Power Flow Solutions of AC/DC Micro-grid Structures

Enrique ACHA and Tom RUBBRECHT
Department of Electrical Engineering
Tampere University of Technology
Tampere, Finland

Luis M. CASTRO
Department of Electrical Engineering
National Autonomous University of Mexico
Mexico city, Mexico

Abstract—This paper presents a new and general frame-of-reference for the unified, power flow solution of AC and DC micro-grids using the Newton-Raphson method, where the quadratic convergence towards the solution is preserved. The cornerstone of this modeling development in power flow theory is the so-called multi-terminal VSC-HVDC system. In this frame-of-reference, an AC micro-grid of arbitrary configuration is connected to the high-voltage side of the LTC transformer of a VSC station. In turn, the DC side of each VSC is linked to a DC system of arbitrary configuration. Any number of AC micro-grids may exist and the DC system may contain single load or generation points such as a PV installation. Each VSC model takes into account, in aggregated form, the phase-shifting and scaling nature of the PWM control. It also accounts for the VSC current design limits, PWM limits within the linear range, switching losses and ohmic losses.

Index Terms—Micro-grids, multi-terminal HVDC systems, Newton-Raphson method, power flows, VSC modeling

I. INTRODUCTION

The global electricity supply industry is undergoing unprecedented change to be able to cope with major challenges arising from an ageing infrastructure, market liberalisation and the availability of renewable generation. Over the past decade, the concepts of active networks, micro-grids and smart grids have been put forward as theoretical frameworks aimed at addressing these challenges [1]-[2]. It is said that a micro-grid will contain de-centralized electricity generation combined with on-site production of heat, bringing about substantial environmental benefits to society. With the use of modern technology, micro-grids enable the integration of renewable energy sources and achieve a good match between generation and load inside the micro-grid, reducing the impact on the neighboring electricity network.

In a more general sense, the consensus is that tomorrow’s power grids must ensure secure and sustainable electricity supplies with low losses and low CO2 emissions [3]. In Europe, these power grids should also comply with new policy imperatives, changing business frameworks and to incorporate the state-of-the-art information technology, communications technology and the latest generation of electrical equipment. Paramount in this array of new technologies is the ubiquitous power electronic converter, which power engineers have used in a variety of forms to enable the instantaneous control of the voltage and current waveforms in the electrical power grid [4]. Power electronics converters are also used in grid connection of renewable sources of electrical energy and storage systems [5].

Current research efforts in the power electronics area concentrate on the development of modular, multi-level power electronic converters, SiC valves, self-monitoring and fault tolerant converters. It is argued that this will result in more efficient, scalable, reliable and inexpensive converters with longer lifetimes and improved performances, paving the way for the common place existence of AC/DC micro-grids. It is surmised that AC/DC micro-grids would be amenable to higher energy yields than AC micro-grids, reducing very considerably carbon footprints and using less material resources. It is envisaged that multi-terminal VSC-HVDC systems are very well placed to be the transmission structures that will be used in the next generation of micro-grids.

The design and operation of AC/DC micro-grids calls for the development of new models, methods and control techniques embedded in software. For instance, the operation of a multi-terminal VSC-HVDC-based micro-grid may be assessed by building a model that comprises a number of VSC units which is commensurate with the number of terminals in the HVDC system, suitably accommodated in an all-encompassing frame-of-reference. This paper introduces such a frame-of-reference, with particular reference to the power flow solution of micro-grids, using the Newton-Raphson algorithm which exhibits quadratic convergence owing to its true unified characteristics. The topic of multi-terminal VSC-HVDC power flows and transient simulations has received a fair amount of research attention over the past five years aimed at bulk power transmission [6-8], as opposed to micro-grid systems, which is the remit of this paper.

II. THE BASIC MODEL

The fundamental frequency, steady-state operation of a Multi-Terminal VSC-HVDC (MT-VSC-HVDC) system may be assessed by building a compound model that comprises a number of basic VSC models, which equals the number of terminals in the HVDC system. By way of example, the three-terminal VSC-HVDC system shown in Fig. 1 illustrates this concept where three AC micro-grids are connected asynchronously through a DC grid.
Each converter unit in the AC/DC system illustrated in Fig. 1 comprises a VSC, a phase reactor and a filter capacitor, in addition to the LTC transformer, to connect to the high-voltage AC network, as illustrated in Fig. 2.

![Diagram of Three-terminal VSC-HVDC system](image)

**Figure 1. Three-terminal VSC-HVDC system**

The intended functionality of the phase reactor and filter capacitor is aimed at harmonic frequencies, to improve the quality of the voltage and current waveforms at the low-voltage side of the connecting transformer. However, their inductance and capacitance parameters affect also the fundamental frequency operation of the VSC station and require representation within the power flow formulation.

The model of the basic VSC unit is the kernel with which the three-terminal VSC-HVDC system shown in Fig. 1 is built. The kernel has been developed in [9] for the case of a STATCOM. It has the nodal admittance matrix given below:

\[
\begin{bmatrix}
Y_{i} & Y_{i} & \cdots & Y_{i} \\
-Y_{i} & -Y_{i} & \cdots & -Y_{i} \\
\vdots & \vdots & \ddots & \vdots \\
-Y_{i} & -Y_{i} & \cdots & -Y_{i}
\end{bmatrix}
\]

where \( Y_i = 1/(R_i + jX_i) \) and \( R_i \) and \( X_i \) account for the ohmic losses and the interface magnetics internal to the VSC. The current–dependent resistor, \( G_{dc} \), accounts for the converter switching power loss and \( B_{dc} \) is an equivalent susceptance which is responsible for the whole of the reactive power production in the VSC’s valve set. The amplitude modulation index, \( m_{dc} \), should be kept within its linear range \((0 < m_{dc} < 1)\), for a smooth operation. The phase angle \( \phi \) is the phase angle of the complex voltage \( V_v \) relative to the system phase reference and \( k_i = \sqrt{3}/2 \) for cases of three-phase converters.

Owing to the nodal admittance nature of the basic VSC model, it becomes quite a straightforward matter to combine it with the representation of the smoothing line reactor and the shunt filter, given rise to a model where nodes \( vi, vi' \) and \( 0i \) are explicitly represented. However, since the external injected current at node \( vi' \) is nil then a more compact representation is arrived at by the mathematical elimination of node \( vi' \), using Kron’s reduction [10]:

\[
\begin{bmatrix}
I_{i} \\
0 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
Y_{i} & \cdots & Y_{i} \\
-Y_{i} & \cdots & -Y_{i} \\
\vdots & \ddots & \vdots \\
-Y_{i} & \cdots & -Y_{i}
\end{bmatrix} \begin{bmatrix}
V_i \\
V_{i} \\
\end{bmatrix}
\]

Expression (2) is combined with the nodal transfer admittance matrix equation of the LTC transformer, which would be connected between nodes \( k \) and \( vi \). The resulting nodal admittance matrix representing the VSC station with ancillary elements, shown in Fig. 2, is:

\[
\begin{bmatrix}
I_{i} \\
0 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
Y_{i} & -Y_{i} & 0 \\
-Y_{i} & Y_{i} + Y_{i} & 0 \\
0 & 0 & \ldots
\end{bmatrix} \begin{bmatrix}
V_i \\
V_{i} \\
E_{dc}
\end{bmatrix}
\]

where \( Y_i \) is the leakage admittance of the \( i \)-th LTC transformer and \( I_i \) is its tap.

### III. THE THREE-TERMINAL POWER FLOW MODEL

By way of example, the nodal matrix equation for the three-terminal system of Fig. 1 is given in eqn. (4).
(a) Nodal power equations for the three-terminal system of Figure 1

The nodal power equations are derived by multiplying the nodal voltages by the conjugate of the nodal currents. Separation of the ensuing equations into real and imaginary parts yields the nodal active and reactive powers expressions.

The nodal power equations for one generic VSC station \( i \) – refer to Fig. 2 – are derived and then by suitable replacement of subscripts, the corresponding nodal power equations of the three VSC stations in Fig. 1 become readily available.

Hence, involving nodes \( i, vi \) and \( 0i \) in the circuit of Fig. 2 and the currents expressions in eqn. (3), we have,

\[
\begin{bmatrix}
    \bar{V}_i \\
    \bar{V}_{vi} \\
    \bar{V}_{0i}
\end{bmatrix} = 
\begin{bmatrix}
    0 & 0 & T_{ii} \\
    0 & 0 & T_{vi} \\
    0 & 0 & T_{0i}
\end{bmatrix} 
\begin{bmatrix}
    I_i \\
    I_{vi} \\
    I_{0i}
\end{bmatrix} \quad (5)
\]

Note that the complex nodal voltages may be expressed either in rectangular coordinates or in polar coordinates. In this paper the former representation will be used.

Following some complex number algebra, we have:

\[
P_i = -k_m E_{0i} \left[ e_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] - f_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] \right] + G_{i1} (e_i + f_i) \quad (6)
\]

\[
Q_i = -k_m E_{0i} \left[ e_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] + f_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] \right] - B_{i1} (e_i + f_i) \quad (7)
\]

\[
P_v = -k_m E_{0i} \left[ e_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] - f_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] \right] + G_{i1} E_{0i} \quad (8)
\]

\[
Q_v = -k_m E_{0i} \left[ e_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] + f_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] \right] - B_{i1} E_{0i} \quad (9)
\]

\[
P_{0i} = -k_m E_{0i} \left[ e_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] + f_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] \right] + \phi \quad (10)
\]

\[
Q_{0i} = -k_m E_{0i} \left[ e_i \left[ G_{i1} \sin \phi + B_{i1} \cos \phi \right] - f_i \left[ G_{i1} \cos \phi + B_{i1} \sin \phi \right] \right] - B_{i1} E_{0i} \quad (11)
\]

From the generic expressions (6)-(11), the nodal active and reactive powers: \( P_{i1}, Q_{i1}, P_{01}, Q_{01}, P_{i2}, Q_{i2}, P_{02}, Q_{02}, P_{i3}, Q_{i3} \) become readily available by simply replacing the subscript \( i \) by 1, 2 and 3. Furthermore, the DC power contributions, in explicit form, are:

\[
P_{i1}^\text{dc} = (G_{i1} + G_{i2}) E_{i1}^2 - G_{i1} E_{i1} E_{i2} - G_{i2} E_{i2} E_{i1} \quad (12)
\]

\[
P_{i2}^\text{dc} = (G_{i1} + G_{i2}) E_{i2}^2 - G_{i1} E_{i1} E_{i2} - G_{i2} E_{i2} E_{i1} \quad (12)
\]

\[
P_{0i}^\text{dc} = (G_{i1} + G_{i2}) E_{0i}^2 - G_{i1} E_{i1} E_{0i} - G_{i2} E_{i2} E_{0i} \quad (12)
\]

\[
P_{0i}^\text{dc} = (G_{i1} + G_{i2}) E_{0i}^2 - G_{i1} E_{i1} E_{0i} - G_{i2} E_{i2} E_{0i} \quad (12)
\]

\[
P_{i3}^\text{dc} = (G_{i1} + G_{i2}) E_{i3}^2 - G_{i1} E_{i1} E_{i3} - G_{i2} E_{i2} E_{i3} \quad (13)
\]

(b) Nodal power equations for the three-terminal system when the DC system is star-connected as opposed to delta

The generic expressions (6)-(11) apply with no change to the nodal active and reactive powers of the three-terminal circuit of Fig. 1, namely, \( P_{i1}, Q_{i1}, P_{01}, Q_{01}, P_{i2}, Q_{i2}, P_{02}, Q_{02}, P_{i3}, Q_{i3} \). However, the DC power contributions differ from those given in (12). They are:

\[
P_{i1}^\text{dc} = G_{i1} E_{i1}^2 - G_{i1} E_{i1} E_{i2} - G_{i2} E_{i2} E_{i1} \quad (12)
\]

\[
P_{i2}^\text{dc} = G_{i1} E_{i1}^2 - G_{i1} E_{i1} E_{i2} - G_{i2} E_{i2} E_{i1} \quad (12)
\]

\[
P_{0i}^\text{dc} = G_{i1} E_{0i}^2 - G_{i1} E_{i1} E_{0i} - G_{i2} E_{i2} E_{0i} \quad (12)
\]

\[
P_{0i}^\text{dc} = G_{i1} E_{0i}^2 - G_{i1} E_{i1} E_{0i} - G_{i2} E_{i2} E_{0i} \quad (12)
\]

\[
P_{i3}^\text{dc} = (G_{i1} + G_{i2}) E_{i3}^2 - G_{i1} E_{i1} E_{i3} - G_{i2} E_{i2} E_{i3} \quad (13)
\]

(c) VSC types

Borrowing the concept used in conventional AC power flows, relating to the bus classification into three different types, namely, slack, PV and PQ, Table I introduces three types of VSC stations which are required to solve the generic DC power grid problem put forward in this paper.

The slack converter VSC\textsubscript{slack} provides voltage control at its DC terminal and it is linked on its AC side to a network which contains synchronous generation; the converter of type VSC\textsubscript{PV} serves the purpose of injecting a scheduled power into the DC grid and it is also linked on its AC side to a network with synchronous generation; the third type of VSC station is the passive converter VSC\textsubscript{PQ} which is used to interconnect the DC grid with an AC network which contains no synchronous generation of its own. In the passive AC power grids the VSC’s internal angle, \( \phi \), provides the angular reference for the network.

(d) Mismatch Powers

The non-linear equation set (6)-(13) is solved by iteration using the Newton-Raphson method. To this end, mismatch power equations are set up where upon convergence of the iterative solution, the mismatch between the specified powers and the calculated powers become smaller than a pre-specified tolerance, at every node of the power system.

\[
\Delta P_i = P^{\text{spec}}_i - P_i^\text{calc} \\
\Delta Q_i = Q^{\text{spec}}_i - Q_i^\text{calc} \\
\Delta P_{0i} = P^{\text{spec}}_{0i} - (P_i^\text{calc} + P_{0i}^\text{calc}) \\
\Delta Q_{0i} = Q^{\text{spec}}_{0i} - Q_{i}^\text{calc} \\
\Delta U_i = V_{i}^\text{spec} - \sqrt{\psi_i + f_i^2}
\]

The superscript \( \text{net} \) is used to signify the power difference between an external injection of power by a source connected at a given node and a load connected at the same node.

Furthermore, a reactive power constraining equation is required for all three types of converters to prevent the flow of reactive power into the DC grid. In connection with the three-terminal network in Fig. 1, this reactive power constraining equation is:
\[ \Delta Q_{t_{in}} = 0 - Q_t \]  \hspace{1cm} (15)

Moreover, converters of type VSC_{Pch} require an active power constraining equation which for the three-terminal networks in Fig. 1, and with no loss of generality, takes the following form:

\[ \Delta P_{t_{in}} = p_{t_{in}}^e - P_t \]  \hspace{1cm} (16)

where \( p_{t_{in}}^e \) is the amount of DC power entering inverter \( i \) at its DC bus.

\((e)\) **Linearized system of equations**

Assuming that VSC\(_1\) operates as a rectifier and VSC\(_2\) and VSC\(_3\) operate as inverters then linearization of (14)-(16) around the following base operating point: \( (\xi_0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \) is suitable to regulate power on the DC bus of the rectifier and to regulate the voltage magnitude at both the rectifier AC bus and the inverters AC buses. The relevant system of linearized equations is arranged, using compact notation, in the structure shown in eqn. (17).

\[
\begin{bmatrix}
F_{\text{VSC}_1} \\
F_{\text{VSC}_2} \\
F_{\text{VSC}_3} \\
F_{\text{DC}}
\end{bmatrix} =
\begin{bmatrix}
J_{11} & 0 & 0 & J_{\text{DC}_1} \\
0 & J_{22} & 0 & J_{\text{DC}_2} \\
0 & 0 & J_{33} & J_{\text{DC}_3} \\
J_{\text{DC}_1} & J_{\text{DC}_2} & J_{\text{DC}_3} & J_{\text{DC}}
\end{bmatrix}
\begin{bmatrix}
\Delta \Phi_{\text{VSC}_1} \\
\Delta \Phi_{\text{VSC}_2} \\
\Delta \Phi_{\text{VSC}_3} \\
\Delta \Phi_{\text{DC}}
\end{bmatrix}
\]

where the \( 0 \) entries are zero-padded matrices of suitable orders.

In addition to the matrix entries corresponding to the three VSCs, \( J_{11}, J_{22}, J_{33} \), there are matrix entries corresponding to the DC grid, \( J_{\text{DC}} \), and mutual matrix terms between the DC nodes and their respective AC nodes: \( J_{\text{DC}_1}, J_{\text{DC}_2}, J_{\text{DC}_3}, J_{\text{DC}_2}, J_{\text{DC}_3} \).

The matrix entries \( J_{11}, J_{22}, J_{33} \) into the higher order matrix in (17), correspond to first order partial derivatives of the power and voltage mismatch vectors in (18)-(20) with respect to the state variables increments in (21)-(23):

\[
F_{\text{VSC}_1} = \begin{bmatrix} \Delta P_t & \Delta Q_t & \Delta U_{t_{in}} & \Delta Q_{t_{in}} & \Delta P_{t_{in}} \end{bmatrix}
\]

\[
F_{\text{VSC}_2} = \begin{bmatrix} \Delta P_t & \Delta Q_t & \Delta U_{t_{in}} & \Delta Q_{t_{in}} & \Delta P_{t_{in}} \end{bmatrix}
\]

\[
F_{\text{VSC}_3} = \begin{bmatrix} \Delta P_t & \Delta Q_t & \Delta U_{t_{in}} & \Delta Q_{t_{in}} & \Delta P_{t_{in}} \end{bmatrix}
\]

\[
F_{\text{DC}} = \begin{bmatrix} \Delta P_{t_{in}} & \Delta Q_{t_{in}} & \Delta U_{t_{in}} & \Delta Q_{t_{in}} & \Delta P_{t_{in}} \end{bmatrix}
\]

\[
\Delta \Phi_{\text{VSC}_1} = \begin{bmatrix} \Delta \Phi_t & \Delta \Phi_t & \Delta \Phi_{t_{in}} & \Delta \Phi_{t_{in}} \end{bmatrix}
\]

\[
\Delta \Phi_{\text{VSC}_2} = \begin{bmatrix} \Delta \Phi_t & \Delta \Phi_t & \Delta \Phi_{t_{in}} & \Delta \Phi_{t_{in}} \end{bmatrix}
\]

\[
\Delta \Phi_{\text{VSC}_3} = \begin{bmatrix} \Delta \Phi_t & \Delta \Phi_t & \Delta \Phi_{t_{in}} & \Delta \Phi_{t_{in}} \end{bmatrix}
\]

\[
\Delta \Phi_{\text{DC}} = \begin{bmatrix} \Delta \Phi_{t_{in}} & \Delta \Phi_{t_{in}} \end{bmatrix}
\]

The mutual matrix entries \( J_{\text{DC}_1}, J_{\text{DC}_2}, J_{\text{DC}_3} \) between the AC nodes and their corresponding DC nodes, correspond to first order partial derivatives of the mismatch vectors in (18)-(20) with respect to the state variables increments in (24):

\[
\Delta E_{\text{DC}} = \begin{bmatrix} \Delta E_{\text{DC}_2} & \Delta E_{\text{DC}_3} \end{bmatrix}
\]

Note that VSC\(_1\) is acting as slack VSC in the DC grid and, therefore, \( \Delta E_{\text{DC}_1} = 0 \).

The mutual matrix entries \( J_{\text{DC}_1}, J_{\text{DC}_2}, J_{\text{DC}_3} \) between the DC nodes and their corresponding AC nodes, correspond to first order partial derivatives of the mismatch vector in (25) with respect to the state variables increments in (21)-(23):

\[
F_{\text{DC}} = \begin{bmatrix} \Delta P_{\text{DC}_2} & \Delta P_{\text{DC}_3} \end{bmatrix}
\]

The entry \( J_{\text{DC}} \) corresponding to the DC part of the system, contains the partial derivatives of (25) with respect to the DC voltages in (24). Note that since VSC\(_1\) is acting as slack VSC in the DC grid the matrix \( J_{\text{DC}_1} \) is 0 but not \( J_{\text{DC}} \).

If no voltage regulation is exerted at the AC bus of any of the VSCs then suitable changes take place in (18)-(20) and the corresponding matrix entries in (17). More explicitly, since the state variable \( m_{t_{in}} \) is charged with regulating the AC voltage at node \( vi \) and voltage no regulation is exerted then \( m_{t_{in}} \) becomes a constant parameter.

Conversely, if voltage regulation takes place at any of the DC buses then the corresponding row and column are deleted from (24), (25) and in the mutual matrix entries in (17). It should be remarked that the voltage must be specified in at least one of the buses of the DC network. Such a node plays the role of reference node in the DC network and in this three-terminal VSC-HVDC example this role has been assigned to VSC\(_1\) which is VSC_{Slack} type.

The increments of the state variables in vector (17), calculated at iteration \( r \), are used to update the state variables, as follows:

\[
e_{t^{(r)}} = e_{t^{(r-1)}} + \Delta e_{t^{(r)}}
\]

\[
f_{t^{(r)}} = f_{t^{(r-1)}} + \Delta f_{t^{(r)}}
\]

\[
m_{t_{in}}^{(r)} = m_{t_{in}}^{(r-1)} + \Delta m_{t_{in}}^{(r)}
\]

\[
B_{t_{in}}^{(r)} = B_{t_{in}}^{(r-1)} + \Delta B_{t_{in}}^{(r)}
\]

\[
\phi_{t_{in}}^{(r)} = \phi_{t_{in}}^{(r-1)} + \Delta \phi_{t_{in}}^{(r)}
\]

Similar expressions exist for updating the state variables of the inverters contained in vector (22) and (23).

The updating of the DC voltages using vector (24) is carried out as follows:

\[
E_{\text{DC}_2}^{(r)} = E_{\text{DC}_2}^{(r-1)} + \Delta E_{\text{DC}_2}
\]

\[
E_{\text{DC}_3}^{(r)} = E_{\text{DC}_3}^{(r-1)} + \Delta E_{\text{DC}_3}
\]

It should be noted that when all entries relating to VSC\(_3\) are removed in (4)-(27) then the three-terminal VSC-HVDC model reduces neatly to the more particular case of the point-to-point VSC-HVDC link model, a case in which it is equivalent to the point-to-point VSC-HVDC link model. However, it should be remarked that the model put forward in this paper is general and handles the DC link in explicit form. It is precisely the explicit representation of all the DC nodes in the formulation of this paper that enables general multi-terminal AC/DC power flow solutions in a truly unified way.
IV. MULTI-Terminal VSC-HVDC MODEL

A straightforward expansion of the linearized structure in (17), to include \( n \) rectifying stations, \( m \) inverting stations and an arbitrary DC network, yields the following result:

\[
\begin{bmatrix}
\mathbf{F}_{\text{AC}} \\
\mathbf{F}_{\text{VSC-R}} \\
\vdots \\
\mathbf{F}_{\text{VSC-I}} \\
\mathbf{F}_{\text{DC}} \\
\mathbf{F}_{\text{AC/DC}}
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{J}_{\text{AC/DC}}
\end{bmatrix}
\begin{bmatrix}
\Delta \Phi_{\text{AC}} \\
\Delta \Phi_{\text{VSC-R}} \\
\vdots \\
\Delta \Phi_{\text{VSC-I}} \\
\vdots \\
\Delta \Phi_{\text{DC}} \\
\Delta \Phi_{\text{AC/DC}}
\end{bmatrix}
\quad (28)
\]

In this expression, each one of the \( n \) and \( m \) terms \( \mathbf{F} \) with subscripts VSC-R and VSC-I are vectors of power mismatches corresponding to \( n \) rectifying stations and \( m \) inverting stations with an assigned operational characteristic of the type: VSC\(_{\text{slack}}, \) VSC\(_{\text{rect}} \) or VSC\(_{\text{inv}} \) according to operational requirements. Likewise, the vectors \( \Delta \Phi \) contain the corresponding incremental state variable terms. By the same token, matrices \( \mathbf{J} \) with subscripts RR and II contain first order partial derivatives of the rectifier and inverter stations’ state variables. The higher order Jacobian terms with subscripts DCR, DCI, RDC and IDC are interfacing terms between the AC and DC sides of the converter stations’ state variables. The terms with subscripts DC contain powers and voltages belonging to the state variables of the DC network.

Since the multi-terminal VSC-HVDC system is used to interconnect a number of otherwise independent micro-grids, the linearised form of the overall electric power system’s equations at a given iteration \((r)\), is:

\[
\begin{bmatrix}
\mathbf{F}_{\text{AC}}^{(r)} \\
\mathbf{F}_{\text{VSC-R}}^{(r)} \\
\vdots \\
\mathbf{F}_{\text{VSC-I}}^{(r)} \\
\mathbf{F}_{\text{DC}}^{(r)} \\
\mathbf{F}_{\text{AC/DC}}^{(r)}
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{J}_{\text{AC/DC}}^{(r)}
\end{bmatrix}
\begin{bmatrix}
\Delta \Phi_{\text{AC}}^{(r)} \\
\Delta \Phi_{\text{VSC-R}}^{(r)} \\
\vdots \\
\Delta \Phi_{\text{VSC-I}}^{(r)} \\
\vdots \\
\Delta \Phi_{\text{DC}}^{(r)} \\
\Delta \Phi_{\text{AC/DC}}^{(r)}
\end{bmatrix}
\quad (29)
\]

V. TEST CASE

The five-terminal VSC-HVDC network shown in Fig. 3 is used to illustrate the applicability of the new frame-of-reference. It represents an AC/DC grid comprising a DC ring that interconnects two distribution systems (DS), a battery energy storage system (BESS), two micro-grids (MG) and a DFIG-based wind farm. For the purpose of this illustrative test case and with no loss of generality, each distribution system comprises 17 nodes, 16 distribution feeders and a total system load of 2.3 MW. The BESS is set to inject 0.5 MW from the DC ring. Each micro-grid draws 5 MW and operates at 0.9 lagging power factor. The wind farm contains four doubly-fed induction generators operating at its nominal power of 2 MW, to give an aggregated power of 8 MW.

Table II gives the parameters in per-unit values of the VSCs, DC ring cables, distribution lines of DS1, DS2 and WF.
The BESS injects 0.5 MW into the DC ring and its corresponding DC voltage is 1.9951 p.u. The angle $\phi$ of converters VSC$_3$, VSC$_4$ and VSC$_5$ take values of zero since these converters provide the angular references for networks MG$_1$, MG$_2$ and WF. As shown in Table IV, the voltage phase angles of these nodes are displaced by -4.0565°, -4.0565° and -6.5750°, respectively. Notice that their amplitude modulation indexes take different values from each other since their voltage set points and reactive power injections are different.

The power loss incurred by each VSC is given in Table III. Converter VSC$_1$ incurs the highest loss and converter VSC$_2$ the lowest since it draws the lowest amount of power from the DC ring. The nodal voltages and active and reactive powers injected at the terminal of each AC network are given in Table IV. It should be noted that the LTC of VSC$_1$ injects 7.0417 MW and the LTC of VSC$_2$ draws 0.5987 MVAr in order to uphold the respective target voltages of DS$_1$ and DS$_2$ at 1 p.u. Also, VSC$_2$ is set to draw 2.5 MW from the DC ring and the power flow through VSC$_1$, from DS$_1$ and towards the DC ring, is 4.3374 MW.

The power control of a converter type VSC$_{PDC}$ applies at its DC bus; hence, the power delivered at its AC terminal will be slightly less due to the power loss incurred within the converter. In this test case, the power delivered to DS$_2$ stands at 2.4933 MW. Also, it should be noticed that the power injected by the LTCs at DS$_1$ and WF carry a negative sign which correctly accounts for the fact that the powers are being injected into the DC ring. The opposite occurs for the case of DS$_2$, MG$_1$ and MG$_2$, which draw power from the DC grid.

The power flows in the DC ring are given in Table V. It is noticed that the heavily loaded ring sector is $d$-$e$, which carries 5.3908 MW. Conversely, the less loaded is sector $c$-$d$, since most of the power imported by DS$_2$ (VSC$_2$) comes from the branches connecting VSC$_1$ and the EV charging station. The total power loss in the DC ring stand at 50.77 kW.

![Figure 3. Multi-terminal VSC-HVDC system with micro-grids](image)

**VI. CONCLUSIONS**

A generalized frame-of-reference for the unified power flow solution of hybrid power systems has been presented and applied to the solution of low-power AC-DC grids (i.e., micro-grids). The new frame-of-reference is a linearised representation of the whole AC and DC power network around a base operating point enabling its iterative solution by means of the Newton-Raphson method, with a quadratic rate of convergence. The modeling flexibility of this computational framework enables a representation of a wide range of micro-grids, e.g., AC, DC and hybrid.

**REFERENCES**


