STATE SPACE ESTIMATION OF ELECTRIC ARC PARAMETERS BASED ON MAXIMUM LIKELIHOOD AND KALMAN FILTERS

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Abstract - This work describes a state space estimation method based on Maximum Likelihood (ML) and Kalman Filters. The methodology is tested against two distinct circuit configuration, one dealing with the arc in a circuit breaker and other dealing with the simulation of a secondary arc. For the circuit breaker case two arc models were considered, namely Mayr and Habedank models. For the second circuit configuration an arc in air(secondary arc) was considered using a Modified Thiel arc model. These tests are needed to assess whether the state space estimation was independent of the arc model. In all simulated cases a white noise was added to the signal in order to evaluate the robustness of the identification/estimation procedure. The procedure is based upon a dual state/parameters estimation used as a first step with a refinement based on ML and Gauss-Newton to achieve an optimum state realization. The results indicate that very accurate results were achieved in all cases regardless of the model to be identified.

Keywords - State Estimation, Parameter estimation, arc modeling, Extended Kalman Filter, Maximum likelihood, Gauss Newton.

1 INTRODUCTION

ALTHOUGH a great effort has already been made to represent an electric arc in Power Systems little has been done with regard to the identification of the arc parameters. The goal of arc modeling is to improve the analysis whenever a circuit breaker or secondary arc is involved. There are several models such as Cassie [2], Mayr [1], Thiel [4], Schavemaker [6], Habedank [3, 11] to represent circuit breaker arc; and Johns [15], Kizilcay [14] and Portela[4] models, just to name a few, to represent secondary arc. Although the technical literature presents several arc models, only recently the issue of arc parameters determination has been addressed [7, 8, 9, 10, 12, 13]. It should be pointed out that in the aforementioned work, one considers a specific arc model. The aim of this paper is slightly different, a state estimation technique is used to identify distinct arc models implemented via simulation. In order to assess the robustness of the proposed procedure white noise is added to the simulation results. The simulated configurations were implemented in EMTDC and MATLAB.

Unlike conventional system identification based upon black box model where one does not consider the dynamics of the system to be identified, here a gray box model was used instead. In a gray box model one must assume some specific dynamics for the system. Thus some prior knowledge of the overall structure of the system to be identified is required, i.e., one must consider a specific differential equation to represent the system. For the case at hand here it implies that one must consider a specific arc model prior to identification of its parameters.

To estimate the arc parameters a state space model with the arc conductance as a state variable was used. The output (input) of the model is the arc voltage (current) depending on the differential equation used for the electric arc. The identification procedure can be understood as dual (state/parameter) estimation as both parameters and state are obtained in the same iteration. This procedure was also implemented in MATLAB.

2 ARC MODELING

For the sake of clarity in this section we briefly review all the arc models considered in this work and their respective characteristics.

For the circuit breaker case the well known Mayr [1] model was used as shown in (1)

$$\frac{dg}{dt} = \frac{1}{\tau_m} \left[ \frac{ui}{P_0} - 1 \right]$$

where $u$ is the arc voltage and $i$ is the arc current, $P_0$ is the arc conductance and $\tau_m$ is the time constant associated with the arc. Another possibility for the circuit breaker is to use a model with a combination of Mayr and Cassie Model [2] as proposed by Habedank [3] which is a cascade connection of two arcs, one modeled using Cassie’s model with a time constant $\tau_c$ and conductance $g_c$ and a second arc using Mayr’s formulation model with a time constant $\tau_m$ and conductance $g_m$ as shown below

$$\frac{dg_c}{dt} = \frac{1}{\tau_c} \left[ \frac{u^2g^2}{U_0^2} - g_c \right]$$

$$\frac{dg_m}{dt} = \frac{1}{\tau_m} \left[ \frac{u^2g^2}{P_0^2} - g_m \right]$$

(2)
Despite the fact that an arc in air may present a similar dynamic behavior in comparison with the one found in circuit breakers, the arc parameters will vary considerably. For the identification of this system a secondary arc case modeled as a Modified Thiel model [4] was considered. It consists of a parallel connection of two arcs, each modelled by a Thiel’s model, where each conductance is given by

\[ g_1 = \xi g \]
\[ g_2 = \eta g \]  

with \( \eta = 1 - \xi \) and \( g \) total arc conductance. The complete arc model can thus be defined as

\[ \frac{dg_1}{dt} = \frac{1}{\tau_1} \frac{u^2 g^2}{P_0} - g_1 \]
\[ \frac{dg_2}{dt} = \frac{1}{\tau_2} \frac{u^2 g^2}{P_0} - g_2 \]  

where \( \tau_1 = A_1 g^\alpha \), \( \tau_2 = A_2 g^\alpha \), being \( A_1 \), \( A_2 \) and \( \alpha \) constants and \( P_0 = B g^\beta \), and \( B \) and \( \beta \) variable parameters. Therefore this model is defined by 5 parameters, 2 variables and 3 constants.

As the arc are connected in parallel, the total conductance of the Modified Thiel arc model is

\[ g = g_1 + g_2 \]  

and the total arc current is given by

\[ i = u (g_1 + g_2) \]  

where \( u \) the voltage across the arc.

It can be noticed that a Thiel arc model can be seen as a modified Mayr arc model in which the “time constant” and the cooling power are dependent on the arc conductance [4].

3 STATE SPACE MODELING OF AN ARC

The modeling of RLC circuit is trivial while the modeling of distributed parameters such as overhead transmission lines or underground cable are also known for a number of years [5]. Unfortunately for a nonlinear models to obtain the state space model a linearization stage is needed. Here the procedure to obtain a state space model of an arc is carried out in detail using the Mayr’s model. To illustrate this matter further consider an electric arc given by the Mayr’s model as previously shown in (1). By a suitable change of variables, i.e. \( \log(g) = x \) and using \( i = u g \) one obtain a nonlinear state space equation as shown in (7).

\[ \frac{dx}{dt} = \frac{\exp(x) u^2}{P_0} - \frac{1}{\tau_m} \]  

The model defined in (7) depends only on time constant \( \tau_m \) and the cooling power \( P_0 \). If a linearization is applied one obtains

\[ \frac{dg}{dt} = \frac{i^2}{P_0} - \frac{g}{\tau_m} \]
\[ i = u g \]  

This procedure allows for a better performance of the linear stage in the parameter estimation and can be applied in cases where more than one sub-arc is involved. For instance, consider an arc described by the Habedank’s model The Habedank’s model can thus be defined as a model with four parameters, two time constants, the arc cooling power \( P_0 \) and the arc voltage \( U_C \). For this model the total arc current is given by

\[ i = u \frac{g_0 g_m}{g_c + g_m} \]  

4 STATE AND PARAMETERS ESTIMATION

Independently of the arc model considered, the identification procedure is divided in two stages. In the first stage a Kalman filter is used to estimate state variable, i.e., arc conductance. To apply Kalman filters one need a linear system, thus after a discrete time arc model is obtained via a “Zero-Order Hold” approach a linearization is carried out, see appendix for details. The linearized equations can then be written in state space as shown in (10).

\[ x(t + \Delta t) = A(t)x(t) + B(t)u(t) \]
\[ y(t) = C(t)x(t) + D(t)u(t) \]  

In the second stage, a Maximum Likelihood using Gauss-Newton method is used to obtain the matrix parameters, \( A(t), B(t), C(t) \) and \( D(t) \) with which after a simple algebraic manipulation one may obtain the arc parameters. This dual state/parameter estimation is summarized in Fig. 1.

![Figure 1: Structure of State and Parameter Estimation](image)

In order to assess the identification and estimation routines an additional stage called output error estimation (OEM) [18, 19] is implemented as shown in Fig. 2.
4.1 Kalman Filter

The Kalman filter [16] is basically a recursive filter which estimates the state of a dynamic system based on incomplete and noisy data. It can be regarded as analogous to the hidden Markov model, with the key difference that the hidden state variables are continuous (as opposed to being discrete in the hidden Markov model). Additionally, the hidden Markov model can represent an arbitrary distribution for the next value of the state variables, in contrast to the Gaussian noise model that is used for the Kalman filter. The Kalman filter is used in a wide range of engineering applications. Kalman filters are based on linear systems, making it inappropriate to nonlinear systems estimation; nevertheless it can be applied to a linearized expression of the arc equation.

4.2 Maximum Likelihood

Maximum likelihood provides a consistent approach to parameter estimation problems. This means that maximum likelihood estimates can be developed for a large variety of estimation situations. For example, they can be applied in reliability analysis. Maximum likelihood methods have desirable mathematical and optimality properties [17, 18, 19, 20, 21]. Unlike some optimization process where the goal is to minimize an objective function, in Maximum likelihood one aims at maximize it. In some cases is more convenient to work with the negative of the maximum likelihood function. A Gauss-Newton method [22] refinement was also implemented in order to achieve an optimized approximation of the state realization. Kalman filter and maximum likelihood implementation are further detailed in references [16, 17, 18, 19, 20, 21].

5 TEST CASES

To analyze the performance of the dual identification scheme a single-phase circuit was used as shown in Fig. 3 where “Ia” is the total arc current and “Va” the arc voltage. The arc is represented as a switch with a controllable conductance, but it could also be modeled as a time-varying resistance. The simulated system data is given in Table 5, while the arc parameters are shown in Table 1 and 2. The circuit and the identification scheme were implemented in Simulink/MATLAB for Mayr and Habedank cases.

For the Modified Thiel case an EMTDC test case was built. In order to present a similar performance to an actual measured current, white noise was added to the simulated signal prior to the application of the estimation procedure.

Four tests were considered. In the first two tests the arc is represented using (1) and the difference among those tests lies in the fact that in the first case a nonlinear identification was used. In the third case the arc was modeled using (2) and the four case was modeling using (4). Fig. 3 shows the circuit used for the circuit breaker arc tests and Fig. 4 shows the single-line diagram for the secondary arc case in a 500 kV system. In the latter case, all transmissions lines were simulated as three-phase untransposed lines and the line is transposed using a 1/6, 1/3, 1/3 and 1/6 scheme, the load is a frequency dependent network which delivers at 60 Hz the line nominal power. For this case there were two estimates tests one with and other without noise.

<table>
<thead>
<tr>
<th>arc model</th>
<th>(\tau_m)</th>
<th>(\tau_c)</th>
<th>(P_0)</th>
<th>(U_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mayr (nonlinear)</td>
<td>0.3(\mu s)</td>
<td>—</td>
<td>30900</td>
<td>—</td>
</tr>
<tr>
<td>Mayr (simplified)</td>
<td>0.3(\mu s)</td>
<td>—</td>
<td>30900</td>
<td>—</td>
</tr>
<tr>
<td>Habedank (nonlinear)</td>
<td>0.3(\mu s)</td>
<td>1.2(\mu s)</td>
<td>30900</td>
<td>3850</td>
</tr>
</tbody>
</table>

Table 1: Circuit breaker arc parameters

<table>
<thead>
<tr>
<th>(A_1)</th>
<th>(A_2)</th>
<th>(\alpha)</th>
<th>(B)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.92e-6</td>
<td>7.65e-5</td>
<td>0.1</td>
<td>0.41e6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 2: secondary arc parameters

![Figure 3: Circuit breaker arc test case](image)

![Figure 4: Secondary arc test case](image)
Table 3: Estimation errors in %

<table>
<thead>
<tr>
<th>Arc Model</th>
<th>$\tau_m$</th>
<th>$\tau_c$</th>
<th>$P_0$</th>
<th>$U_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mayr</td>
<td>0.04</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Mayr (simpl)</td>
<td>0.255</td>
<td>—</td>
<td>0.07</td>
<td>—</td>
</tr>
<tr>
<td>Habedank</td>
<td>0.88</td>
<td>1.12</td>
<td>1.45</td>
<td>1.45</td>
</tr>
</tbody>
</table>

For the identification of the Modified Thiel model the parameters $B$ and $\tau_c$ values were fixed, with the arc model data shown in Table 2. In this paper we considered the following parameters $\xi = 0.41$ and $\eta = 0.59$ respectively. Therefore for this case we must obtain the following parameters $A_1$, $A_2$, $\alpha$. The “time constants” $\tau_1$ and $\tau_2$ are estimated using

\[
\log(\tau) = \log(A) + \alpha \log(\tau),
\]

\[
\log(P_0) = \log(B) + \beta \log(g)
\]

The results of the state/parameter identification are summarized in Table 4.

<table>
<thead>
<tr>
<th>Case</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\alpha$</th>
<th>$B$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>without noise</td>
<td>2.34</td>
<td>1.78</td>
<td>1.24</td>
<td>2.56</td>
<td>2.78</td>
</tr>
<tr>
<td>with noise</td>
<td>4.66</td>
<td>3.35</td>
<td>3.43</td>
<td>4.89</td>
<td>5.22</td>
</tr>
</tbody>
</table>

Table 4: Parameters errors in % for the secondary arc identification

The Fig. 6 and Fig. 7 shows voltage and current for the modified Thiel arc model when the simulated current does not have any noise and Fig. 8 shows the results of voltage and current for the modified Thiel arc with white noise.

Table 5: Simulated system data

<table>
<thead>
<tr>
<th>Circuit Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_s(H)$</td>
<td>$3.52 \times 10^{-3}$</td>
</tr>
<tr>
<td>$R_e(\text{Ohms})$</td>
<td>29.8</td>
</tr>
<tr>
<td>$L_e(H)$</td>
<td>$5.28 \times 10^{-3}$</td>
</tr>
<tr>
<td>$C_e(\text{F})$</td>
<td>1.98e-6</td>
</tr>
<tr>
<td>$R_L(\text{Ohms})$</td>
<td>450</td>
</tr>
<tr>
<td>$L_L(H)$</td>
<td>$6.256 \times 10^{-4}$</td>
</tr>
<tr>
<td>$C_L(\text{F})$</td>
<td>$1.93 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS

This paper has presented a dual state/parameter identification procedure involving an electric arc. Two main configurations were considered, one where the arc equations are used to model a circuit breaker and the other where the arc equations are used to model an secondary arc. For the modeling of the circuit breaker two models were considered either using Mayr’s or Habedank’s equations while for the modeling of arc in air a Modified Thiel model was used. The identification procedure is based on Kalman filter and Maximum likelihood. A Gauss-Newton refinement was implemented in the Maximum likelihood stage to improve the estimations. The results indicate that fairly accurate results can be achieved although the presence of noise usually implies in a higher number of iterations for the convergence of the estimation procedure as show in Table 4.
iterations required are around 20 or higher. The proposed scheme was able to identify fairly distinct arc model, although further studies are still needed in order to assure the generality of the propose procedure. As the secondary arc estimation presented a relatively higher errors, future work will focus in improving the identification procedure when the arc time constants also depend on the arc conductivity.

Validation of the methodology for estimation of arc parameters from laboratory test results will be discussed in a future paper.

REFERENCES


