On the Problem Formulation of Model Predictive Control for Demand Response of a Power-to-Heat Home Microgrid

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Abstract—In this paper, MPC problem formulations for demand response of a power-to-heat residential microgrid are investigated. Two demand response operation strategies using MPC are proposed and thoroughly evaluated. The simulation results indicate that the problem formulation always poses a trade-off between comfort of the inhabitants, peak power reduction at point of common coupling and CPU time for the optimizer. These aspects have to be taken into consideration for a roll-out of demand response technology for residential buildings.

While for the case presented in this paper, a quadratic objective function is the most promising one, the necessity of further investigations is derived from the results.

Index Terms—Energy storage, heat pumps, load management, optimization methods, photovoltaic systems

I. INTRODUCTION

The goal of reducing CO₂ emissions and the need for a reliable and secure energy supply imply two main challenges: the utilization of all possible renewable energy sources (RES) and the improvement of energy efficiency. The utilization of new low carbon electrical technologies offers opportunities to overcome these challenges. However, due to the rapid penetration of these technologies, the electrical supply system is constantly being exposed to changes in its operation. Distribution systems are being confronted with reverse power flows as a result of the simultaneous feed-in of photovoltaic (PV) plants during low consumption periods. Furthermore energy efficient power-to-heat appliances, such as heat pumps, are being incorporated in single and multi-family houses. The activation of these new loads at times of high electricity demand leads to further challenges in the grid operation [1]. Demand response (DR) is considered one key element to facilitate the integration of such new technologies into the power system. DR enables the intelligent control of customer electrical demand to respond to variable supply conditions.

In general, residential space heating and space cooling, domestic hot water (DHW) are the end uses with the largest flexibility for DR [2], [3]. The implementation of DR using thermal energy storage (TES) have benefits for customers and utilities, as reported in [4]. In [5] and [6], the potential of using residential heating systems with TES for balancing fluctuations resulting from renewable energy sources (RES) is investigated, indicating their positive contribution to increase demand-side flexibility. Such residential power-to-heat systems can be operated as a home microgrid to respond as a single unit to grid requirements. More importantly, before exploiting the capabilities of a power-to-heat home microgrid to perform DR, first, an optimal microgrid operation has to be assured and customer thermal comfort has to be guaranteed. Intelligent energy management systems (EMS) are appropriate for enabling DR. An intelligent EMS optimizes the interaction of components at the end-user level and also satisfies other operational goals. The design and implementation of an optimal EMS to enable the DR of residential power-to-heat systems, is an actual and important topic for investigation.

To address this concern, several approaches have been proposed and discussed in literature. One promising proposal is the use of model predictive control (MPC) as EMS for DR purposes. In [8], the use of MPC for the DR of a micro combined heat and power system (µ-CHP) is presented. The authors formulate the optimization problem as a mixed-integer linear problem, where thermal comfort satisfaction is defined as a constraint for the optimization problem, given a thermal demand prediction. A different way to consider thermal comfort satisfaction is to include a building’s thermal model in the optimization problem and define upper and lower room temperature boundaries as constraints [11], [10], [12], [13]. A load management strategy using MPC for PV with electrical heaters was investigated by [7], in which the thermal comfort satisfaction issue was treated as a temperature set point tracking problem using the $l_1$-norm (see [9] for further information) in the objective function to minimize absolute deviations of the actual room temperature to the given set-point temperature. Another characteristic of the operation strategy adopted in the cited references is that, the DR activity
was defined as the minimization of operation costs with an assumption that the utility offers a variable price signal.

In this paper, a home microgrid of a single family house (SFH) is investigated, which combines an HP, a TES and a PV plant. We consider a DR activity where the objective is to minimize extreme power peaks (in generation and consumption) at the point of common coupling (PCC) between the home microgrid and the distribution grid. This type of DR activity could be promoted by the distribution system operator (DSO), by requiring limitations on PV feed-in at the PCC in order to avoid reverse power flows from the low voltage to the medium voltage grid as well as large consumption peaks, which are normally not desired in the grid operation. For an energy retailer this kind of DR could also be interesting, if electricity tariff based on demand rates on peak power consumption is desired. To achieve this DR objective, we minimize the instantaneous difference between PV generation and house electricity consumption (household load and heat pump consumption). For thermal comfort satisfaction, we follow the idea of temperature set-point tracking problem as discussed in [7], but we investigate an alternative problem formulation. Three operation strategies are benchmarked regarding different indicators. To sum up, the purpose of this paper is to provide new insights regarding the problem formulation of MPC for DR of residential PV-heat pump systems. The contribution of this paper will specifically be on the following:

- Focusing on residential PV-Heat pump systems and formulating the optimization problem for such systems considering heat pump operational characteristics, thermal dynamics of the house and a thermal energy storage unit, which allows further investigation aspects of these kinds of systems to be investigated.
- Providing conclusive results on the best problem formulation for these specific systems by comparing different operating modes using MPC, namely a heat-led mode, a DR-L1 mode and a DR-L2 mode

The paper is organized as follows: First, the model description for the PV-HP-TES system and the formulation of the optimization problem for MPC are presented. Then, three operational strategies are developed and formulated as objective functions. Evaluation criteria and simulation setup are presented next. In the end, numerical results are illustrated and discussed.

II. Problem Formulation

In this section dynamics and constraints for the MPC are formulated, followed by the description of the objective functions for each operation strategy. Finally, indicators for evaluation and simulation setup are defined. One advantage of MPC is that the concept can be implemented at different layers of an automation system i.e. MPC can used at the low controller level or at the supervisory level [14]. In this paper, MPC is considered at the supervisory level of the home energy management; low level control loops are not investigated. MPC determines optimal power set-points and sends them to the low level controllers. It is assumed that these controllers are able to track the given set-points. Fig. 1 shows an MPC schematic structure for buildings, and Table I depicts the notation used in this paper.

A. Principle of MPC

The optimization problem is considered from an arbitrary initial time \( k_i \) to a final time \( k_i + N_p \), where \( N_p \) is the prediction horizon. To predict future behavior of the system, MPC considers thermal dynamic models represented in state space form and forecast for disturbances. At the start of the time window, initial values of output and controllable input variables, together with disturbance forecasts for the whole prediction horizon are obtained. Then the optimization problem is solved and input trajectories are obtained. Once an input trajectory is computed, just the first sequence of the solution is implemented and the rest are neglected. In the next time step, the time window is moved one step ahead, initial values are updated with actual information, new disturbance forecasts are obtained and the process is repeated. This procedure is called receding horizon control [15]. The MPC strategy is applied to dynamic models of a single family house (SFH) and a TES. Measurement data for PV, load and weather from a field test in Germany serve as input data. The goal of MPC is to find a future trajectory for the TES and the HP, over a finite prediction horizon, to minimize the instantaneous difference between PV generation and electrical consumption (household loads plus HP) by shifting the operation times of the heating appliance using the flexibility provided by the TES and the storage capacity of the house.

B. Models

In this section, models used in the optimization problem to represent the main thermal characteristics of the considered system are presented. For the formulations below, the relative time within the prediction horizon is indicated with the index \( k = 1, \ldots, N_p \). The Table I below gives the notation used for variables and parameters in the upcoming subsections.
TABLE I: Used notation in this study

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{th} \in \mathbb{R}_+$</td>
<td>Thermal power</td>
<td>kW$_{th}$</td>
</tr>
<tr>
<td>$P_{el} \in \mathbb{R}_+$</td>
<td>Electrical Power</td>
<td>kW$_{el}$</td>
</tr>
<tr>
<td>$T \in \mathbb{R}$</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$\Phi \in \mathbb{R}$</td>
<td>Solar radiation</td>
<td>kW/m$^2$</td>
</tr>
<tr>
<td>SoC$ \in [0,100]$</td>
<td>State of Charge</td>
<td>-</td>
</tr>
<tr>
<td>$\eta^{Tes} \in [0,1]$</td>
<td>Charging efficiency</td>
<td>-</td>
</tr>
<tr>
<td>$\eta^{Tes} \in [0,1]$</td>
<td>Discharging efficiency</td>
<td>-</td>
</tr>
</tbody>
</table>

![Figure 2: Overview of the system components in the residential setting](image)

1) Heat pump: In [17], the economic addressable market for HPs and μ-CHP in Germany is estimated, concluding that for SFH, heat pumps are more cost efficient. The heating system is assumed to be monovalent, consisting of just the HP as heating source without auxiliary boiler. The HP is directly connected to the TES, which feeds the residential heating circuit. Fig. 2 shows a schematic representation of the considered system.

The relationship between the generated thermal power $P_{th,k}$ and consumed electrical power $P_{el,k}$ in a HP is given by the coefficient of performance (COP) as shown in the equation below:

$$P_{th,k}^{HP} = P_{el,k} \cdot COP$$ (1)

In this work a brine/water HP is considered. Brine/water HPs use heat from the ground as source, at a relative constant temperature over the year [18]. Therefore, a constant COP for the entire year is assumed, neglecting variations due to weather conditions. The HP is driven by a frequency inverter, which implies that it can be modulated. A binary variable $b_k^{HP}$ is used to indicate the ON/OFF status of the HP. The following inequality is used to restrict the operation of the HP to its maximum allowable power $P_{el,max}^{HP}$ and minimum allowable power $P_{el,min}^{HP}$:

$$b_k^{HP} \cdot P_{el,min}^{HP} \leq P_{el,k} \leq b_k^{HP} \cdot P_{el,max}^{HP}$$ (2)

2) House thermal model: The thermal behavior of the house is represented by the model proposed in [19] which has already been used for demand side management with heating systems [16], [20], [21]. The storage capacity of the house is dominated by the air in the room and a large heat-accumulating medium, composed mainly by the walls and the floor as shown in (4).

$$\begin{bmatrix} T_{k+1}^m \\ T_{k+1}^{room} \end{bmatrix} = A \cdot \begin{bmatrix} T_{k}^m \\ T_{k}^{room} \end{bmatrix} + B \cdot \begin{bmatrix} P_{Sh}^m \\ P_{th,k}^{Sh} \end{bmatrix}$$ (4)

The states of the system are $T^m$ and $T^{room}$, where $T^m$ is the temperature of the large heat-accumulating medium in the house, and $T^{room}$ is the house air temperature. Model inputs are outside ambient temperature $T_{Amb}$, solar radiation $\Phi^{Solar}$ and the space heating power $P_{th,k}^{Sh}$.

The matrices $A$ and $B$ are composed of a building physical parameters, such as the heat capacity of the large heat-accumulating medium $c_m$, heat capacity of the room air $c_i$, the resistance against heat transfer from the house air to ambient air $r_a$, and $r_i$ as the resistance against heat transfer between the house air and the large heat-accumulating medium. Besides, other parameters like the window area facing south $A_w$, and the share of the solar radiation which is directly affecting the temperature over the year [18]. Therefore, a constant COP as shown in (1).
With equations (1) to (8) all dynamics and constraints for the MPC are defined. Next we describe the objective functions, that define the different operation modes.

C. Objective functions

As said before, different control strategies are contrasted. We take a heat-led control strategy as reference, and propose two formulations for the DR operation. A formulation using the $l_1$-Norm and a formulation using the squared $l_2$-Norm.

1) Heat-led operation: In this operation mode the only objective is to satisfy thermal comfort. The HP and the TES are operated in a way that the given temperature set-point is satisfied. For this purpose the objective function is:

$$J_1 = \beta \left( \sum_{k=1}^{N_p} [T_k^{\text{Setpoint}} - T_k^{\text{Room}}] \right)$$

(9)

2) DR-L1: In this operation mode the two objectives are the minimization of deviations of the room temperature to the desired set point temperature and the minimization of peak power flows at PCC. As positive and negative deviations have to be minimized, the absolute value of the difference is formulated in the objective function using the $l_1$-Norm.

$$J_2 = \alpha \left( \sum_{k=1}^{N_p} \left| P_{\text{el},k}^{PV,ac} - P_{\text{Load}} - P_{\text{el},k}^{HP} \right| \right)$$

$$+ \beta \left( \sum_{k=1}^{N_p} \left| T_k^{\text{Setpoint}} - T_k^{\text{Room}} \right| \right)$$

(10)

The formulation presented in [9] is used to treat the $l_1$-Norm problem as a linear programming problem.

3) DR-L2: Similar to the previous operation mode, deviations in thermal comfort and peaks at PCC have to be minimized. Here, not the absolute difference is accounted but instead the square of the difference is used. This is done using the squared $l_2$-Norm in the objective function.

$$J_3 = \alpha \left( \sum_{k=1}^{N_p} \left( P_{\text{el},k}^{PV,ac} - P_{\text{Load}} - P_{\text{el},k}^{HP} \right)^2 \right)$$

$$+ \beta \left( \sum_{k=1}^{N_p} \left( T_k^{\text{Setpoint}} - T_k^{\text{Room}} \right)^2 \right)$$

(11)

Table II summarizes all three considered objective functions.

D. Evaluation Criteria and Scenario

To evaluate the performance of the three control strategies, the strategies are compared with regard to customer comfort, power peak reduction as well as computational effort. The evaluation is based on a 30 day long simulation with a time resolution of 10 Minutes ($\Delta T = 10$ min) from an initial simulation time $i$ to a final simulation time $N$. The objective is to guarantee thermal comfort to the inhabitants as well as to operate in a grid friendly way.

As a first approach, a 1°C difference in temperature is assumed to be equally important as a 1kW difference at the PCC. Thus, to perform a fair comparison of all simulations we set $\alpha = 0.5$ and $\beta = 0.5$. Although the optimal value of these parameters is a whole optimization problem in itself and therefore falls out of the scope of this paper, these values will be briefly discussed as follows:

For the sensitivity of people to a change in the temperature, the international standard ISO 7730 [25] can be utilized to evaluate the corresponding neutral temperature levels that go along with a minimum of dissatisfaction. Combining typical household conditions and applying building category Class A results in a desirable, comfortable temperature of 21.0°C and an appropriate allowed temperature variation of roughly 1.0K. The linear combination of temperature and power, through the coefficients $\alpha$ and $\beta$ in the objective function, directly depends on the magnitude of the power and temperature values (e.g., the optimization may lead to different results if the temperature values are in Fahrenheit instead of degrees Celsius). One possible solution to this issue would be a normalization of data (set-points and measured) before optimization.

1) Local PV usage: To quantify the local usage of PV two indicators are defined: a self-consumption quota and a self-sufficiency quota. First the amount of PV power, which is instantaneously and directly used by the local loads at each time simulation step is calculated using:

$$P_{\text{el},i}^{PV,used} = \min \{ P_{\text{el},i}^{PV,ac}, P_{\text{el},i}^{HP} + P_{\text{Load}} \}$$

(12)

The self-consumption quota $q_{sc}$ is calculated as the ratio of total PV used to total PV generated for the whole simulation period as expressed below:

$$q_{sc} = \frac{\sum_{i=1}^{N} P_{\text{el},i}^{PV,used}}{\sum_{i=1}^{N} P_{\text{el},i}^{PV,ac}}$$

(13)

The self-sufficiency quota $q_{ss}$ is calculated as the ratio of total PV used to total electrical consumption for the whole simulation period as expressed below:

$$q_{ss} = \frac{\sum_{i=1}^{N} P_{\text{el},i}^{PV,used}}{\sum_{i=1}^{N} P_{\text{el},i}^{HP} + P_{\text{Load}}}$$

(14)

<table>
<thead>
<tr>
<th>Optimization strategy</th>
<th>Motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>heat-led</td>
<td>Thermal comfort is the only objective so the heating system will act accordingly. As there is no trade-off, this is the <strong>reference</strong> scenario.</td>
</tr>
<tr>
<td>DR-L1</td>
<td>Thermal comfort and the net energy flow at the PCC are evaluated in an integrated, linear $l_1$-norm metric. The weights of the two components can be defined at will. (default weights: $\alpha = 0.5, \beta = 0.5$)</td>
</tr>
<tr>
<td>DR-L2</td>
<td>like DR-L1, but with a squared $l_2$-norm metric. (default weights: $\alpha = 0.5, \beta = 0.5$)</td>
</tr>
</tbody>
</table>

TABLE II: Three operation strategies form the scenarios
2) Mean Thermal Comfort: The temperature mean absolute error (MAE) serves as key performance indicator for customer comfort. The mean error is given by:

$$ MAE = \frac{1}{N} \sum_{i=1}^{N} |T_i^{\text{Setpoint}} - T_i^{\text{Room}}| $$  \hspace{1cm} (15)

Within the simulated time horizon, all ten-minute time steps are treated equally so that the absolute error is evaluated statistically by giving the mean for the absolute temperature error between set point and actual indoor temperature.

3) Further indicators: Since every time step is of different computational effort, the mean of the CPU time for each new optimization has to be evaluated as well. For the evaluation of the DR objective maximum value and minimum value of all observations are compared.

E. Simulation setup

Building physical characteristics are taken from [5], for a SFH with an area of 140m², constructed in 1990 with low thermal modernization. Weather data consist of ambient temperature and global radiation, which are used as inputs for the building thermal model. Time series data for weather and PV power generation are for Germany for 30 days of April. Profiles for electrical household load, and DHW are taken from [23] for a SFH with an annual thermal consumption of 136kWh/(m²a) and an annual electrical consumption (without HP) of 5000kWh/a according to [24]. Profiles are obtained for 30 days in 10 min resolution. A temperature set-point is given, which varies according to the hour in the day. From 22.00 to 6.00, the temperature is set to 18°C, whereas during the day (i.e. from 6.00 to 22.00) a room temperature of 21°C is desired. Table III gives the parameters used for the simulation. The implementation is done in Python using CPLEX [26].

III. Numerical Results

In this section numerical simulation results are presented and discussed. First a qualitative analysis for two exemplary day is given considering interactions and side-effects of the considered operation strategies. Next, an extensive 30 day study is discussed based on performance indicators, TES usage and local PV usage.

![Figure 3: Exemplary day profiles for temperature, electrical consumption, and PCC power flow](image)

Looking at the temperature profile for the DR-L2 strategy, the fluctuation becomes present over the whole heating time.
frame, which therefore makes the restrictions of the whole system and the necessity of a detailed analysis much clearer. There is a clear deterioration in performance concerning the temperature fidelity. Another thing to notice is that even during the setback period, there are undershoots that might not be expected beforehand. It also becomes clear that during the morning time when the heating period starts again, such deviations are no problem at all. The capacity of the heat pump fits the corresponding heat consumption quite well and is able to quickly counterbalance the undershoots that results from the system operation at night. However, the temperature deviation can be as high as two degrees, although normally in the range of 0.5 degrees.

B. Interaction and side-effects of operation strategies

In the second subplot, electrical profiles for PV generation and electrical consumption of the heat pump are illustrated. In general, the heat pump is operated with a power level that is sufficient to just cover the thermal demand (or as much of it as possible), while the PV generation profile is completely ignored. In the example shown, the heat pump reduces its electrical power consumption around noon on both days, although there is a clear peak in the PV generation. This results in large peaks at the PCC for both electrical consumption and feed-in.

DR-L1 strategy: As described above, when applying the DR-L1 strategy, the temperature set point is well tracked, even in direct comparison with the reference scenario. However, it has to be noticed that the temperature sag is visible at an earlier point of time. This is because the optimization according to the $l_1$-norm has to see about both the temperature and the peak power at the PCC at the same time. As there is no PV generation in the evening, it is more favorable to lower the heat pump activity than to invest high power to achieve higher temperatures. For the temperature peak, the optimization strategy dictates to rather use the given power from PV generation and accept the overshoot than to have high power flows at the PCC. It can be stated that the heat pump runs more often and at higher average powers than in the reference scenario. That is because the TES is loaded in a sensible way whenever there is power from PV generation but no instant space heating demand. However, even though the heat pump is operated in a smarter way than before, short activation times of the heat pump are present at full power. Although this is appreciated during periods of high insolation, this is also visible for the night times. This clearly affects the balance of demand and in-feed at the PCC. On the one hand, peaks are clearly reduced at the generation side to less than 1kW. So, the optimization by the $l_1$-norm achieves a significant improvement of the correlation between activation of the heat pump and the given power from PV generation. On the other hand, this optimizer strategy is not able to gain much flexibility by charging the TES, because the heat pump is already running for long periods.

DR-L2 strategy: Using the DR-L2 optimization strategy, the deviation of the room temperature rises although the quadratic approach behind the squared $l_2$-norm generally favors smaller values in comparison to the $l_1$-norm. However, looking at the electrical profiles at the PCC, it can be seen that the correlation between PV generation and heat pump operation significantly improves the power flow at the PCC. Therefore, large activation peaks of the heat pump at times of no PV generation are effectively avoided when employing the squared $l_2$-norm, which is a clear advantage of the squared $l_2$-norm. Another important factor is that in contrast to the DR-L1 strategy, the heat pump is only operated at minimal power (roughly 1kW) when there is no insolation present. So, the charging of the TES is bound in a more restrictive way than for the DR-L1 strategy. Here, the MPC-algorithm anticipates the heat demand of the day and tries to include the TES without overburdening the power budget at the PCC. In fact, this results in a flat power flow profile at the PCC as can be seen in the third subplot.

C. Extensive 30 day study

To provide a better analysis, a quantitative evaluation is done for 30 days simulation. The distribution of all observations for net power at the PCC is displayed in box plots in Fig. 4a. It can be seen how the DR-L2 operation mode is able to reduce peaks from heat-led and peaks from DR-L1.

What is not directly visible from the above discussion is in how far the TES is sensibly used by the operation strategies. It should again be stated that the TES usage is only a boundary condition and not an objective itself. In this regard, Fig. 4b shows a discrete histogram of the TES charging states. The heat-led strategy does not make proper use of the capabilities of the TES, as the state of charge (SoC) shows a dramatic peak at zero and mostly lies below 40\%. This is possible because the heat pump operation is only limited by its technical maximum power of 5kW, which does not pose a big constraint on the target to follow the heat temperature with the given building.

The DR-L1 strategy reduces the peak in the completely discharged state (where $SoC_{TES} = 0\%$) and therefore shifts the tail towards higher values. It has to be said though, that the TES still seems to be oversized and the utility from more TES capacity is extremely low.

For the most sophisticated optimization strategy (DR-L2), we find a completely different situation: The peak at $SoC_{TES} = 0\%$ is only half as high as for the reference scenario, and the tail is much more balanced along the whole spectrum. On the contrary, there is a small but noticeable peak at $SoC_{TES} = 100\%$, which signifies that the optimizer had to deal with a constraint here. Even though the question of the right TES size cannot be answered directly from this finding, the side-effects of optimization strategy and chosen boundary conditions becomes clear now, and is subject to further evaluations in the future.

Table IV summarizes the resulting performance indicators for all strategies.

As mentioned before, DR-L2 achieves a notorious reduction in max and min peaks at PCC. However, there are two important aspects to discuss here, namely the increments in
MAE and mean CPU time. The distribution of CPU time for all runs shows a large number of occurrences with a duration of around 1 sec, and some extreme cases when the optimization takes 9 min or more, as illustrated in Fig. 4c.

It can be appreciated that with 75% of probability the computational time for the optimization using this operation mode will take less than 1 sec. On the other hand there is 21% of probability that the computation time will take 9 min or more. In the simulation, a time limit for the optimization of 540 sec was set, and if this time was reached, available results provided by the solver at that moment were used. This long computation times can be explained by the complexity of the MIQP problem, and by the use of the \textit{mipgap}. We set our relative \textit{mipgap} tolerance to 0.01. Meaning that the solver only stops when it finds a feasible integer solution proved to be within 1% of optimal. If after 9 minutes, this \textit{mipgap} has not been reached, the optimization was stopped, and available results were used. Even though, the available results are still not the optimal, they are good enough to be used, compared to the results obtained with heat-lead and with DR-L1. In our work, we optimized every 10 minutes, and we sent the resulting optimal sequence as set-point to the lower controllers. In this case, getting the optimization results in 9 minutes is an issue and must be further investigated for implementation purposes.

On the other hand, an MAE of almost 0.6°C may be not acceptable in practical situations. By adjusting the weight of $\beta$ in the objective functions i.e. prioritizing comfort over peak reduction, smaller deviations can be reached. Resulting indicators for adjusted $\beta$ values are presented in Table IV.

From this table it can be concluded that by adjusting $\beta$ to 1, the temperature error is around 0.45°C, which is a more realistic value to be admissible by customers. Also, even when thermal comfort weight $\beta$ is three times larger than peak reduction weight, resulting maximal and minimal power peaks at PCC are still lower than the ones obtained by heat-led and DR-L1 operations.

Self-consumption quota and self-sufficiency quota are also calculated and analyzed in Table V. In theory a self-consumption quota of 100% is always possible. Using demand response operation, the self-consumption quota improves around 28 points in percent in comparison to the heat-led operation. As expected, the DR-L2 operation presents the highest self-consumption quota.

The self-sufficiency quota depends on the total PV production, total household load consumption and total heat pump electrical consumption. For the considered dates there is a total PV-production of 670kWh and a total household load consumption of 380kWh. The heat pump electrical consumption varies for each operation mode, with a total consumption of 754kWh, 747kWh and 745kWh for the heat-led, DR-L1 and DR-L2 respectively. This is explained due to the fact that in heat-led mode thermal comfort is well satisfied, which means that thermal energy produced by the heat pump is higher. As the DR operation modes, have a second objective to optimize, the thermal comfort is reduced, which results also in a reduction in the thermal production and accordingly in the electrical consumption of the heat pump. For the heat-led operation there is a theoretical maximum self-sufficiency quota of 59.02%, which means that the heat-led operation achieves a self-sufficiency quota 26 points below its theoretical maximum. For the DR-L1 and DR-L2 there are theoretical maximums for self-sufficiency quota of 59.40% and 59.50% respectively. Both strategies achieve a self-sufficiency quotas around ten points below the theoretical maximum.

IV. Conclusion

In this paper the question was posed to what extent a domestic space heating system can contribute to demand response applications while also maintaining a sensible thermal comfort. The problem formulation not only makes the paper fully replicable but also takes into consideration the acceptability of trade-offs in the residential sector.

In a first step, the simulation results are debated with the help of two exemplary days that are able to reveal the interac-
TABLE IV: Performance indicators with adjusted weights

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Heat-led</th>
<th>DR-L1 (α = 0.5, β = 0.5)</th>
<th>DR-L1 (α = 0.5, β = 0.5)</th>
<th>DR-L2 (α = 0.5, β = 1.0)</th>
<th>DR-L2 (α = 0.5, β = 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean absolute error (MAE, °C)</td>
<td>0.21</td>
<td>0.32</td>
<td>0.55</td>
<td>0.41</td>
<td>0.36</td>
</tr>
<tr>
<td>Mean CPU time (sec)</td>
<td>0.05</td>
<td>0.06</td>
<td>125.87</td>
<td>133.73</td>
<td>128.97</td>
</tr>
<tr>
<td>Max peak PCC (kW(_{el}))</td>
<td>6.05</td>
<td>6.05</td>
<td>2.24</td>
<td>2.69</td>
<td>2.96</td>
</tr>
<tr>
<td>Min peak PCC (kW(_{el}))</td>
<td>-6.29</td>
<td>-6.13</td>
<td>-2.61</td>
<td>-2.61</td>
<td>-2.61</td>
</tr>
</tbody>
</table>

TABLE V: Energy indicators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Heat-led</th>
<th>DR-L1</th>
<th>DR-L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_{sc}) (%)</td>
<td>55.93</td>
<td>82.91</td>
<td>84.99</td>
</tr>
<tr>
<td>(q_{as}) (%)</td>
<td>33.02</td>
<td>49.26</td>
<td>50.57</td>
</tr>
<tr>
<td>Energy import (kWh)</td>
<td>295.31</td>
<td>114.47</td>
<td>100.57</td>
</tr>
<tr>
<td>Energy export (kWh)</td>
<td>760.49</td>
<td>572.39</td>
<td>556.67</td>
</tr>
</tbody>
</table>

tions of the system components and the operating principle of the MPC optimization strategies. With the help of this example it becomes clear that even for smaller interconnected systems, the effects are not always directly foreseeable. Thus, both the weight of individual objectives and the norm have to be determined with care.

Afterwards, an extensive 30-day study was conducted to depict the range of electrical power flows at the PCC, as these are an indicator of the DR applicability. Here, the squared \(l_2\)-norm was the most promising means for peak reductions, while \(l_1\)-norm and the heat-led mode respectively showed higher and less reliable power flows.

Further steps are the evaluation of MPC sensitivity to uncertainties coming from forecasts and model mismatches. The approach will be extended to several systems in a distribution network including other technologies such as mini-CHP and batteries. A benchmark between a decentralized MPC approach and a centralized MPC approach for grid operation with DR will be conducted.

REFERENCES


