bus voltages. λ is the loading parameter with λ=0 at the base loading as defined in figure 1. Superscript (k) is a contingency number.

Typical loci of the solutions of the equations (2) are shown in figure 1, which are called as P-V curves. The conditions of saddle node bifurcations are given as follows.

$$f^{(k)}(x,\lambda,p^{(k)}) = 0 \tag{3}$$

$$w^T D_x g = 0 \tag{4}$$

$$\| w \| \neq 0 \tag{5}$$

or

$$f^{(k)}(x,\lambda,p^{(k)}) = 0 \tag{6}$$

$$D_x g v = 0 \tag{7}$$

$$\| v \| \neq 0 \tag{8}$$

where w and v are left and right eigenvectors respectively corresponding to the zero eigenvalue of the singular Jacobian $D_x g$ of the power flow equations (3). Note that conditions (3), (4), (5) (or (6), (7), (8)) is useful for pre-fault ($p^{(k)}=0$) as well as post-fault ($p^{(k)}=1$). In the proposed method, we first solve the above equations under the pre-fault conditions to obtain $\underline{x}$, $\underline{\lambda}$, $\underline{w}$, $\underline{v}$, the solutions for pre-fault system. Then, the problem becomes how to estimate the post-fault load power margins λ from $\underline{x}$, $\underline{\lambda}$, $\underline{w}$, $\underline{v}$ efficiently.

3 CONTINGENCY EVALUATION IN TERMS OF LOAD POWER MARGINS

In this section, a linear sensitivity relation is presented first to estimate the post-fault solutions. Then, a formula is proposed to improve the linear estimations by taking into account the nonlinearities of the power flow equations. Finally, a line search technique with optimal multipliers is given to further decrease estimation errors.

3.1 Linear Estimations based on Sensitivities

Equations (3) and (4) respectively are linearized around the pre-fault solutions, $\lambda = \underline{\lambda}$, $x = \underline{x}$, $p^{(k)}=0$ to obtain

$$\Delta\lambda y_d - D_x g^{(k)}\big|_{\underline{x}}\, \Delta x - D_{p^{(k)}} g^{(k)}\big|_{\underline{x}}\, \Delta p^{(k)} = 0 \tag{9}$$

$$\Delta w^T D_x g^{(k)}\big|_{\underline{x}} + \underline{w}^T D_{xp^{(k)}} g^{(k)}\big|_{\underline{x}}\, \Delta p^{(k)} + \Delta x^T \sum_i \underline{w}_i D_{xx} g_i^{(k)}\big|_{\underline{x}} = 0 \tag{10}$$

Equation (10) multiplied by v and using (7) yields.

$$\underline{w}^T D_{xp^{(k)}} g^{(k)}\big|_{\underline{x}}\, \underline{v} \Delta p^{(k)} + \underline{v}^T \sum_i \underline{w}_i D_{xx} g_i^{(k)}\big|_{\underline{x}}\, \Delta x = 0 \tag{11}$$

Equations (9) and (11) can be written in the matrix form as

$$H \begin{bmatrix} \Delta x^{(k)} \\ \Delta\lambda^{(k)} \end{bmatrix} = r^{(k)} \tag{12}$$

where

$$H = \begin{bmatrix} D_x g^{(k)}\big|_{\underline{x}} & -y_d \\ \underline{v}^T \sum_i \underline{w}_i D_{xx} g_i^{(k)}\big|_{\underline{x}} & 0 \end{bmatrix} \tag{13}$$

$$r^{(k)} = \begin{bmatrix} D_{p^{(k)}} g^{(k)}\big|_{\underline{x}} \Delta p^{(k)} \\ \underline{w}^T D_{xp^{(k)}} g^{(k)}\big|_{\underline{x}}\, \underline{v} \Delta p^{(k)} \end{bmatrix} \tag{14}$$

Note that matrix H is a constant square matrix and is independent of contingencies, while vector $r^{(k)}$ depends on contingency (k). The dimension of (12) is n+1, where n stands for the dimensions of the power flow equations. The post-fault condition is expressed with $\Delta p^{(k)}=1$ in (12), which is solved to obtain the post-fault voltages $\hat{x}^{(k)}$ and load power margin $\hat{\lambda}^{(k)}$ as follows.

$$\hat{x}^{(k)} = \underline{x} + \Delta x^{(k)} \tag{15}$$

$$\hat{\lambda}^{(k)} = \underline{\lambda} + \Delta\lambda^{(k)} \tag{16}$$

Since matrix H is constant, the LU factorization of H is required only once for all contingencies. Only forward and backward substitutions are required with different $r^{(k)}$ for each contingency (k). The linear estimation by (16) is exactly equivalent to those obtained by the method proposed in [1-4], where a more efficient formula has been presented to estimate the load power margin only.

3.2 Nonlinear Formula

As will be shown in the latter section, the linear estimations by (16) are quite erroneous. Therefore, we will take into account the nonlinearities of the power flow equations. The post-fault system must satisfy:

$$\underline{w}^T(y_0 + \lambda y_d - g(x,1)) = 0 \tag{17}$$

Rearranging (17), we have the following equation for the post-fault load power margin $\lambda^{(k)}$ as follows.

$$\lambda^{(k)} = \frac{\underline{w}^T(g(x^{(k)},1) - y_0)}{\underline{w}^T y_d} = F^{(k)}(x^{(k)}) \tag{18}$$

Note that $\lambda^{(k)}$ is a function of the post-fault bus voltages. A direct substitution of the linear estimates $\hat{x}^{(k)}$ into (18) will provide an improved estimation. However, more exact estimations are obtainable when combining (18) with another technique to be proposed in the following section.

3.3 Line Search

In order to further improve the estimations, a line search technique with optimal multipliers is applied to this problem. Two variants of the method will be developed.

Proposed Method 1:

Instead of using (15), we assume the following voltage estimation:

$$\tilde{x}^{(k)} = \underline{x} + \mu^{(k)}\Delta x^{(k)} \tag{19}$$

where $\mu^{(k)}$ is an optimal multiplier to be determined. Method 1 is to determine $\mu^{(k)}$ by solving the following minimization problem.

$$\min_{\mu^{(k)}} \varepsilon^{(k)}(\mu^{(k)}) \tag{20}$$

where

$$\begin{aligned}&\varepsilon^{(k)}(\mu^{(k)})\\ &= f(\tilde{x}^{(k)}(\mu^{(k)}),\hat{\lambda}^{(k)},1)^T W f(\tilde{x}^{(k)}(\mu^{(k)}),\hat{\lambda}^{(k)},1)\\ &= \mu^{(k)4}a_4^{(k)} + \mu^{(k)3}a_3^{(k)} + \mu^{(k)2}a_2^{(k)} + \mu^{(k)}a_1^{(k)} + a_0^{(k)}\end{aligned} \tag{21}$$

where $f^{(k)}(\tilde{x}^{(k)},\hat{\lambda}^{(k)},1)$ is the post-fault power flow mismatch whose norm is used as the objective function. W is a constant diagonal weight matrix, in which diagonal elements are set to unity except the two elements corresponding to the buses connected to the faulted branch. The values of these two elements are denoted as σ which will be examined numerically in the latter section. Since (21) is a scalar fourth order polynomial, a very fast solution is obtainable to yield the voltage estimation of (19), which will be substituted into (18) to obtain the final estimation of the load power margin. Note that the objective function (21) of method 1 utilizes a fixed value of $\hat{\lambda}^{(k)}$, which has been obtained as the linear estimation by (15) in advance.

Proposed Method 2:

Method 2 differs from method 1 in the treatment of $\hat{\lambda}^{(k)}$ only. A modified objective function is written as follows.

$$\begin{aligned}&\varepsilon^{(k)}(\mu^{(k)})\\ &= f(\tilde{x}^{(k)}(\mu^{(k)}),\tilde{\lambda}^{(k)}(\mu^{(k)}),1)^T W f(\tilde{x}^{(k)}(\mu^{(k)}),\tilde{\lambda}^{(k)}(\mu^{(k)}),1)\\ &= \mu^{(k)4}a_4^{(k)} + \mu^{(k)3}a_3^{(k)} + \mu^{(k)2}a_2^{(k)} + \mu^{(k)}a_1^{(k)} + a_0^{(k)}\end{aligned} \tag{22}$$

where

$$\begin{aligned}\tilde{\lambda}^{(k)} &= \frac{\underline{w}^T(g(\underline{x} + \mu^{(k)}\Delta x^{(k)},1) - y_0)}{\underline{w}^T y_d}\\ &= F^{(k)}(\underline{x} + \mu^{(k)}\Delta x^{(k)})\end{aligned} \tag{23}$$

Note that $\tilde{\lambda}^{(k)}$ is assumed as a function of $\mu^{(k)}$ using (18). This treatment will give a different solution $\mu^{(k)}$ of the minimization problem from that of method 1. Using this solution, the final estimation is obtained by (18).

3.4 Computational Procedure

The procedure of the proposed method is summarized as follows.

1) Specify the conditions of the operating point.
2) Compute $\underline{x}$, $\underline{\lambda}$, $\underline{w}$ and $\underline{v}$ for pre-fault system.
3) Compute matrix H and perform the factorization.
4) Repeat below for all the contingencies.
   a. Compute $r^{(k)}$.
   b. Compute $\Delta x^{(k)}$, $\Delta\lambda^{(k)}$ by solving (12) with the factorized H.
   c. Proposed Method 1:
   Solve (20) with (21) for $\mu^{(k)}$.
   Proposed Method 2:
   Solve (20) with (22) for $\mu^{(k)}$.
   d. Evaluate $\tilde{x}^{(k)}$ by (19) and compute the post-fault load power margin $\tilde{\lambda}^{(k)}$ using (18).

The total computation time for K branch outage contingencies is α+K β, where α corresponds to a preliminary computation, which is equivalent to 4~10 ordinary power flow computations. The preliminary computation corresponds to steps (1) to (3) above, the computation of the saddle node point for pre-contingency conditions and a LU factorization of the power flow Jacobian. β is the computation time for a single contingency, which is equivalent to a single iteration in a power flow computation. A major parts of this computation is step (4)-b, the forward and backward substitutions for the factorized Jacobian, while the other computations including the nonlinear estimation with the line search are negligibly fast.

4 NUMERICAL EXAMINATION

In this section, we examine the performance of the proposed method through IEEE 14, 30, 57 bus systems. The pre-fault maximum loading point is computed using the Point of Collapse method [11], where reactive power limits of generators are taken into account. All the possible route open branch outages are tested except the cases where islanding occurs.

A parameter σ in weight matrix W in (21) and (22) will be examined by changing its value as 1, 10, 100, 1000, 10000 in order to evaluate the accuracy of the proposed method.

The results are examined in terms of the prediction errors in load power margins ΔP in p.u. given as

$$\Delta P = \lambda P_0 \tag{23}$$

where P_0 is the base load at the operating point corresponding to λ=0 in figure 1. The above ΔP directly indicates the active power margin of the studied system.

Tables 1~3 lists the computed values of ΔP [p.u.] for 14, 30, 57 bus systems respectively, where all the possible contingencies have been tested and numbered by severity order based on their exact values of ΔP, then the most severe cases are listed. The exact value implies the result obtained by the Point of Collapse method for every contingency. The tables also compare the performances of different methods, which include the conventional sensitivity method [1~4] as well as the proposed methods. It is observed that the most severe case for each system shows a negative value of ΔP, which implies the voltage collapse. When the exact value is compared with the estimated value by each method, superior performance of the proposed method may be seen. The observation shows that the estimation error tends to increase for cases with more severe contingencies. Although this is the case for all the methods, the characteristic is especially abysmal for the conventional method, which is no more useful for severe contingencies. On the other hand, the degradation of the performance of the proposed methods is quite limited.

Contingency	Faulted Branch	Exact Value	Sensitivity	Method 1 (σ=1)	Method 2 (σ=1)
1	1-2	-0.0605	1.2151	-0.1916	-0.3581
2	2-3	0.7698	1.4948	0.9181	0.9407
3	1-5	1.0175	1.2255	1.0229	1.0169
4	7-9	1.0846	1.8051	1.1554	1.0615
5	2-4	1.5059	1.6386	1.5038	1.5054

Table 1: Load power margins for 14 bus system ΔP [p.u.]

Contingency	Faulted Branch	Exact Value	Sensitivity	Method 1 (σ=1)	Method 2 (σ=1)
1	1-2	-0.3195	0.8094	-0.4221	-0.5409
2	2-5	0.3803	1.0731	0.4718	0.4984
3	1-3	0.6649	0.9567	0.6732	0.6631
4	3-4	0.6817	1.3836	0.6747	0.6521
5	2-6	1.0743	1.1956	1.0723	1.0719

Table 2: Load power margins for 30 bus system ΔP [p.u.]

Contin-gency	Faulted Branch	Exact Value	Sensitivity	Method 1 σ=1000	Method 2 σ=100
1	35-36	-0.9831	7.2636	-2.6045	-2.0312
2	25-30	2.2825	7.1955	3.5971	3.7513
3	34-35	2.9318	7.2964	2.8168	2.8167
4	37-38	3.6354	7.0881	3.4155	3.2074
5	1-15	4.3447	5.6082	4.3961	4.4106

Table 3: Load power margins for 57 bus system ΔP [p.u.]

In order to examine the performances of the methods more exactly, examination will be performed in terms of the errors defined as:

$$\%Error = \frac{P|_{estimate} - P|_{exact}}{P|_{exact}} \times 100 \qquad (24)$$

where $P|_{estimate} = P_0 + \Delta P$, which is the estimated real power at the collapse point, while $P|_{exact}$ is its exact value.

		14 Bus System		30 Bus System		57 Bus System	
		Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Proposed Method	σ=1	5.18	11.76	7.63	8.80	73.32	21.33
	σ=10	9.33	13.98	7.56	9.56	66.28	29.36
	σ=100	9.78	14.24	7.56	9.63	37.22	10.31
	σ=1000	9.83	14.26	7.56	9.64	14.78	34.65
	σ=10000	9.83	14.26	7.56	9.64	14.78	34.65
Sensitivity		50.42		44.90		75.14	

Table 4: Maximum estimation errors for branch outage contingencies (%)

		14 Bus System		30 Bus System		57 Bus System	
		Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Proposed Method	σ=1	1.13	1.32	0.59	0.74	3.68	2.06
	σ=10	1.39	1.56	0.65	0.76	2.79	1.30
	σ=100	1.46	1.58	0.66	0.76	1.41	0.73
	σ=1000	1.47	1.59	0.66	0.76	0.83	1.16
	σ=10000	1.47	1.59	0.66	0.76	0.83	1.23
Sensitivity		8.28		4.45		5.49	

Table 5: Mean estimation errors for branch outage contingencies (%)

Table 4 and 5 show respectively the maximum and mean values of the estimation errors of (24) for all the tested contingencies. The performance of the proposed method for different setting of parameter σ may be examined as well as the performance of the conventional method. It is observed that the estimation errors are clearly much smaller for the proposed methods compared with the conventional sensitivity method in all the cases.

Observations for the proposed methods are:

(1) To obtain the maximum performance for a specific system, a suitable selection of method and the value of σ are important. For example, method 1 with σ =1 shows the best performance for the 14 and 30 bus systems, where the maximum estimation error is 7.63%.

(2) Method 2 with σ =100 shows allround performance. The maximum errors are less than 15% for all the systems. This setting may be used as the default option for the proposed method.

It has been also confirmed that the computation time of the proposed methods roughly agrees with the theoretical computation time, which has been given in the previous section.

5 CONCLUSION

In this paper, a fast and accurate computation method has been proposed to evaluate the load power margins for contingencies. The computation time for one contingency case is less than a single iteration time of the power flow computation. The accuracy is high enough to evaluate voltage stabilities. The errors were less than 15% for all the branch outage (route open) contingencies including voltage collapse cases.

The accuracy and the computation time are very important factors for a contingency evaluation method. Specially, a critical issue is the maximum error of the method since an estimation error in general tends to increase for a severe contingency. In this sense, the proposed method shows a quite robust performance and therefore the method may be useful for real time contingency evaluations.

On the other hand, the conventional linear sensitivity method is inaccurate but much faster than the proposed method. This implies that a proper combination of the methods, such as a strategy with multi-layer screening, may considerably improve the efficiency and accuracy of on-line computations.

Although the saddle node bifurcations have been treated in this paper, a further study is required for the immediate instabilities induced by reactive power limits of generators since the existing methods to compute the instabilities such as [12] are still time consuming when using for contingency analysis.

REFERENCE

[1] I. Dobson, "Computing an optimal direction in control space to avoid saddle node bifurcation and voltage collapse," IEEE Trans. On Automatic Control, AC-37, No.10, pp1616-1620, 1992.

[2] S. Greene, I. Dobson, and F. L. Alvarado, "Sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters," IEEE Trans. On Power System, PWRS-12, No.1, pp.262-272, 1997.

[3] N. Yorino, K. Hayashi, S. Harada, H. Sasaki, "On contingency analysis based on maximum loading point," Proceeding of the Eighth Annual Conference of Power & Energy Society, IEE of Japan, No.208-209, 1997.

[4] S. Greene, I. Dobson, F. L. Alvarado, "Contingency Ranking for Voltage Collapse via Sensitivities from a Single Nose Curve," IEEE Trans. on Power System, Vol.14, No.1, pp.232-240, 1999.

[5] H. D. Chiang, C. S. Wang, A. J. Flueck, "Look-ahead voltage and load margin contingency selection functions for large-scale power system, " IEEE Trans. on Power System, Vol.12, No.1, pp.173-180, 1997.

[6] T. Van Cutsem, C. Moisse, R. Mailhot, "Determination of secure operating limits with respect to voltage collapse, " IEEE Trans. on Power System, Vol.14, No.1, pp.327-335, 1999.

[7] F. Capitanescu, T. Van Cutsem, "Evaluation of reactive power reserves with respect to contingencies," Bulk Power System Dynamics and Control V, August 26-31, 2001, Onomichi, Japan.

[8] A. Berizzi, Y. G.Zeng, P. Marannino, A. Vaccarini, "A Second Order Method for Contingency Severity Assessment with Respect to Voltage Collapse," IEEE Trans. on Power System, Vol.15, No.1, pp.81-87, 2000.

[9] H. Liu, A. Bose, V. Venkatasubramanian, "A Fast Voltage Security Assessment Method Using Adaptive Bounding," IEEE Trans. on Power System, Vol.15, No.3, pp1137-1141, 2000.

[10] N. Yorino, S. Harada, K. Hayashi, H. Sasaki, "A method of voltage stability evaluation for contingencies," Bulk Power System dynamics and Control IV, Santorini, Greece, August, 1998.

[11] C. A. *Cañizares* , F. L. Alvarado, "Point of Collapse and Continuation Method for Large AC/DC System," IEEE Trans. on Power System, Vol.8, No.1, pp.1-8, 1993.

[12] N. Yorino, H. Sasaki, "A Continuation Power Flow Tracing Q-Limit Points on Bifurcation Diagrams in Power Systems," Trans IEE of Japan, Vol. 119-B, No. 2, pp. 259-267, 1999.