COMBINED HEAT AND POWER ECONOMIC DISPATCH BY AUGMENTED LAGRANGE HOPFIELD NETWORK

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Abstract – This paper proposes an augmented Lagrange Hopfield network (ALHN) for combined heat and power economic dispatch (CHPED) problem. The ALHN method is the continuous Hopfield neural network with its energy function based on augmented Lagrangian relaxation. In the proposed ALHN, the energy function is augmented by Hopfield terms from Hopfield neural network and penalty factors from augmented Lagrangian function to damp out oscillation of the Hopfield network during the convergence process, leading to a fast convergence. The proposed ALHN has been tested on various systems and compared to Lagrangian relaxation (LR), genetic algorithm (GA), improved ant colony search algorithm (IACSA), evolutionary programming (EP), improved genetic algorithm with multiplier updating (IGA-MU) and harmony search algorithm (HSA). Test results indicate that the proposed neural network is better than the other methods due to a lower total cost and faster computational time, especially for large-scale CHPED problems.

Keywords: Augmented Lagrange Hopfield network, combined heat and power, economic dispatch

NOMENCLATURE

\begin{align*}
F_i(P_j) & \quad \text{cost function of pure power generating unit } i \\
F_j(P_j, H_j) & \quad \text{cost function of co-generation units } j \\
g_i(V) & \quad \text{inverse sigmoid function of continuous neurons} \\
H_D & \quad \text{system heat demand (MWth)} \\
H_j & \quad \text{heat production of co-generation unit } j \quad \text{(MWth)} \\
H_j^{\max}(P_j) & \quad \text{maximum heat output of co-generation unit } j, \text{a function of output power (MWth)} \\
H_j^{\min}(P_j) & \quad \text{minimum heat output of co-generation unit } j, \text{a function of output power (MWth)} \\
H_k & \quad \text{heat production of pure heat production unit } k \quad \text{(MWth)} \\
H_k^{\max} & \quad \text{maximum output of pure heat production unit } k \quad \text{(MWth)} \\
H_k^{\min} & \quad \text{minimum output of pure heat production unit } k \quad \text{(MWth)} \\
N_c & \quad \text{number of co-generation units} \\
N_h & \quad \text{number of pure heat production units} \\
N_p & \quad \text{number of pure power generating units} \\
P_D & \quad \text{system power demand, in MW} \\
P_i & \quad \text{output power of pure power generating unit } i \quad \text{(MW)} \\
P_{j}^{\max}(H_j) & \quad \text{maximum output power of co-generation unit } j, \text{a function of heat production (MW)} \\
P_{j}^{\min}(H_j) & \quad \text{minimum output power of co-generation unit } j, \text{a function of heat production (MW)} \\
V_{ih} & \quad \text{output of multiplier neuron representing } \lambda_p \\
V_{ip} & \quad \text{output of multiplier neuron representing } \lambda_h \\
V_{ij} & \quad \text{output of continuous neuron } j \text{ representing } H_j \\
V_{ik} & \quad \text{output of continuous neuron } k \text{ representing } H_k \\
V_{it} & \quad \text{output of continuous neuron } i \text{ representing } P_i \\
\beta_p, \beta_h & \quad \text{penalty factors for power and heat demand balances, respectively} \\
\lambda_p, \lambda_h & \quad \text{Lagrange multipliers for power and heat demand balances, respectively}
\end{align*}

1 INTRODUCTION

Combined heat and power generation (co-generation) units have an increasingly important role in energy production technology recently [1]-[3]. A co-generation unit can provide not only power but also heat to customers. For most co-generation units, there is a mutual dependency between heat and power, e.g. the heat production capacity depends on power generation and vice versa. Therefore, the combined heat and power economic dispatch (CHPED) problem introduces complexities in the integration of co-generation units into the power system economic dispatch. The objective of the CHPED problem is to determine both power and heat production from units so that total operation cost is minimized satisfying both power and heat demands.

Many methods have been applied to solve the CHPED problems. An exploitation of the high degree of separability of the cost function and the constraint has been applied in [4]. The method has proved to converge faster than conventional procedure based on quadratic programming. A method based on two dimensional probability load density functions for performing probabilistic production simulation involving combined heat and power units has been developed in [5]. In this method, the equivalent load functions, expected energy generation of units, expected unserved energy, and expected overfl ow are determined by convolution of the combined heat and power units. In [6], the CHPED problem is decomposed into two sub-problems, the heat
dispatch and the power dispatch, connected through the heat-power feasible region constraints of co-generation units. In [7], the CHPED problem has been solved by an improved penalty function for genetic algorithm (GA). An improved ant colony search algorithm (IACSA) with positive feedback, distributed computation and the use of constructive greedy heuristic incorporated with other search techniques has been proposed for solving the CHPED problem in [8]. This method has advantages of rapid discovery of good solutions, avoidance of premature convergence and find of acceptable solutions in the early stages of the search process. However, the solution obtained by this method is usually near global optimum in a simple problem with a slow convergence. An evolutionary programming (EP) for the CHPED has been developed in [9]. In this method, the methods for ensuring the satisfaction of the power and heat demands and determining the dispatch order of the units in the cogeneration system are developed and included in the algorithm. An improved genetic algorithm with multiplier updating (IGA-MU) for the CHPED problem has been proposed in [10]. In this method, IGA equipped with an improved evolutionary direction operator and migration operation is incorporated with multiplier updating method which is introduced to avoid deforming of augmented Lagrange function. This hybrid system requires only a small size population for the problem but still suffers slow convergence. In [11], a harmony search algorithm (HSA) has been presented for CHPED problem. HAS can obtain near optimal solution but slowly converges due to large number of iterations.

In this paper, an augmented Lagrange Hopfield network (ALHN) is proposed for solving CHPED problem. ALHN is the continuous Hopfield neural network with its energy function based on augmented Lagrangian relaxation. In the proposed ALHN, the energy function is augmented by Hopfield terms from Hopfield neural network and penalty factors from augmented Lagrangian function to damp out oscillation of the Hopfield network during the convergence process, leading to a fast convergence. The effectiveness of the proposed ALHN is demonstrated by comparing to LR [6], GA [7], IACSA [8], EP [9], IGA-MU [10] and HAS [11] on various test systems.

2 CHPED PROBLEM FORMULATION

The problem considers three types of units including pure power, combined power and heat, and pure heat units. For co-generation units, the heat-power feasible operation region of a combined power and heat unit is shown in Figure 1, where the boundary curve ABCDEF determines the feasible region. Along the boundary there is a trade-off between power generation and heat production from the unit. It can be seen that along the curve AB the unit reaches maximum output power. In contrast, the unit reaches maximum heat production along the curve CD. Therefore, power generation limits of co-generation units are the combined functions of the unit heat production and vice versa.

Mathematically, the problem is formulated as follows

\[
\text{Min} \left\{ \sum_{i=1}^{N_i} F_i(P_i) + \sum_{j=1}^{N_j} F_j(P_j, H_j) + \sum_{k=1}^{N_k} F_k(H_k) \right\} 
\]

subject to

(a) power balance constraint

\[
P_D - \sum_{i=1}^{N_i} P_i - \sum_{j=1}^{N_j} P_j = 0
\]

(b) heat balance constraint

\[
H_D - \sum_{j=1}^{N_j} H_j - \sum_{k=1}^{N_k} H_k = 0
\]

(c) generation and heat limits constraints

\[
P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}}
\]

\[
P_j^{\text{min}} (H_j) \leq P_j \leq P_j^{\text{max}} (H_j)
\]

\[
H_j^{\text{min}} (P_j) \leq H_j \leq H_j^{\text{max}} (P_j)
\]

\[
H_k^{\text{min}} \leq H_k \leq H_k^{\text{max}}
\]

where \(F_i(P_i), F_j(P_j, H_j)\) and \(F_k(H_k)\) are convex functions.

3 AN ALHN FOR CHPED

For the implementation of ALHN, the augmented Lagrangian function for the problem is firstly formulated, and then energy function of ALHN is constructed based on the formulated augmented Lagrangian function and enhanced by Hopfield terms. The augmented Lagrangian function is formulated:
\[ L = \sum_{i=1}^{N_c} F_i(P_i) + \sum_{i=1}^{N_m} F_i(P_i, H_j) + \sum_{i=1}^{N_h} F_i(H_k) \]
\[ + \lambda_\alpha \left( P_D - \sum_{i=1}^{N_c} P_i - \sum_{i=1}^{N_m} P_i \right) + \frac{1}{2} \beta_\lambda \left( P_D - \sum_{i=1}^{N_c} P_i - \sum_{i=1}^{N_m} P_i \right)^2 \]  
\[ + \lambda_\alpha \left( H_D - \sum_{j=1}^{N_h} H_j - \sum_{j=1}^{N_h} H_j \right) \]
\[ + \frac{1}{2} \beta_\lambda \left( H_D - \sum_{j=1}^{N_h} H_j - \sum_{j=1}^{N_h} H_j \right)^2 \]  

In the ALHN method, the neurons associated with continuous variables are called continuous neurons and neurons associated with Lagrangian multipliers are called multiplier neurons. To implement in ALHN, \( N_c + 2 * N_m + N_h \) continuous neurons and two multiplier neurons are required.

Energy function of ALHN is defined as follows:
\[ E = \sum_{i=1}^{N_c} F_i(V_{pi}) + \sum_{i=1}^{N_m} F_i(V_{pi}, V_{pj}) + \sum_{i=1}^{N_h} F_i(V_{ph}) \]
\[ + V_{dp} \left( P_D - \sum_{i=1}^{N_c} V_{pi} - \sum_{i=1}^{N_m} V_{pi} \right) \]
\[ + \frac{1}{2} \beta_\lambda \left( P_D - \sum_{i=1}^{N_c} V_{pi} - \sum_{i=1}^{N_m} V_{pi} \right)^2 \]
\[ + V_{dh} \left( H_D - \sum_{j=1}^{N_h} V_{pj} - \sum_{j=1}^{N_h} V_{pj} \right) \]
\[ + \frac{1}{2} \beta_\lambda \left( H_D - \sum_{j=1}^{N_h} V_{pj} - \sum_{j=1}^{N_h} V_{pj} \right)^2 \]
\[ + \sum_{i=1}^{N_c} \int g^{-1}(V) dV + \sum_{i=1}^{N_m} \int g^{-1}(V) dV \]
\[ + \sum_{i=1}^{N_h} \int g^{-1}(V) dV + \sum_{i=1}^{N_h} \int g^{-1}(V) dV \]

where \( g^{-1} \) is the inverse function of sigmoid function of continuous neurons and the four last terms in (9) are Hopfield terms where their global effect is a displacement of solutions toward the interior of the state space [12].

Dynamics of the neurons in ALHN are defined:
\[ \frac{dU_c}{dt} = -\frac{\partial E}{\partial V_c} \]  
\[ \frac{dU_m}{dt} = -\frac{\partial E}{\partial V_m} \]  

where \( U_c \) and \( V_c \) represent inputs and outputs of continuous neurons, respectively and \( U_m \) and \( V_m \) represent inputs and outputs of multiplier neurons, respectively. In (10) and (11), \( V_c \) represents \( V_{pi}, V_{pj} \) and \( V_{ph} \), and \( V_m \) represents \( V_{dp} \) and \( V_{dh} \). The inputs \( U_c \) and \( U_m \) represent the corresponding inputs of \( V_{pi}, V_{pj} \) and \( V_{ph} \) and \( V_{dp} \) and \( V_{dh} \), respectively.

Inputs of neurons at iteration \( n \) are updated:
\[ U_c^{(n)} = U_c^{(n-1)} - \alpha_c \frac{\partial E}{\partial V_c} \]  
\[ U_m^{(n)} = U_m^{(n-1)} - \alpha_m \frac{\partial E}{\partial V_m} \]  

where \( \alpha_c \) and \( \alpha_m \) are updating step sizes for continuous and multiplier neurons, respectively.

Outputs of continuous neurons are determined by a sigmoid function [13]:
\[ V_c = g(U_c) = \left( PH_{max} - PH_{min} \right) \left( \frac{1 + \tan \sigma U_c}{2} \right) + P_{min} \]  

where \( PH_{max} \) and \( PH_{min} \) are the maximum and minimum output values of power generation or heat production, and \( \sigma \) is the slope of the sigmoid function. The sigmoid function with different slopes for continuous neurons is shown in Figure 2.

\[ \sigma = 0.005 \]  
\[ \sigma = 0.01 \]  
\[ \sigma = 100 \]  

\[ \text{Figure 2: Sigmoid function of continuous neurons with different values of the slope.} \]

Based on Figure 1, the dependent maximum and minimum power generation and maximum and minimum heat production of co-generation units are determined:
\[ P_j^{\max}(H_j) = \min \left\{ P_j(H_j)_{\text{AB}}, P_j(H_j)_{\text{IC}} \right\} \]  
\[ P_j^{\min}(H_j) = \max \left\{ P_j(H_j)_{\text{EF}}, P_j(H_j)_{\text{CD}} \right\} \]  
\[ H_j^{\max}(P_j) = \min \left\{ H_j(P_j)_{\text{IC}}, H_j(P_j)_{\text{CD}} \right\} \]  
\[ H_j^{\min}(P_j) = 0 \]

The outputs of multiplier neurons are determined via a linear transfer function as follows:
\[ V_m = g_m(U_m) = U_m \]  

The proof of convergence for the proposed ALHN is given in [14].
3.1 Selection of parameters

The parameters of ALHN are easily tuned for better convergence. Different set of parameters will lead to the same final result in a different computational time. A proper parameter selection will guarantee rapid convergence to ALHN. For simplicity, updating step sizes of continuous neurons associated with power outputs and penalty factors are fixed at 1 and 0.01, respectively. The values of others function slope and penalty factors are fixed at 100 and 1. By experiments, the values of sigmoid and transfer functions, respectively.

For the continuous neurons, the outputs are initiated: initial outputs of multiplier neurons are determined as follows:

\[ V_{pi}^{(0)} = \frac{P_i^{\text{max}}}{\sum_{j=1}^{N_i} P_j^{\text{max}} + \sum_{j=1}^{N_j} P_j^{\text{max}}} P_D \]  

\[ V_{pj}^{(0)} = \frac{P_j^{\text{max}}}{\sum_{i=1}^{N_i} P_i^{\text{max}} + \sum_{j=1}^{N_j} P_j^{\text{max}}} P_D \]  

\[ V_{hj}^{(0)} = \frac{H_j^{\text{max}}}{\sum_{j=1}^{N_j} H_j^{\text{max}} + \sum_{k=1}^{N_k} H_k^{\text{max}}} H_D \]  

\[ V_{hi}^{(0)} = \frac{H_i^{\text{max}}}{\sum_{j=1}^{N_j} H_j^{\text{max}} + \sum_{k=1}^{N_k} H_k^{\text{max}}} H_D \]  

The initial outputs of multiplier neurons are determined by solving the dynamics (11) neglecting inputs of neurons and penalty factors. The outputs of multiplier neurons are initiated:

\[ V_{p}^{(0)} = \frac{\partial F_{i}(V_{pi}, V_{hj})}{\partial V_{pi}} \bigg|_{V_{pi}, V_{hj}^{(0)}} \]  

\[ V_{h}^{(0)} = \frac{\partial F_{i}(V_{pi}, V_{hj})}{\partial V_{hi}} \bigg|_{V_{pi}, V_{hj}^{(0)}} \]  

The initial inputs of continuous and multiplier neurons are calculated via the invered functions of the sigmoid and transfer functions, respectively.

3.3 Termination criteria

In the proposed ALHN, the algorithm will be terminated when either the maximum error \( \text{Err}_{\text{max}} \) is lower than a pre-specified tolerance \( \epsilon \) or the maximum number of iterations \( N_{\text{max}} \) is reached.

4 NUMERICAL RESULTS

The proposed ALHN is tested on different benchmarked systems from [6] and [11]. The results from the proposed method here are compared to all other methods available in the literature for the tested systems. Based on the study on these systems, the proposed method can be easily applied to the realistic test cases. The CPU times here are to estimate the efficiency of methods rather than the exact comparison since the difference becomes obvious for large-scale systems.

The algorithm of the proposed ALHN is coded in MATLAB and run on 1.1GHz Celeron PC. The maximum tolerance for ALHN \( \epsilon \) is set to \( 10^{-3} \).

4.1 Case 1

The test system in [6] has one pure power generation unit, two co-generation units and one pure heat production unit. Data from test system is described as follows:

\[ F_{d}(P_i) = 50P_i \quad \text{($/h$)} \]

\[ F_{d}(P_i,H_j) = 2650 + 14.5P_2 + 0.0345P_2^2 + 4.2H_2 + 0.03H_2^2 + 0.031P_3H_3 \quad \text{($/h$)} \]

\[ F_{d}(P_i,H_j) = 1250 + 36P_2 + 0.0435P_2^2 + 0.6H_3 + 0.027H_3^2 + 0.011P_3H_3 \quad \text{($/h$)} \]

\[ F_{d}(H_j) = 23.4H_j \quad \text{($/h$)} \]

\[ 0 \leq P_i \leq 150 \text{ MW} \]

\[ 0 \leq H_i \leq 2695.2 \text{ MWth} \]

Output limits of heat and power for co-generation units 2 and 3 are given in Figures 3 and 4. The system power and heat demands are 200 MW and 115 MWth, respectively.

The results obtained from the proposed ALHN are compared to LR [6], GA [7], IACSA [8], EP [9], IGA-MU [10] and HSA [11] in Table 1. Due to a small sys-
tem, the methods can easily find an optimal solution. The total production cost from the proposed method is much closer to that of LR, EP, IGA-MU and HSA, and less than GA and IACSA.

Figure 3: Heat-power feasible operation region for co-generation unit 2 in Cases 1 and 2.

4.2 Case 2

A steam turbine is added to the test system in Case 1 to form combined cycle CHP [16]. The steam generating unit always generates a fixed output power of 80 MW while consuming 120 MWth of heat. The power and heat demands for this case are 250 MW and 115 MWth, respectively.

Table 2 shows the comparison of the results obtained by the proposed method and Lagrange relaxation and sequential quadratic programming (LR-SQP) method [16]. The proposed ALHN has obtained less total cost than LR-SQP for this case.

Figure 4: Heat-power feasible operation region for co-generation unit 3 in Cases 1 and 2, and unit 2 in Case 3.

4.3 Case 3

The test system consists of one pure power unit, three co-generation units and one pure heat unit from [16], in which the co-generation unit 2 is similar to co-generation unit 3 in Case 1. The cost functions of units are:

\[
F_1(P_1) = 254.8863 + 7.6997P_1 + 0.00172P_1^2 + 0.000115P_1^3 \quad ($/h)
\]

\[
F_2(P_2, H_2) = 1250 + 36P_2 + 0.0435P_2^2 + 0.6H_2 + 0.027H_2^2 + 0.011P_2H_2 \quad ($/h)
\]

\[
F_3(P_3, H_3) = 1565 + 20P_3 + 0.072P_3^2 + 2.3H_3 + 0.027H_3^2 + 0.011P_3H_3 \quad ($/h)
\]

\[
F_4(P_4, H_4) = 950 + 2.0109H_4 + 0.038H_4^2 \quad ($/h)
\]

\[
35 \leq P_1 \leq 135 \text{ MW}
\]

\[
0 \leq H_5 \leq 60 \text{ MWth}
\]

Table 1: Total cost comparison with other methods for 4-unit system in Case 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>(P_1) (MW)</th>
<th>(P_2) (MW)</th>
<th>(P_3) (MW)</th>
<th>(H_2) (MWth)</th>
<th>(H_3) (MWth)</th>
<th>(H_4) (MWth)</th>
<th>Cost ($/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR [6]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9,257.10</td>
</tr>
<tr>
<td>GA [7]</td>
<td>0</td>
<td>159.23</td>
<td>39.94</td>
<td>40.77</td>
<td>75.06</td>
<td>0</td>
<td>9,267.20</td>
</tr>
<tr>
<td>IACSA [8]</td>
<td>0.08</td>
<td>150.93</td>
<td>48.84</td>
<td>49.00</td>
<td>65.79</td>
<td>0.37</td>
<td>9,452.20</td>
</tr>
<tr>
<td>EP [9]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9,257.10</td>
</tr>
<tr>
<td>IGA-MU [10]</td>
<td>0</td>
<td>160.00</td>
<td>40.00</td>
<td>39.99</td>
<td>75.00</td>
<td>0</td>
<td>9,257.08</td>
</tr>
<tr>
<td>HSA [11]</td>
<td>0</td>
<td>160.00</td>
<td>40.00</td>
<td>40.00</td>
<td>75.00</td>
<td>0</td>
<td>9,257.07</td>
</tr>
<tr>
<td>ALHN</td>
<td>0</td>
<td>159.9994</td>
<td>40</td>
<td>39.9993</td>
<td>75</td>
<td>0</td>
<td>9,257.05</td>
</tr>
</tbody>
</table>

Table 2: Total cost comparison with other method for 5-unit system in Case 2.
<table>
<thead>
<tr>
<th>Case</th>
<th>Method</th>
<th>Demand</th>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
<th>Unit 4</th>
<th>Unit 5</th>
<th>Total cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GA [11]</td>
<td>300</td>
<td>150</td>
<td>135.00</td>
<td>70.81</td>
<td>80.54</td>
<td>10.84</td>
<td>39.81</td>
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<tr>
<td></td>
<td>HAS [11]</td>
<td>134.74</td>
<td>48.20</td>
<td>81.09</td>
<td>16.23</td>
<td>23.92</td>
<td>100.85</td>
<td>6.29</td>
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<td></td>
<td>ALHN</td>
<td>135.00</td>
<td>40.77</td>
<td>73.59</td>
<td>19.23</td>
<td>36.78</td>
<td>105.00</td>
<td>14.40</td>
</tr>
<tr>
<td>2</td>
<td>GA [11]</td>
<td>250</td>
<td>175</td>
<td>119.22</td>
<td>45.12</td>
<td>78.94</td>
<td>15.82</td>
<td>22.63</td>
</tr>
<tr>
<td></td>
<td>HAS [11]</td>
<td>134.67</td>
<td>52.99</td>
<td>85.69</td>
<td>10.11</td>
<td>39.73</td>
<td>52.23</td>
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<tr>
<td></td>
<td>ALHN</td>
<td>135.00</td>
<td>40.00</td>
<td>75.00</td>
<td>10.00</td>
<td>40.00</td>
<td>65.00</td>
<td>14.40</td>
</tr>
<tr>
<td>3</td>
<td>GA [11]</td>
<td>160</td>
<td>220</td>
<td>37.98</td>
<td>76.39</td>
<td>106.00</td>
<td>10.41</td>
<td>38.37</td>
</tr>
<tr>
<td></td>
<td>ALHN</td>
<td>35.00</td>
<td>42.07</td>
<td>82.07</td>
<td>17.31</td>
<td>43.47</td>
<td>65.62</td>
<td>34.46</td>
</tr>
</tbody>
</table>

Table 3: Total cost comparison with other methods for 5-unit system in Case 3.

The feasible operation zone for co-generation units 2-4 are depicted in Figures 4-6. The test system is considered for three load and heat demands.

The total costs obtained from ALHN for the three load levels are compared to those from GA and HAS from [11] shown in Table 3. In this case, the proposed ALHN also obtains less total costs than the others.

4.4 Case 4

The proposed method is also implemented on large scale CHPED problems. In this implementation, four systems are considered with 20, 40, 80 and 120 units based on the basis of the four-unit system in Case 1. To obtain the system with 20 units, the basic four units are duplicated five times for number of units and load demands.

<table>
<thead>
<tr>
<th>No. of units</th>
<th>CGA [10] (S/h)</th>
<th>IGA-MU [10] (S/h)</th>
<th>ALHN (S/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>47,984.048</td>
<td>46,285.375</td>
<td>46,285.409</td>
</tr>
<tr>
<td>40</td>
<td>99,643.170</td>
<td>92,570.764</td>
<td>92,570.775</td>
</tr>
<tr>
<td>80</td>
<td>207,066.240</td>
<td>185,145.060</td>
<td>185,141.509</td>
</tr>
<tr>
<td>120</td>
<td>321,895.820</td>
<td>277,815.840</td>
<td>277,712.791</td>
</tr>
</tbody>
</table>

Table 4: Total costs comparison for systems up to 120 units in Case 4.

<table>
<thead>
<tr>
<th>No. of units</th>
<th>CGA [10] (s)</th>
<th>IGA-MU [10] (s)</th>
<th>ALHN (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>264.47</td>
<td>86.95</td>
<td>0.37</td>
</tr>
<tr>
<td>40</td>
<td>534.54</td>
<td>168.67</td>
<td>0.38</td>
</tr>
<tr>
<td>80</td>
<td>1,053.69</td>
<td>343.01</td>
<td>0.42</td>
</tr>
<tr>
<td>120</td>
<td>1,489.14</td>
<td>471.27</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 5: CPU times comparison for systems up to 120 units in Case 4.
The total costs and computational times for the large scale systems from the proposed method are compared to those from conventional GA (CGA) and IGA-MU in [10] given in Tables 4 and 5. The total costs from the proposed method are much less than CGA and slightly less than IGA-MU for systems of 80 and 120 units. This shows that the proposed method is very efficient for the CHPED problem since it is simple for implementation and obtains better results than GA based methods.

The CPU times obtained from CGA and IGA-MU in [10] were from a PIII (700MHz) PC which is a little slower than the hardware of the computer used in this study. However, the CPU times obtained from the proposed method are vastly faster than the others. It is shown that for the large-scale problems the efficiency of methods in terms of total cost and computational time are more obvious.

5 CONCLUSION

ALHN is efficiently solving CHPED problem. The proposed ALHN is a combination of Hopfield neural network and Lagrangian relaxation. In ALHN, augmented Lagrangian function used its energy function can easily handle the problem constraints in addition to sigmoid function of Hopfield network which can properly handle unit limits. Moreover, the Hopfield terms and penalty factors from the augmented Lagrangian function have damped out the oscillation of Hopfield network during the convergence process. On the other hand, an appropriate selection of parameters for ALHN will guarantee rapid convergence among different sets of parameters, leading to the convergence of ALHN. The proposed network results are better than other methods in terms of less total costs and shorter computational times than the other methods in the literature, especially for large systems. Therefore, the proposed ALHN is very favorable for solving large scale CHPED problems. CHPED problem considering ramp rate will be further investigated by ALHN.

REFERENCES