SOLVING THE HYDROTHERMAL UNIT COMMITMENT PROBLEM VIA LAGRANGIAN RELAXATION AND AUGMENTED LAGRANGIAN

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Abstract – This paper considers the hydrothermal unit commitment problem of power systems with high proportion of hydraulic generation. In this sense, a large-scale mixed-integer nonlinear programming problem for the two day cost-optimal generation of system is developed. The model involves a large number of mixed-integer decision variables and constraints linking time periods and operation units. A Lagrangian Relaxation - LR scheme, based on variable splitting technique, is designed by assigning multipliers to all constraints coupling. The resulting dual problem is solved by a Bundle method. As the LR method fails to find a feasible solution, a primal recovery phase is also developed. Starting of the dual solution, we use an Augmented Lagrangian technique to find a near-optimal primal feasible solution. We assess our approach on real-life hydrothermal configuration extracted from Brazilian Hydrothermal power system, proving the conceptual and practical feasibility of the proposed algorithm.

Keywords: Hydrothermal Unit Commitment, Lagrangian Relaxation, Augmented Lagrangian.

1 INTRODUCTION

The Hydrothermal Unit Commitment – HTUC is a very complex optimization problem and it has been a subject of intense research in the power systems area in the last decades. In this problem, specifically for hydro plants, production decisions are coupled along time stages (reservoirs storage depends on the past system operation) and distinct plants are also coupled in same cascade. Furthermore, this problem presents other type of coupling among generators by means of demand and reserve constraints. Due to the presence of these multiple coupling, decomposition techniques [1,2] appear as a natural approach for solving the HTUC problem. In this direction, an approach, based on the duality theory, is the Lagrangian Relaxation – LR, which is one of the most powerful decomposition methods to solve the HTUC problem for large scale hydrothermal systems [3].

The classical application form of LR relaxes coupling constraints, such as demand and reserve requirements. The reason is that these constraints couple all generators at every time stage. As a result, for a given set of Lagrange Multipliers the dual problem can be separated into a number of smaller subproblems. Nevertheless it is worth to observe that even using this type of decomposition, the hydro subproblem is still very complex because time and space linked constraints still remain.

An efficient approach to deal with this difficulty consists in combining LR with Variable Splitting – LRVS method [4], where the decomposition is achieved by duplicating some variables. In this approach, artificial variables are used to replace the original ones and equality constraints are added to the problem. These equality constraints are relaxed in following.

In this paper, the LRVS is used to duplicate thermal and hydro generation variables, and also, the turbine outflow and spillage variables. The advantage of our approach is to obtain four separate subproblems: thermal, hydro, hydrothermal and hydraulic. The first two subproblems take into account the unit commitment constraints (thermal and hydro, respectively). The hydrothermal subproblem takes into account the demand and reserve requirements, and the transmission system limits. Finally, all of the reservoirs constraints are modelled into the hydraulic subproblem.

This decomposition allows to model HTUC problem in detail. In particular, of outmost importance for large scale hydrothermal systems with high proportion of hydro generation, requires a detail description of the so-called hydro unit production function. In our model, we related the amount of hydropower generation to nonlinear tailrace levels, and also, take into account hydraulic losses, turbine-generator efficiencies, as well, the multiple states related with forbidden operating zones. Our formulation also includes several constraints associated to the reservoirs operation, and it is suitable for a system with large hydro valleys and several plants in cascade.

The resulting nondifferentiable dual problem is solved by a robust Bundle method [5]. As the LRVS method proposed usually fails to find a feasible solution, a Primal Recovery – PR phase is required after the dual solution. In this sense, an Augmented Lagrangian – AL approach is used to search a feasible solution. We take the advantage the same artificial constraints, which are relaxed in the LR phase, but now we add a quadratic term in the objective function of each subproblem to force the primal feasibility. In order to maintain the decomposition of the dual function we apply the Auxiliary Problem Principle [6] in such a way that the local subproblems have a same structure as in the LR phase. It is important remember that the dual function in PR phase is differentiable, so a gradient-like method is applied to maximize it, starting from a dual solution obtained in the LR phase.

We assess the performance of our approach in terms of the accuracy of the obtained solution. We also analyze the quality of the pseudo-optimal primal point – corresponding to the dual solution – in relation to its feasibility. We used a real-life hydroelectric configurat-
2 PROBLEM FORMULATION

In this work, the objective function for the HTUC problem has the expression:

$$\min F = \sum_{i=1}^{I} \sum_{r} c_{i}(p_{i})$$  \hspace{1cm} (1)

The planning horizon is composed by \( T \) time stages (hours). The thermal mix has \( I \) plants, where \( c_{i}(.) \) represents the operating cost of the \( i \)-th thermal plant at time stage \( t \), and includes fixed costs as well as fuel costs related to start-up and nominal operation of units; see [7] for more details.

To help us with the exposition we formulate the HTUC constraints by splitting them into three different subsets, \( C_{ii} \), \( C_{iH} \) and \( C_{iHT} \), corresponding to the respective variables involved in each subset namely, hydraulic, thermal, or both.

2.1 Hydraulic Constraints (\( C_{ii} \))

$$v_{r,t+1} + Q_{r,t} + s_{n} - \sum_{w \in \mathcal{W}(r,t)} \left( Q_{w,t} - s_{w} - t_{w} \right) - v_{r,t} = y_{r,t} \hspace{1cm} (2)$$

Constraints (2) represent the stream-flow balance equations. We use the index \( t \) for time stage and index \( r \) for a specific reservoir; \( v \) is the reservoir storage, \( y \) is the incremental inflow, \( \mathcal{W}(r,t) \) is a set gathering all reservoirs upstream the \( r \)-th, and \( t_{w} \) is the water travel time between reservoirs \( w \) and \( r \).

$$v_{r,t}^\text{max} \leq v_{r,t} \leq v_{r,t}^\text{min}, \hspace{1cm} 0 \leq s_{n} \leq s_{r,t}^\text{max} \hspace{1cm} (3)$$

Constraints (3) include the maximum \( v_{r,t}^\text{max} \) and minimum \( v_{r,t}^\text{min} \) storage, the storage of reservoir \( r \) at the end of the study horizon (imposed by the long-term model), \( v_{r,t}^\text{LFL} \), and maximum spillage \( s_{r,t}^\text{max} \) per reservoir.

$$Q_{r,t} - \sum_{j=1}^{J(r)} q_{r,t} = 0 \hspace{1cm} (4)$$

Constraints (4) represent the penstock water balance in each reservoir, where \( J(r) \) is the number of units in reservoir \( r \). \( Q_{r,t} \) is plant turbined outflow in reservoir \( r \) during stage \( t \) and \( q_{r,t} \) is the turbined outflow of hydro unit \( j \) of reservoir \( r \) during stage \( t \).

$$\sum_{j=1}^{J(r)} p_{j,t}^\text{min} \leq p_{j,t} \leq p_{j,t}^\text{max}, \hspace{1cm} (5)$$

Constraints (5) are related with power limits for each non-forbidden zone of the unit. \( \Phi_{j} \) denotes the total number of non-forbidden zones of the \( j \)-th unit in reservoir \( r \). \( k \) is the corresponding index, and \( p_{j,t}^\text{min,max} \) stand for the minimum and maximum power limits. The binary variable \( z_{j,t} \) is 1 if the \( j \)-th unit in reservoir \( r \) is operating in the \( k \)-th non-forbidden zone at time stage \( t \), and it is set to 0 otherwise.

In (5), hydro production function, \( p_{j,t}(\cdot) \), is represented by a high order nonconvex polynomial, allowing modeling all relevant factors that affect the output. This function takes account hydraulic losses, nonlinear tailrace levels, nonlinear turbine-generator efficiencies, and forbidden operation zones simultaneously; thus, we can obtain an accurate solution for the problem, which is significant in systems with high participation of hydropower generation. Details about the hydro production function used in this work can be found in [8].

$$\left( \sum_{j=1}^{J(r)} p_{j,t}^\text{max} - PH_{r,t} \right) \geq rh_{r,t} \hspace{1cm} (6)$$

Constraints (6) refer to reserve hydro requirement of reservoir \( r \) at stage \( t \), \( rh_{r,t} \), and the reservoir power balance. \( PH_{r,t} \) is the hydro plant generation.

$$z_{j,t} \in [0,1], \sum_{r=1}^{R} z_{j,t} \leq 1. \hspace{1cm} (7)$$

Equations (7) represent the integrality constraints.

In the sequel, to alleviate the notation, we write constraints \( C_{ii} \) above in the abstract form \( C_{ii} = C_{ii}(Q,s,V) \cap C_{HUC}(z,q,S,PH) \). The vectors \( z \), \( q \), \( S \), \( PH \) and \( V \) gather the respective variables. The set \( C_{HH} \) represents constraints given by (2)-(3), modeling the reservoirs, while \( C_{HUC} \) represents the unit constraints, i.e., (4)-(7).

2.2 Thermal Constraints (\( C_{iH} \))

$$p_{i,t}^\text{min,min} \leq p_{i,t} \leq p_{i,t}^\text{min,max} \hspace{1cm} (8)$$

Constraints (8) represent the power limit for each unit. Here \( p_{i,t}^\text{min,max} \) stand for the minimum and maximum power limits of unit \( i \). The binary variable \( u_{i} \) is 1 if the unit is operating at time stage \( t \) and 0 otherwise.

$$p_{i,t} \leq u_{i}(p_{i,t}^\text{min} - rt_{i}) \hspace{1cm} (9)$$

Constraints (9) refer to the reserve requirements, where \( rt_{i} \) is the reserve of unit \( i \) at time stage \( t \).

$$u_{i} = \begin{cases} 1 & \text{if } 1 \leq x_{u} < t_{i}^u, \\ 0 & \text{if } -1 \geq x_{u} > -t_{i}^\text{down}, \\ 0 \text{ or 1 otherwise} & \end{cases} \hspace{1cm} (10)$$

$$x_{u} = \begin{cases} \max(x_{u+1,0},0) + 1, & \text{if } u_{i} = 1, \\ \min(x_{u,0}, -1), & \text{if } u_{i} = 0. \end{cases} \hspace{1cm} (11)$$

Equations (10) are the minimum up-time, \( t_{i}^u \), and downtime, \( t_{i}^\text{down} \), for each unit; the state variable \( x_{u} \) is equal to the number of time steps that the unit has been up/down until stage \( t \).

$$\delta_{i}(u_{i,t-1}, x_{u}) \leq p_{i,t} - p_{i,t-1} - \Delta_{i}(u_{i,t-1}, x_{u}) \hspace{1cm} (11)$$

Equations (11) represent the ramp constraints, where \( \delta(.) \) and \( \Delta(.) \) are the maximum allowed variations of
generation of the unit between two subsequent time stages.

In a abstract formulation, constraints in the set \( C_T \) correspond to \( C_u(u, pt) \), where \( u \) and \( pt \) are vectors gathering all binary and continuous thermal variables, respectively.

2.3 Hydrothermal Constraints \((C_{HT})\)

\[
\sum_{i} p_{ti} + \sum_{r} P_{hi} + \sum_{l} (I_{tal} - I_{tal}) = D_{al}
\]  

This set represents the satisfaction of the demand, per time stage and subsystem. The interconnected hydrothermal system is divided into subsystems, indexed by \( e \). Accordingly, all thermal units (reservoirs) of subsystem \( e \) are gathered in the index set \( I_e(R_e) \). There are \( \Omega_e \) subsystems interconnected with subsystem \( e \); the exchange of energy at stage \( t \), is denoted by \( I_{tal} \) (\( I_{tal} \) respectively) when it goes from subsystem \( l \) to \( e \) (from \( e \) to \( l \), respectively). Finally, \( D_{al} \) is the demand of subsystem \( e \) and at stage \( t \): \( 0 \leq I_{tal} \leq I_{tal}^{\max} \), \( 0 \leq I_{tal} \leq I_{tal}^{\max} \).

Constraints (13) model the subsystems exchange limits, from \( e(l) \) to \( e(e) \), at time \( t \), \( I_{tal}^{\max} \), \( I_{tal}^{\max} \).

In our abstract notation, the set \( C_{HT} \) is written as \( C_{HT} \) \((pt,PH,Int)\), where the vector \( Int \) gathers the subsystem exchanges. We now address the solution strategy adopted in this work.

3 SOLUTION STRATEGY

3.1 Phase 1: Decomposition by Lagrangian Relaxation

Considering the abstract notation, the HTUC problem \((1)-(13)\) becomes:

\[
\text{minimize } c(pt)\text{ s.t.: } C_u(u, pt) \cap C_{HT}(pt, PH, Int) \cap C_{HH}(Q, s, V) \]  

\[
C_{HH}(z, q, s, PH)
\]

To achieve the decomposition, we introduce artificial variables \( pta \) and \( PHa \), which duplicate, respectively, \( pt \) and \( PH \). Variables \( pta \) and \( PHa \) are used in constraints \( C_{HH} \) to replace \( pt \) and \( PH \). In addition, artificial variables \( Qa \) and \( sa \) duplicate \( Q \) and \( s \), respectively. \( Qa \) and \( sa \) replace \( Q \) and \( s \) in \( C_{HH} \). With these additional variables, \((14)\) is rewritten as follows:

\[
\text{minimize } c(pt)\text{ s.t.: } C_u(u, pt) \cap C_{HT}(pta, PHa, Int) \cap C_{HH}(Qa, sa, V) \cap C_{HH}(z, q, s, PH) \]  

\[
pt = pta, \quad PH = PHa, \quad Q = Qa, \quad s = sa.
\]

At this point, the artificial constraints that still keep the coupling the problem are relaxed by introducing the associated Lagrange multipliers \( \lambda_{pt}, \lambda_{PH}, \lambda_{Qa}, \lambda_{sa} \).

Next, we relax the artificial constraints shown in \((15)\) constructing the corresponding dual problem \(\Theta()\):

\[
\text{max } \Theta(\lambda_{pt}, \lambda_{PH}, \lambda_{Qa}, \lambda_{sa})
\]

where,

\[
\Theta() = \min_{u, pta, s, Qa, sa, PH, PHa, Int} c(pt) + \langle \lambda_{pt}^T, pt - pta \rangle + \langle \lambda_{PH}^T, PH - PHa \rangle
\]

\[+ \langle \lambda_{Qa}^T, Q - Qa \rangle + \langle \lambda_{sa}^T, s - sa \rangle \]

\[
s.t.: C_u(u, pt) \cap C_{HT}(pta, PHa, Int) \cap C_{HH}(Qa, sa, V) \cap C_{HH}(z, q, s, PH)
\]

where \( \langle , \rangle \) denotes the dot product. For each Lagrange multiplier vector, the minimization problem needed to assess \(\Theta()\) can be decomposed into four subproblems, as follows:

\[
\text{max } \Theta_i(\lambda_{pt}, \lambda_{PH}, \lambda_{Qa}, \lambda_{sa}) = \Theta_i(\lambda_{pt}, \lambda_{PH}) + \Theta_i(\lambda_{Qa}, \lambda_{sa})
\]

\[
\text{where,}
\]

\[
\Theta_i(\lambda_{pt}, \lambda_{PH}) = \min_{u, pta, PH, Int} c(pt) + \langle \lambda_{pt}^T, pt \rangle
\]

\[
\text{s.t.: } C_u(u, pt) \cap C_{HT}(pta, PHa, Int) \cap C_{HH}(Qa, sa, V) \cap C_{HH}(z, q, s, PH)
\]

Subproblem \((19)\) is a nonlinear mixed-integer optimization problem, coupled along time stages, but not along plants. This subproblem corresponds to the thermal unit commitment, and it is solved by a classic Dynamic Programming method. Subproblem \((20)\) is a standard Linear Programming - LP problem, coupled along plants, but not along times stages, which can be solved by any LP commercial solver. Subproblem \((21)\) is also an LP problem, coupled both in time stages and space via the stream-flow constraints given by \((2)\). Even though \((21)\) can be a large-scale problem but an LP solver can still solve it efficiently. Finally, subproblem \((22)\) is a nonlinear mixed-integer optimization problem, uncoupled both in time and hydro plants variables. This subproblem corresponds to the commitment of hydro units, for a given reservoir and time stage.

Each sub-subproblem in \((22)\), for each time stage and for a given power plant, is a mixed-integer nonlinear programming problem, with binary variables corresponding to different operating modes in the plant. The total number of possible operating modes is given by the product of all combinations of the operating modes of all the units composing the plant. Each combination of a unit is a configuration where the corresponding binary variables are fixed to one of the feasible values. Once the binary values are fixed, the problem becomes a nonlinear program, whose size is dependent on \(J(R)\). In this work we solve \((22)\) by enumeration of operating modes at plant \( r \) and stage \( t \). An alternative strategy to avoid the enumeration is described in \([9]\).
Still regarding (22), with the objective of solving the nonlinear programming problems in an efficient manner, this work makes use of a Sequential Quadratic Programming – SQP algorithm. A version of the SQP algorithm called Quasi-Newton BFGS is developed. Additional details of this SQP algorithm can be seen in [8,9].

3.2 Phase 2: Primal Recovery by Augmented Lagrangian

To recover primal solutions we use an inexact Augmented Lagrangian - AL technique, similar to the ones in [10,11]. We now explain the decomposition scheme chosen. Starting from (15), the same artificial constraints are relaxed, but now we add a quadratic term in the objective function to create the AL function. In this sense, the new dual function is show below.

$$\Phi(\lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T) = \min_{\lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T} c(pt) + \langle \lambda^T, pt - pta > + \langle \lambda^T, PH - PHa > + \langle \lambda^T, Q - Qa > + \langle \lambda^T, s - sa > + \frac{c}{2} \|pt - pta\|^2 + \frac{c}{2} \|PH - PHa\|^2 + \frac{c}{2} \|Q - Qa\|^2 + \frac{c}{2} \|sa - s\|^2$$

(23)

where $\|\cdot\|$ denotes the Euclidian norm in $\mathbb{R}^n$ and $c$ is a positive parameter which increases along iterations in order to force satisfaction of the relaxed constraints. Due to the introduction of the quadratic term, AL yield differentiable dual functions. Furthermore, a dual solution gives a primal feasible point.

Note, however, that the same quadratic term yielding differentiability prevents the dual problem from being decomposable. By applying the Auxiliary Problem Principle [6], the following approximations are made:

$$\frac{c}{2} \|pt - pta\|^2 \approx \frac{c}{2} \|pt - pta + pt^k + pt^k - pta\|^2$$

$$\frac{c}{2} \|PH - PHa\|^2 \approx \frac{c}{2} \|PH - PHk^i + PHk^i - PHa\|^2$$

$$\frac{c}{2} \|Q - Qa\|^2 \approx \frac{c}{2} \|Q - Qa + Qa^k + Qa^k - Qa\|^2$$

$$\frac{c}{2} \|sa - s\|^2 \approx \frac{c}{2} \|sa - s + s^i + s^i - s\|^2$$

(24)

where all variables with index $\xi$ correspond to the values obtained for these variables at the previous iteration.

Problem (23) can be rewritten then as the sum of four local subproblems, as follows:

$$\max_{\xi, \xi, \xi, \xi, \xi, \xi, \xi, \xi} \Phi(\lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T, \lambda^T)$$

$$\Phi() := \phi(\lambda^T, c) + \phi_{int}(\lambda^T, \lambda^T, \lambda^T, c) + \phi_{int}(\lambda^T, \lambda^T, \lambda^T, c) + \phi_{int}(\lambda^T, \lambda^T, \lambda^T, c)$$

(25)

where,

$$\phi_{int}(\lambda^T, c) = \min_{\xi, \xi, \xi, \xi, \xi, \xi} - \frac{c}{2} \|pt - pta + pt^k + pt^k - pta\|^2 + \frac{c}{2} \|PH - PHk^i + PHk^i - PHa\|^2 + \frac{c}{2} \|Q - Qa + Qa^k + Qa^k - Qa\|^2 + \frac{c}{2} \|sa - s + s^i + s^i - s\|^2$$

(26)

Note that the local subproblems (26)-(29) have the same structure as in the LR phase, with the important difference that the LPs (20) and (21) are transformed into Quadratic Programming problems (27) and (28). The nature of subproblems (15) and (22) is not altered by the quadratic term inclusion; therefore, the same strategy solution of RL phase is used.

Since the approximation of the quadratic term is valid only locally, it is fundamental to have a good starting point available. The dual function is differentiable, so a gradient-like method is applied to maximize it, starting from the solution found by the LR phase. The procedure increases $c$ along iterations in a rate which best balances search for feasibility and search for optimality. Additional details will be presented in the Section 4, in the sequence.

4 NUMERICAL RESULTS

We assess the solution strategy scheme on a real-life hydrothermal configuration extracted from the Brazilian power system. More precisely, we consider a system with 121 hydro and 12 thermal units whose maximum installed capacity is 38,227,0 MW.

Five cascades make up the hydro system; the largest one possesses seven plants with 58 units and the smallest one is composed by two plants with 12 units. The two largest plants have 20 units, whereas the smallest one has only two. One plant possesses two groups of
units with different operating characteristics. One plant presents three identical units with two forbidden zones. All the remaining plants of the hydro system have identical units with only one forbidden zone. Detailed data for the hydro system constraints are too lengthy to list in this paper; for more detail see [9]. We show now in Table 1 and 2 the thermal data.

<table>
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<tr>
<th>Plant</th>
<th>Fuel</th>
<th>$a_1$ (S/MW)</th>
<th>$a_2$ (S/MW²)</th>
<th>Fixed (S)</th>
<th>Cold start-up (S)</th>
<th>Cooling constant (h)</th>
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<td>NUCLEAR</td>
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<td>0.0002</td>
<td>45,000</td>
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<tr>
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<td>0.0002</td>
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<td>21,000</td>
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<td>101.6</td>
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Table 1: Thermal operation costs.

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<th>Plant</th>
<th>$p^{max}$ (MW)</th>
<th>$p^{max}$ (MW)</th>
<th>$c^0$ (h)</th>
<th>$c^max$ (h)</th>
<th>$S$ (MW/h)</th>
<th>$A$ (MW/h)</th>
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</table>

Table 2: Thermal constraints.

The two days planning horizon is discretized hourly, yielding $T=48$. Initial reservoir volumes were taken at 50% of usable volumes, whilst the incremental inflows were considered null. The storage of the reservoirs at the end of the horizon are adjusted to prevent that if it uses more of the one than 5% of the initial useful volumes. Nevertheless, in three reservoirs these values are set at approximately 10%. The interconnected hydrothermal system is divided into four subsystems.

The numerical experiments are organized in two parts. Firstly, we present the LR phase convergence and respective dual solution. In the LR optimization we used the subroutine N1CV2 Code [12]. Secondly, the Primal Recovery – PR, the computational time and the feasible solution are analyzed.

In the PR phase, the initial value of parameter $c^0$ is equal to $1 \times 10^{-8}$ and it is increased along the iterations $\xi$ as follow:

$$c^{0+1} = \begin{cases} c^0 \times 1.5 & \text{if } c^0 < 1 \times 10^{-4} \\ c^0 + 2 \times 10^{-6} c^0 & \text{if } c^0 \geq 1 \times 10^{-4} \end{cases}$$

Regarding the multipliers of (26)-(29), in this paper, they are updated proportionally to the mismatch of each artificial constraint, $g(x^\xi)^2$:

$$\lambda_{i}^{\xi+1} = \lambda_{i}^{\xi} + 0.9 \frac{g(x^\xi)^2}{\|g(x^\xi)^2\|}$$

The PR algorithm is stopped with a relative tolerance between the originals and duplicated variables less than 1%.

4.1 Optimization by LR

The initial Lagrange multipliers are $\lambda_{PT} = \lambda_{PH} = \lambda_{Q} = \lambda_{S} = 1.0$. We found an optimal value of $116,178.41$, after 699 iterations which took 100 minutes of CPU times in an AMD Athlon 64 3000+ Processor, 1.0 GB of RAM memory. In the Figure 1 we show, at each iteration, the value of the subgradient norm and the best of value of the dual function. Although the dual function is almost at its optimal value by iteration 200, the subgradient norm is still dropping in further iterations.

Figure 1: LR phase convergence.

In order to assess the solution quality obtained by the LR optimization, in terms of primal feasibility, we show in Figure 2 the difference between the demand and the sum of all generations for each stage in the horizon planning.

Note that the deviations are not so small. For this reason, this dispatch is unrealistic and it is necessary to realize a primal recovery (described in Section 4.2). These differences cannot be simply adjusted in real time operation. Great part of this infeasibility is related to the oscillatory aspect inherent to the LR technique when applied to a linear formulation, such as subproblems (20) and (21).
We found an optimal value of $3,223,963.0$, after 234 iterations that took 66 minutes of CPU times. In the last dual iteration (Fig. 1), the subgradient norm is equal a $6.31 \times 10^5$; on the other hand, in the last iteration of RP phase this value decrease to 4.8. Figure 5 shows the difference between the demand and the sum of all generations for each hour in the horizon planning. Note that it is available a high precision feasible solution.

Figure 2: Infeasibility in the demand attainment.

Figure 3 shows the differences ($PH_a - PH$) along the horizon between subproblems (21) and (22) for an important hydro plant of Brazilian system.

Figure 3: Primal infeasibility – Hydro plant generation.

It can be seen that $PH_a$ takes mostly two values: 0 and 1,148.0 MW. This bang-bang behavior is explained by the linear nature of Subproblem (20).

4.2 Primal Recovery

The focus now is on the PR phase results. Figure 4 shows the performance of the PR approach proposed to obtain a feasible solution, where are evidenced the evolution of the augmented dual objective function and, also, the norm of the infeasibility vector.

Figure 4: Primal recovery convergence.

We found an optimal value of $3,223,963.0$, after 234 iterations that took 66 minutes of CPU times. In the last dual iteration (Fig. 1), the subgradient norm is equal a $6.31 \times 10^5$; on the other hand, in the last iteration of RP phase this value decrease to 4.8. Figure 5 shows the difference between the demand and the sum of all generations for each hour in the horizon planning. Note that it is available a high precision feasible solution.

Figure 5: Feasible solution.

At this point, an interesting analysis consists of evaluating the differences between the optimal Lagrange multipliers calculated for LR and PR phases. In this sense, we choose to show the Lagrange multipliers associated with the ($pta$) artificial constraints. For simplification, we show the values associated with thermal unit 08.

Figure 6: Lagrange multipliers – LR and PR phases.

Note that the multipliers have larger magnitudes in the RP phase. This explains the thermal generation differences shown in the Figures 2 and 5.

4.3 Sensitivity Analysis

A new demand scenario (named Case II) was performed to realize a sensitivity analysis of the methodology. In this sense, the system demand was reduced in 5%. This new configuration can represent either a typical weekend load profile or a high value of incremental inflows to the reservoirs (wet period).

The Table 3 summarizes the main aspects regarding the computational performance of the cases. In this
table, the values of iterations and computation burden are obtained by the two phases of the algorithm.

<table>
<thead>
<tr>
<th>Case</th>
<th>Gradient Norm</th>
<th>Iterations</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.55</td>
<td>1035</td>
<td>209</td>
</tr>
<tr>
<td>II</td>
<td>4.98</td>
<td>470</td>
<td>144</td>
</tr>
</tbody>
</table>

Table 3: Algorithm performance - 1% tolerance relative.

The cases I and II, above, were performed considering a high precision degree (relative tolerance between the originals and duplicated variables is less or equal to 1%). Table 4 shows the algorithm performance in the PR phase considering that this tolerance value is increased to 5%.

<table>
<thead>
<tr>
<th>Case</th>
<th>Gradient Norm</th>
<th>Iterations</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>40.94</td>
<td>815</td>
<td>136</td>
</tr>
<tr>
<td>II</td>
<td>32.75</td>
<td>239</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 4: Algorithm performance – 5% tolerance relative.

The relative tolerance variation in the PR phase caused an approximate reduction of 40% in the computational burden. It is important to mention that the gradient norm values shown in the Table 4 represent 0.25% of the system average demand.

5 CONCLUSIONS

The HTUC formulation and solution with simultaneous dual optimization and primal recovery are presented in this paper. Lagrangian Relaxation is used to decompose the problem into four subproblems. The dual maximization step is done using a bundle method. As a result of nonconvex primal problem, the dual solution is not feasible. To recover primal solution we applied the Auxiliary Problem Principle, which is an inexact form of Augmented Lagrangian technique. The numerical results demonstrated that on the solution of the linear subproblems, the variables have as resulted its minimum or maximum values. On the primal recovery, the introduction of a quadratic parameter eliminated this bang-bang. Also, the duplicated variable constraints are satisfied and a feasible solution is obtained. These preliminary results show promising results for the application of this methodology for bigger problems like the whole Brazilian power systems.

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