Long term nodal pricing and transmission costing

Nicolas Omont, Arnaud Renaud
Artelys
Paris, France
http://www.artelys.com

Patrick Sandrin
Réseau de Transport d'Electricité
Paris, France
http://www.rte-france.com

Abstract - This article questions costing principles through their links with long term nodal pricing and discusses remaining steps to the implementation of the associated model. The long term nodal pricing model differs from the standard nodal pricing model in that it considers transmission capacities as decision variables.

We show that a tariff based on long term nodal prices on an optimal network is equivalent to a tariff based on the compensation of each network element development costs proportionally to their usage by network transactions (balanced production–demand sets), i.e. based on the proportional costing principle.

In a multi–situation framework, we show that development costs of each line are mostly allocated to situations in which the line is saturated. However, we show that cost allocation between saturating situations is complex as it entangles spatial and temporal effects. Intrinsically linked with the fact that the network is dimensioned to face contingencies, the analysis of multiple saturating situation cost allocation is the key remaining step to the model implementation.

Keywords - Long Term Nodal Pricing, Marginal Pricing, Proportional Transmission Costing, Multi–period Transmission Cost Allocation.

1 Introduction

The revenues of Transmission System Operators (TSOs) are usually based on two paradigms. On the one hand, market mechanisms lead to fix prices for the usage of network elements. On the other hand, tariffs are built from cost evaluations. They are adjusted so as to ensure economic sustainability. Each TSO uses each paradigm to a different extent, based on their respective advantages and drawbacks. Few links have been established between them.

Many market structures rely on the theory of nodal pricing [1]. This theory asserts that the maximization of the social welfare resulting from the network usage is reached by choosing a market structure that leads to fix energy prices at each network node (nodal prices) at the value of specific dual variables of an optimal power flow problem ([2], See [3] for an introduction). This advocates the fact that such nodal prices would provide the right incentives to each market actor. However, in its usual formulation in which transmission capacities are parameters, it computes short term prices which do not allow TSOs to recover their expenses [4]. Therefore, it has to be completed by other mechanisms like access or "club membership"[5].

Conversely, costing principles used to establish tariffs are numerous (See [6] for review) partly because arguments in favor of one or another often lack of theoretical grounds. In particular, two questions are regularly debated: (a) Transaction based power flow methods face the problem of counter flow pricing. Should flows whose direction is opposite to the line net flow get credit for lightening the flow, pay according to their absolute value, or pay 0? (b) How should periodic (annual) costs be allocated to each time slot? Should they be allocated to the peak one, to each of them (if yes, with which weights?), or based on each line peak state?

In this article, we introduce a new framework in order to compute long term nodal prices. Its properties allow to show links with specific costing principles, theoretically supporting them while pointing out the non–optimality of others. As developed below, the model differs from the short–term nodal price framework on two points: (a) Network expansion; (b) Simultaneous optimization over several situations. It induces a peak load pricing [7] in a model with fixed production and variable transmission capacities.

Firstly, considering network expansion is intrinsically linked with long term nodal prices and full cost recovery. Indeed, the model minimizes development costs, which includes both investment and operation costs. These development costs would be the prices paid by a TSO if it rented the assets to third party owners. On the one hand, these costs would reflect the expected asset usage during its full economical life, i.e. they would be long term costs. On the other hand, at the beginning of an optimization period, capacity renting would effectively be a decision variable in hands of the TSO. We show that a nodal pricing based on such an optimization scheme leads to the full recovery of development costs. This reconciliation property induced by long term transmission expansion planning has already been noted by [8]. However, given the choice of a continuous model to the expense of realism, interpretation of dual variables as nodal prices is possible. Thanks to this property, we show that this nodal pricing is equivalent to compensation of each network element development cost proportionally to its usage by transactions occurring on the network. This founds costing principles based on: (a) Transactions; (b) Cost allocation proportional to network usage; (c) Credit to counter flows. To the best of our knowledge, this result is original.

Secondly, several situations need to be considered because networks are dimensioned to face operating conditions resulting from: (a) Normal variations of demand and production, both in quantity and location; (b) Contingencies [9]. In the standard framework, optimization over several normal situations is equivalent to optimization over each situation separately because all variables are specific to each situation. However, in our model, as line capacities are chosen once for all situations, separate optimization on each of them is impossible. Besides, multi–situation long term nodal pricing leads to allocate development costs to each situation. As a primary result, if only normal situations are considered, we show the usual result that most of each element development costs are allocated to the situations which saturate it. It backs cost allocation to the peak situation of each line. However, as suggested in [4], taking into account contingencies will probably result in less concentrated allocations. Indeed, the cost allocated to contingency situations derived from a normal situation should be reported on it. Consequently, even if a line is not saturated in a normal situation, it might be saturated in some associated contingency situations, leading to a significant cost allocation to the normal situation.

The paper is organized as follows. After the model presentation in Section 2, we will analyze optimality conditions, focusing on consequences on conditions of transmission capacity development in Section 3. In Section 4, the equivalence between nodal pricing and proportional costing is developed. In Section 5, the first steps towards establishing a sound situation cost allocation are presented through a three node three situation example.

2 Model

2.1 System Representation

The graph underlying the power system network is modeled as a set of nodes $n \in \mathcal{N}$ and a set of edges $e \in \mathcal{E}$.

Edges represent either physical lines or abstraction of them. They are arbitrarily oriented. An edge belongs to the downstream edge set $\mathcal{E}_n^-$ of its upstream node n_e^+ and to the upstream edge set $\mathcal{E}_n^+$ of its downstream node n_e^-:

$$\left\{ \begin{array}{l} e \in \mathcal{E}_n^- \Leftrightarrow n_e^+ = n \\ e \in \mathcal{E}_n^+ \Leftrightarrow n_e^- = n \end{array} \right. \tag{1}$$

For each edge e, the cost of developing one or several lines of total capacity X_e is $K_e(X_e) \stackrel{\text{def}}{=} \mathrm{k}_e X_e$, i.e. $\partial K_e(X_e)/\partial X_e = \mathrm{k}_e$. To simplify equations, the load level t_e of a line of capacity X_e transited by a flow Z_e is defined such that:

$$t_e X_e \stackrel{\text{def}}{=} Z_e \tag{2}$$

The availability level $f_e \in [0, 1]$ of the line represents the upper limit of the load level ($|t_e| < f_e$). It can be strictly below 1 due to technical constraints, or even 0 if the line is out of order.

The loss coefficient α_e is defined as the proportion of power lost if the line is saturated ($|t_e| = 1$). The existence of this loss coefficient independent of the capacity requires that the resistance is inversely proportional to it. Assuming a fixed phase shift, Pouillet's law asserts that the resistance is inversely proportional to cable section, while the capacity is proportional to it. These two relationships allow to define this independent loss coefficient.

A set of producers $g \in \mathcal{G}_n$ and a set of consumers $d \in \mathcal{D}_n$ are attached to each node n. For each producer, the cost $\mathrm{c}_g^g(G_g)$ of producing a given quantity of energy G_g is defined as a convex function without fixed costs ($\mathrm{c}_g^g(0) = 0$). The production is limited by Y_g. Equivalently, the demand D_d and the cost $\mathrm{c}_d^u(U_d)$ of unserved demand U_d are given for each consumer. The set of producers and consumers are respectively noted $\mathcal{G} \stackrel{\text{def}}{=} \bigcup_n \mathcal{G}_n$ and $\mathcal{D} \stackrel{\text{def}}{=} \bigcup_n \mathcal{D}_n$. Finally, the net injection at each node is defined as:

$$I_n \stackrel{\text{def}}{=} \sum_{g \in \mathcal{G}_n} G_g - \sum_{d \in \mathcal{D}_n} (D_d - U_d) \tag{3}$$

2.2 Objective function

Given demands D_d, the objective function is the sum of production and transmission costs and of the cost of unserved energy:

$$\sum_{e \in \mathcal{E}} \mathrm{k}_e X_e + \sum_{d \in \mathcal{D}} \mathrm{c}_d^u(U_d) + \sum_{g \in \mathcal{G}} \mathrm{c}_g^g(G_g) \tag{4}$$

The minimization of the criterium with respect to X_e, t_e, U_d and G_g is submitted to the following constraints, defined along with their corresponding dual variables (Greek letters are used to denote them).

Energy conservation: Given the assumption on losses, they are equal to $\alpha_e t_e^2 X_e$. Assuming losses are split between each end node, the energy balance that should be satisfied at each node n is:

$$\begin{array}{rl} & \sum_{e \in \mathcal{E}_n^-} X_e t_e \left(1 + \frac{\alpha_e}{2} t_e\right) \\ - & \sum_{e \in \mathcal{E}_n^+} X_e t_e \left(1 - \frac{\alpha_e}{2} t_e\right) - I_n \end{array} = 0 \longleftarrow \lambda_n \tag{5}$$

The voltage law is not taken into account due to the non-convexity implied by its coupling with capacity development [10]. The model is therefore a flow model as in [11], except that it includes losses.

Load level: The available capacity of each line should not be exceeded.

$${t_e}^2 \leq f_e^2 \qquad \longleftarrow \theta_e \tag{6}$$

Capacity: The capacity of each line is positive.

$$X_e \geq 0 \qquad \longleftarrow \xi_e \tag{7}$$

Unserved demand: The unserved demand of each consumer is positive and limited by its demand.

$$0 \leq U_d \leq D_d \qquad \longleftarrow \underline{\delta}_d, \overline{\delta}_d \tag{8}$$

Generation: The generation of each producer is positive and limited by its capacity.

$$0 \leq G_g \leq Y_g \qquad \longleftarrow \underline{\gamma}_g, \overline{\gamma}_g \tag{9}$$

Overall, the Lagrangian of the system is:

$$\mathscr{L}\left(X_e, t_e, U_d, G_g, \lambda_n, \theta_e, \xi_e, \underline{\gamma}_g, \overline{\gamma}_g, \overline{\delta}_d, \underline{\delta}_d\right) = \tag{10}$$

$$\sum_{e\in\mathcal{E}} \mathrm{k}_e X_e + \sum_{d\in\mathcal{D}} \mathrm{c}_d^u(U_d) + \sum_{g\in\mathcal{G}} \mathrm{c}_g^g(G_g)$$

$$+ \sum_{n\in\mathcal{N}} \lambda_n \left(\begin{array}{l} \sum_{e\in\mathcal{E}_n^-} X_e t_e \left(1+\frac{\alpha_e}{2}t_e\right) \\ - \sum_{e\in\mathcal{E}_n^+} X_e t_e \left(1-\frac{\alpha_e}{2}t_e\right) - I_n \end{array} \right)$$

$$+ \sum_{e\in\mathcal{E}} \theta_e \left({t_e}^2 - f_e^2\right) - \sum_{e\in\mathcal{E}} \xi_e X_e$$

$$+ \sum_{d\in\mathcal{D}} \overline{\delta}_d (U_d - D_d) - \sum_{d\in\mathcal{D}} \underline{\delta}_d U_d$$

$$+ \sum_{g\in\mathcal{G}} \overline{\gamma}_g (G_g - Y_g) - \sum_{g\in\mathcal{G}} \underline{\gamma}_g G_g$$

2.3 Multi–situation extension

Networks are dimensioned to face different operating conditions, resulting from hourly/seasonal demand variations and contingencies. The following extension of the previous model allows to take into account both types of situations: Demands D_d^s, unserved demand costs $\mathrm{c}_d^{u,s}(U_d^s)$, production limits Y_g^s, production costs $\mathrm{c}_g^{g,s}(G_g^s)$, and availability levels f_e^s are defined for each situation $s\in\mathcal{S}$. Consequently, load levels t_e^s, productions G_g^s and unserved demand U_d^s are also indexed by the situations, as are associated constraints and dual variables. On the contrary, development costs k_e, developed capacities X_e, and positive capacity constraint dual variables ξ_e are the same for all situations. Each situation is assigned a probability p^s representing its expected duration during the optimization period. The modified objective has the following shape :

$$\sum_{e\in\mathcal{E}} \mathrm{k}_e X_e + \sum_{s\in\mathcal{S}} p^s \left(\sum_{d\in\mathcal{D}} \mathrm{c}_d^{u,s}(U_d^s) + \sum_{g\in\mathcal{G}} \mathrm{c}_g^{g,s}(G_g^s) \right) \tag{11}$$

3 Optimality condition analysis

3.1 Mono–situation optimality analysis

Let us introduce the following notations:

$$\left\{ \begin{array}{rcl} \Delta\lambda_e &=& \lambda_{n_e^-} - \lambda_{n_e^+} \\ \lambda_e &=& (\lambda_{n_e^+} + \lambda_{n_e^-})/2 \end{array} \right. \tag{12}$$

Given Equations (1) and (10), the associated first–order K.K.T. (Karush–Kuhn–Tucker) necessary optimality conditions are:

$$\frac{\partial\mathscr{L}}{\partial X_e} = 0 = -\Delta\lambda_e t_e + \lambda_e \alpha_e t_e^2 + \mathrm{k}_e - \xi_e \tag{13}$$

$$\frac{\partial\mathscr{L}}{\partial t_e} = 0 = X_e(-\Delta\lambda_e + 2\lambda_e\alpha_e t_e) + 2\theta_e t_e \tag{14}$$

$$\frac{\partial\mathscr{L}}{\partial U_d} = 0 = \frac{\partial \mathrm{c}_d^u}{\partial U_d}(U_d) + \overline{\delta}_d - \underline{\delta}_d - \lambda_n \tag{15}$$

$$\frac{\partial\mathscr{L}}{\partial G_g} = 0 = \frac{\partial \mathrm{c}_g^g}{\partial G_g}(G_g) - \underline{\gamma}_g + \overline{\gamma}_g - \lambda_n \tag{16}$$

with the complementary slackness conditions:

$$\left\{ \begin{array}{lcl} \theta_e\left({t_e}^2 - f_e^2\right) &=& 0 \\ \xi_e X_e &=& 0 \\ \overline{\delta}_d(U_d - D_d) &=& \underline{\delta}_d\, U_d = 0 \\ \overline{\gamma}_g(G_g - Y_g) &=& \underline{\gamma}_g\, G_g = 0 \end{array} \right. \tag{17}$$

and the positivity of θ_e, ξ_e, $\overline{\delta}_d$, $\underline{\delta}_d$, $\overline{\gamma}_g$ and $\underline{\gamma}_g$.

3.1.1 Node analysis

From Equation (16), for an active unsaturated producer ($G_g \in]0, Y_g[$) :

$$\frac{\partial \mathrm{c}_g^g}{\partial G_g}(G_g) = \lambda_n \tag{18}$$

Therefore λ_n is equal to the marginal cost of producing energy at node n. If the λ_n is below the initial marginal production cost ($\lambda_n < \frac{\partial \mathrm{c}_g^g}{\partial G_g}(0)$), then the producer is inactive ($G_g = 0$). If the production reached its maximum ($G_g = Y_g$), λ_n is above this marginal cost ($\lambda_n \geq \frac{\partial \mathrm{c}_g^g}{\partial G_g}(Y_g)$). For a more comprehensive discussion of optimality for producers and consumers, see [3]. It hints at interpreting λ_n as the energy price at node n.

3.1.2 Edge analysis

Equations (13) and (14) allow to discuss whether a capacity X_e is developed on edge e, how it is loaded (t_e) and how its development cost k_e is related to energy price difference $\Delta\lambda_e$ and mean energy price λ_e.

If a capacity is installed ($X_e > 0$), the expression of the relationship between k_e and X_e depends on whether the development cost k_e is higher than the cost of losses at saturation $\lambda_e\alpha_e f_e^2$ or not. If it is, the line is saturated, i.e. its load level is maximum ($|t_e| = f_e$). It may be pointed out that saturations resulting from this modeling are different from short-term congestions since the line capacities are always optimal. The relationship between $\Delta\lambda_e$ and k_e is linear:

$$\left\{ \begin{array}{l} \mathrm{k}_e = |\Delta\lambda_e| f_e - \lambda_e\alpha_e f_e^2 \\ t_e = \mathrm{sgn}(\Delta\lambda_e)\, f_e \end{array} \right. \quad : \mathrm{k}_e \geq \lambda_e\alpha_e f_e^2 \tag{19}$$

If the development cost is strictly lower than the cost of losses, the line is unsaturated ($|t_e| < f_e$). The relationship between $\Delta\lambda_e$ and k_e is quadratic:

$$\left\{ \begin{array}{l} \mathrm{k}_e = \Delta\lambda_e^2/4\lambda_e\alpha_e \\ t_e = \Delta\lambda_e/2\lambda_e\alpha_e \end{array} \right. \quad : \mathrm{k}_e < \lambda_e\alpha_e f_e^2 \tag{20}$$

In both cases, the load level t_e and the energy price difference $\Delta\lambda_e$ are oriented in the same direction, which means that the energy is transmitted from low price nodes to high price nodes:

$$t_e \Delta\lambda_e \geq 0 \tag{21}$$

If $X_e = 0$, the first order conditions are degenerated because Equation (14) is easily satisfied. Summarizing

Equations (19) and (20), let us define the development cost at which a capacity can be developed:

$$K_e(\Delta\lambda_e, \lambda_e) = \quad (22)$$

$$\begin{cases} |\Delta\lambda_e| f_e - \lambda_e \alpha_e f_e^2 & : |\Delta\lambda_e| \geq 2\lambda_e\alpha_e f_e \\ \Delta\lambda_e^2 / 4\lambda_e\alpha_e & : |\Delta\lambda_e| < 2\lambda_e\alpha_e f_e \end{cases} \quad (23)$$

The study of the stability of solution shows that the development cost is higher than this cost :

$$X_e = 0 \Longrightarrow \mathrm{k}_e \geq K_e(\Delta\lambda_e, \lambda_e) \quad (24)$$

Finally, at optimum, the development cost k_e cannot be lower than $K_e(\Delta\lambda_e, \lambda_e)$. The graph of Figure (1) sums up this discussion by representing the relationship in the $(\Delta\lambda_e, \mathrm{k}_e)$ plane.

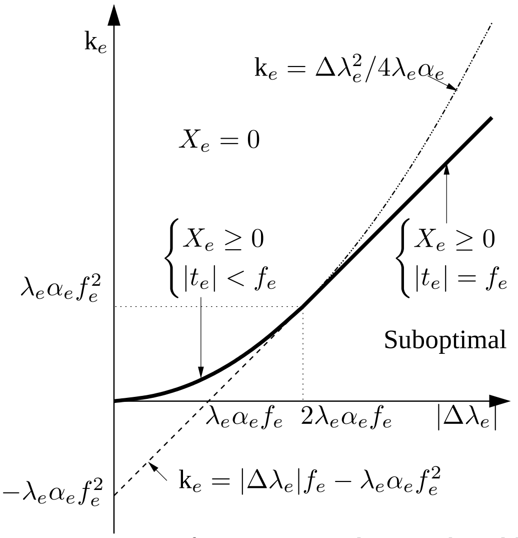


Figure 1: Existence of capacity at optimality given the nodal price difference $|\Delta\lambda_e|$ and the unit capacity development cost k_e. The black thick curve represents the relationship $K(\Delta\lambda_e, \lambda_e)$ existing when capacities are developed ($X_e > 0$) between the development cost k_e, the energy price difference between line extremities $\Delta\lambda_e$, and the mean energy price λ_e. If the development cost k_e is lower than the cost of losses at saturation $\lambda_e\alpha_e f_e^2$, the developed capacities are unsaturated (lower part of the graph, parabolic part of the thick curve), else they are saturated (upper part of the graph, linear part of the curve). Above the thick curve, no capacities are developed because they are too expensive. Below this curve, the situation is suboptimal.

A direct consequence of the previous discussion, particularly of the existence of a development cost $K_e(\Delta\lambda_e, \lambda_e)$ at which a capacity can be developed as a function of dual variables at edge extremities, is the following theorem:

Theorem 1. *Excluding degeneration in parameters, the optimal network is a tree. If parameters are degenerated, there is always a tree in the set of solutions.*

Without losses, the theorem is a classical result of linear programming [12]. Introduction of losses does not change the result because the model defines the resistance as inversely proportional to the capacity.

3.1.3 Price and cost analysis

Classically, the dual variable λ_n is interpreted as the price of energy at node n obtained as a market equilibrium in which transmission of energy is charged at marginal price [1]. It casts a new light on the previous discussion on edge existence: Let us assume that the capacity owner of edge e is able to buy or sell energy at the price $\lambda_{n_e^+}$ and $\lambda_{n_e^-}$ respectively to each end node. It can transmit energy provided it purchases half of its losses at each end node. This paragraph discusses the cost and the revenue associated to a small variation around the optimum of the transmitted energy. It shows how it implies that the total revenue of transmission compensates not only losses but also development costs, even if the line is unsaturated.

First of all, due to Equation (21) and to the arbitrary orientation of edges, we assume without loss of generality that $\Delta\lambda_e \geq 0$ and $t_e \geq 0$. At optimum, the opportunity to transmit a small additional quantity of energy δZ_e is null. Let us assume that the edge is unsaturated. This energy could have been transmitted either by increasing the edge capacity X_e or the load level t_e.

In the first case ($\delta Z_e = t_e \delta X_e$), the opportunity is null because the transmission revenue $\Delta\lambda_e \delta Z_e$ is exactly compensated by two costs: (a) The development cost increase $\mathrm{k}_e \delta X_e$; (b) The cost of losses due to the additional flow $\lambda_e \alpha_e t_e \delta Z_e$. As the load level is constant, the mean cost of losses remains $\lambda_e \alpha_e t_e$.

On the contrary, in the second case ($\delta Z_e = X_e \delta t_e$), the opportunity is null because transmission revenue $\Delta\lambda_e \delta Z_e$ is exactly compensated by two costs: (a) The increase of the cost of losses for the original flow $\lambda_e \alpha_e Z_e \delta t_e$, due to the increase of the load level; (b) The cost of losses due to the additional flow $\lambda_e \alpha_e t_e \delta Z_e$. As already noted in [4], due to the quadratic nature of losses, both costs are coincidentally equal: $\lambda_e \alpha_e Z_e \delta t_e = \lambda_e \alpha_e t_e \delta Z_e$. This discussion is summed up in the two following equations. The first one, derived from Equation (13), is related to an increase of the capacity. The second one, derived from Equation (14), is related to an increase of load level:

$$\Delta\lambda_e \delta Z_e = \Delta\lambda_e t_e \delta X_e = \mathrm{k}_e \delta X_e + \lambda_e \alpha_e t_e \delta Z_e \quad (25)$$

$$\Delta\lambda_e \delta Z_e = \Delta\lambda_e X_e \delta t_e = \lambda_e \alpha_e Z_e \delta t_e + \lambda_e \alpha_e t_e \delta Z_e \quad (26)$$

As development costs are linear ($K_e(X_e) \overset{\text{def}}{=} \mathrm{k}_e X_e$), it is possible to integrate previous equations in order to obtain the financial balance associated to edge e:

$$\begin{cases} \Delta\lambda_e Z_e & = & \mathrm{k}_e X_e + \alpha_e t_e \lambda_e Z_e \\ \Delta\lambda_e Z_e & = & 2\alpha_e t_e \lambda_e Z_e \end{cases} \quad (27)$$

Consistently with the opportunity analysis, the revenue $\Delta\lambda_e Z_e$ is equally split into development costs $\mathrm{k}_e X_e$ and loss purchase $\alpha_e t_e \lambda_e Z_e$. Indeed, assuming that the capacity is given, the energy is transmitted at marginal cost $2\alpha_e t_e \lambda_e$, whereas the first units transmitted generate less losses. The difference is allocated to development costs.

Finally, let us consider a saturated edge ($|t_e| = f_e$). A similar financial analysis leads to:

$$\begin{cases} \Delta\lambda_e Z_e & = & \mathrm{k}_e X_e + \alpha_e f_e \lambda_e Z_e \\ \Delta\lambda_e Z_e & = & 2\alpha_e f_e \lambda_e Z_e + 2\theta_e \end{cases} \quad (28)$$

In the saturated as in the unsaturated case, the energy transmission does not generate any additional economic rent because the development costs are linear and developable capacities unlimited. Overall, this paragraph sums up in the following theorem:

Theorem 2. *In the long term nodal pricing model, the revenue of transmission recovers exactly the development costs and the cost of losses. The development cost share is at least equal to cost of losses one. It is equal if the capacity is unsaturated and higher if it is.*

3.2 Multi–situation optimality analysis

With the multi–situation extension, the optimality conditions are similar to the previous ones, except the derivative with respect to the edge capacity:

$$\frac{\partial \mathscr{L}}{\partial X_e} = 0 = \mathrm{k}_e - \xi_e + \sum_{s \in \mathcal{S}} \left(-\Delta\lambda_e^s t_e^s + \lambda_e^s \alpha_e t_e^{s\,2} \right) \quad (29)$$

Using Equation (22) which defines the nodal price–development cost relationship in the mono–situation case, let us define k_e^s and rewrite Equation (29) when $X_e > 0$:

$$\begin{aligned} \mathrm{k}_e^s &\stackrel{\text{def}}{=} K_e(\Delta\lambda_e^s, \lambda_e^s) \\ \mathrm{k}_e &= \sum_{s \in \mathcal{S}} \mathrm{k}_e^s \end{aligned} \quad (30)$$

This cost decomposition allows to assess the value of each edge developed capacities in each situation. As in the mono-situation case, if the edge is unsaturated, the development costs allocated to the situation are equal to the cost of losses. It means that every used edge ($Z_e^s > 0$) has a value, even if it is unsaturated. It is false in the short term nodal price framework in which nodal price difference covers only losses on unsaturated edges. The fraction of development costs allocated to a situation cannot exceed the cost of losses unless the edge is saturated. In usual networks, the cost of losses represent only a fraction of the development costs (around one fifth). As a result, most of development costs are allocated to the saturating situation or to saturating situations.

The topology of the network is not a tree anymore, even if availability constraints are not used ($f_e^s = 1$). This fact is illustrated by the example developed in Section 5.

4 Nodal pricing and proportional costing equivalence

Despite many contributions, the question of counter flow pricing in transaction based power flow costing methods remains open. In this section, we show the original result that the long term nodal pricing is equivalent to a transaction based costing method in which counter flows get credit for lightening the flow. This method is based on the proportional costing principle, i.e. transactions compensate the development costs and the losses of each edge proportionally to their usage of it (Theorem 3). Moreover development costs are exactly recovered on the optimal network.

Let us assume that, for a given network, a node potential λ_n representing a price of energy at node n is given. Let us assume that a loss function $L_e(Z_e)$ satisfying $L_e(0) = 0$ is given for each edge. In the previously exposed model, $L_e(Z_e) = \alpha_e Z_e^2 / X_e$. Let us also define the mean loss factor as :

$$\ell_{Z_e} = \begin{cases} L_e(Z_e)/Z_e & : \quad Z_e \neq 0 \\ 0 & : \quad Z_e = 0 \end{cases} \quad (31)$$

Let us define a network usage u through sets of productions G_g^u, demands D_d^u, lost loads U_d^u and the corresponding set of net injections I_n^u, partially loading lines of the network at level Z_e^u such that:

$$\begin{cases} I_n^u \stackrel{\text{def}}{=} & \sum_{e \in \mathcal{E}_n^-} Z_e^u + \ell_{Z_e} Z_e^u/2 \\ \quad\quad - & \sum_{e \in \mathcal{E}_n^+} Z_e^u - \ell_{Z_e} Z_e^u/2 \\ X_e = 0 \Longrightarrow Z_e^u = 0 \end{cases} \quad (32)$$

If the network is a tree, there is only one network usage set Z_e^u per injection set I_n^u. On the contrary, if the network has cycles, there can be several usage sets for one injection set, especially if losses are not taken into account. Injection sets correspond to usual transactions, including losses at each node. Due to this definition, the revenue from a nodal tariff of network usage u can be rewritten in the form of edge compensations:

$$\begin{aligned} & \sum_{g \in \mathcal{G}} -\lambda_n G_g^u + \sum_{d \in \mathcal{D}} \lambda_n (D_d^u - U_d^u) \\ & = \sum_{e \in \mathcal{E}} Z_e^u \left(\Delta\lambda_e - \ell_{Z_e} \lambda_e \right) \end{aligned} \quad (33)$$

This equivalence is valid for every node potential, every network topology, and every loss function satisfying Equation (31).

Additionally, in the long term nodal pricing model, due to Equation (13), this relationship between development cost and energy prices is valid for edges with $X_e > 0$:

$$\mathrm{k}_e X_e = Z_e(\Delta\lambda_e - \ell_{Z_e} \lambda_e) \quad (34)$$

Combining Equations (33) and (34) leads to the following equivalence theorem:

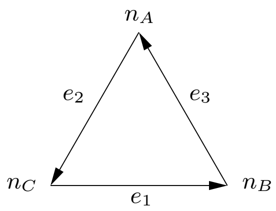
Theorem 3. *For any given network usage u, the nodal tariff based on the optimal dual variables λ_n of the long term nodal pricing model is equivalent to a tariff based on the proportional compensation of development costs on the optimal network:*

$$\sum_{g \in \mathcal{G}} -\lambda_n G_g^u + \sum_{d \in \mathcal{D}} \lambda_n (D_d^u - U_d^u) = \sum_{e \in \mathcal{E} | t_e > 0} Z_e^u \frac{k_e}{t_e} \quad (35)$$

In particular, given that $t_e = 0$ and $X_e > 0$ implies $\mathrm{k}_e = 0$, the overall financial balance is:

$$\sum_{g \in \mathcal{G}} -\lambda_n G_g + \sum_{d \in \mathcal{D}} \lambda_n (D_d - U_d) = \sum_{e \in \mathcal{E}} \mathrm{k}_e X_e \quad (36)$$

(A)



(B)

Edge	X_e	k$_e$	α_e
e_1	0.51	**1**	**2%**
e_2	1.52	**1**	**2%**
e_3	1.01	**1**	**2%**
Tot.	3.03	-	-

(C)

Node	$p^1 \frac{\partial c_g^{g,1}}{\partial G_n}(G_g^1)$	$p^2 \frac{\partial c_g^{g,2}}{\partial G_n}(G_g^2)$	$p^3 \frac{\partial c_g^{g,3}}{\partial G_n}(G_g^3)$
n_A	**10**	**-**	**10**
n_B	**10**	**10**	**-**
n_C	**-**	**10**	**-**

(D)

Sit.	1				2				3				Tot.
N.	G_g^1	D_d^1	λ_n^1	pay.	G_g^2	D_d^2	λ_n^2	pay.	G_g^3	D_d^3	λ_n^3	pay.	pay.
n_A	1.53	**0**	10	15.30	**0**	**2.5**	10.63	-26.57	1.53	**0**	10	15.34	4.07
n_B	0.51	**0**	10	5.10	1.02	**0**	10	10.19	**0**	**1.5**	10.79	-16.18	-0.89
n_C	**0**	**2**	10.76	-21.53	1.53	**0**	10	15.31	**0**	**0**	10.14	0	-6.21
Tot.	2.04	**2**	-	-1.13	2.55	**2.5**	-	-1.06	1.53	**1.5**	-	-0.84	-3.03

Sit.	1				2				3				Tot.
E.	Z_e^1	loss.	$\Delta\lambda_e^1$	comp.	Z_e^2	loss.	$\Delta\lambda_e^2$	comp.	Z_e^3	loss.	$\Delta\lambda_e^3$	comp.	comp.
e_1	-0.51	0.01	-0.76	0.28	0	0	0	0	0.51	0.01	0.65	0.22	0.51
e_2	1.52	0.03	0.76	0.84	-1.52	0.03	-0.63	0.64	0.52	0.01	0.14	0.03	1.52
e_3	0	0	0	0	1.01	0.02	0.63	0.42	-1.01	0.02	-0.79	0.59	1.01
Tot.	-	0.04	-	1.13	-	0.05	-	1.06	-	0.03	-	0.84	3.03

Table 1: Three node three situation example. Data is bold characters and results are in standard characters. Figure (A) sketches the network with three nodes n_A, n_B, and n_C and three potential lines e_1, e_2, and e_3. Arrows on edges indicate the arbitrary direction of positive flow. Table (B) presents capacity development results X_e along with development costs k$_e$ and loss coefficient α_e. Table (C) presents active producers and their production costs in each situation. The production cost (10) is chosen so that the cost of losses is below the development costs so that each line is saturated at least once. Table (D) presents detailed situation results associated to nodes in the upper part and to edges in the lower part. The production G_g^s, demand D_d^s, nodal price λ_n^s, and associated TSO payment (pay. $= G_g^s \lambda_n^s$) are given for each node. The flow Z_e^s, losses (loss. $= \alpha_e t_e Z_e^s$), nodal price difference $\Delta\lambda_e$, and associated TSO compensation after loss purchase (pay. $= \Delta\lambda_e Z_e^s - \lambda_e^s$.loss) for each edge. N. = Node; E. = Edge; Sit. = Situation; Tot. = Total.

In other words, the revenues from a nodal tariff (or from the equivalent proportional costing tariff) are exactly equal to development costs. Indeed, as already mentioned in Theorem 2, there is no additional economic rent for capacity owner. This theorem can be easily extended to the multi–situation extension.

Finally, the reverse of Theorem (3) is true when the network is a tree:

Theorem 4. *The proportional compensation of development costs is equivalent to a nodal tariff if the network is a tree and provided that all edges with capacities and no flow have null development costs (if $X_e > 0$ and $Z_e = 0$, i.e. $t_e = 0$, then k$_e = 0$).*

5 Multi–situation cost allocation

The multi–situation model entangles spatial and temporal aspects so that theoretical analysis have not yet been thoroughly explored. In particular, the underlying principle of edge development cost allocation to each situation is not straightforward. In order to illustrate it, let us consider the three node three situation example described in Table (1-A). Each node n_A, n_B, n_C holds one producer and one consumer. In each situation numbered from 1 to 3, only one consumer is active: The positive demands are D_C^1, D_A^2 and D_B^3 {Table (1-D, Col. D_d^s)}. So as to forbid local production, producers are disabled ($Y_g^s = 0$) when the corresponding consumer is active {Table (1-D) Col G_g^s}. Furthermore, in situation 3, producer in n_C is also deactivated so that the only active producer is located in n_A. Capacities may be installed between each node on three edges noted e_1, e_2, and e_3. Capacity development costs and losses are uniform on the three edges {Table (1-B) Col. k$_e$ and α_e}. Active producer marginal energy cost is chosen so that each edge with developed capacity will have at least one saturating situation ($p^s \frac{\partial c_g^{g,s}}{\partial G_n}(G_g^s) = 10$){Table (1-C)}, i.e. the development costs are sufficiently higher than the costs of losses.

The optimization results in developing network capacity on each edge {Table (1-B, Col. X_e)}. The network is not a tree, even if all edges are always fully available.

The sub graph used in each situation is not necessarily a tree either: In the last situation, all edges are used to send energy from n_A to n_B {Table (1-D, Col. Z_e^3)}. In the two other situations, the edge linking producing nodes is not used ($Z_3^1 = Z_1^2 = 0$). Besides, the analysis of situation development cost allocation confirms the theoretical analysis and shows that usual cost allocation techniques are not optimal with respect to the current model. First of all, as expected, development costs are exactly compensated by the nodal price tariff {Tables (1-D, Col. *Tot. comp.*) and (1-B Line *Tot.*)}. All three edges are saturated in two of the three situations. As expected, for each edge, most development costs are allocated to these saturated situations and no costs are allocated to unused edges {Table (1-D, Col. *comp.*)}.

However, the allocation of costs between saturated situations is not related to usual allocation principles. For example, let us consider three allocation principles: Uniform (U); Proportional to the total demand in MW (D); Proportional to the network load in MW.km, assuming that all edges have the same length (L); According to the current model (N). Table (2) presents the resulting cost allocation results and confirms that all schemes are different.

Sit. \ Meth.	U	D	L	N
1	1.01	1.01	0.93	1.13
2	1.01	1.26	1.16	1.06
3	1.01	0.76	0.94	0.84
Tot.	3.03	3.03	3.03	3.03

Table 2: Development cost allocation to situation 1,2 and 3 using different methods. U = Uniform; D = Proportional to the total demand in MW; L = Proportional to the network load in MW.km assuming that all edges have the same length; N = Long term nodal pricing.

Surprisingly, the current model does not even allocate costs monotonously with the total demand in MW or with the network load in MW.km: costs allocated to situation 2 are lower than costs allocated to situation 1. This is linked with the fact that node n_C cannot produce in situation 3. Indeed, if this constraint is relaxed, the cost allocation resulting from the model (N) is equivalent to the cost allocation proportional to the total demand (D). Applying the constraint rises the compensation of edge e_3 in situation 3 because it directly links the producer node n_A to the consumer node n_B. As a counter effect, it relieves the compensation of the same edge in situation 2, thus lowering the global situation cost allocation.

6 Conclusion

In this article, we show that, provided a long term multi–situation framework is used, nodal pricing theory gives a useful theoretical perspective on the variety of existing costing principles. Inside this perspective, we show that a tariff based on long term nodal prices is equivalent to a tariff based on the proportional compensation of development costs on the optimal network. We also show that the development costs are mostly allocated to the saturating situations. However, among saturating situations, the allocation of costs does not seem to follow simple rules.

The remaining steps before a full implementation of the model are related to security modeling and multiple saturating situations. Indeed, the network is dimensioned to face contingency situations, derived from normal operating situation. A reasonable scheme consists in transferring to the normal situation the costs allocated to derived unobserved contingency situations. In normal situations, all demand is served, so that it is unlikely that an edge is saturated more than once. However, in contingency situations, due to their low probability, there may be unserved demand. As a result, multiple saturating situations might frequently occur. The analysis of cost allocation in such cases is therefore the main milestone towards implementation.

References

[1] J. Boucher and Y. Smeers. Alternative models of restructured electricity systems, part 1: No market power. *Operations Research*, 49:821–838, 2001.

[2] F.C. Schweppe, M.C Caramanis, R.D. Tabors, and R.E Bohn. *Spot Pricing of Electricity*. Kluwer Academic Publishers, London, 1988.

[3] M. Hsu. An introduction to the pricing of electric power transmission. *Utilities Policy*, 6:257–270, 1997.

[4] I.J. Perez-Arriaga, F.J. Rubio, J.F. Puerta, J. Arceluz, and J. Marin. Marginal pricing of transmission services: an analysis of cost recovery. *IEEE Transactions on Power Systems*, 10:546–553, 1995.

[5] W.W. Hogan. Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4:211–242, 1992.

[6] Z. Jing, X. Duan, F. Wen, Y. Ni, and F.F. Wu. Review of transmission fixed costs allocation methods. *IEEE Power Engineering Society General Meeting*, 4:2585–2592, 2003.

[7] M. Boiteux and P. Stasi. Sur la détermination des prix de revient de développement dans un système interconnecté de production–distribution. *Document VI.10 du Congrès de Rome de l'U.N.I.P.E.D.E.*, reprinted in G. Morlat et F. Bessière (eds.) *Vingt-cinq ans d'économie électrique*, Dunod, Paris, 361–400, 1952.

[8] A.S D. Braga and J.T. Saraiva. Long term marginal prices – solving the revenue reconciliation problem of transmission providers. In *Proceedings of the 15th Power Systems Computation Conference*, 2005.

[9] F. Bouffard, F.D. Galiana, and J.M. Arroyo. Umbrella contingencies in security-constrained optimal power flow. In *Proceedings of the 15th Power System Computation Conference*, 2005.

[10] L. Bahiense, G.C. Oliveira, M. Pereira, and S. Granville. A mixed integer disjunctive model for transmission network expansion. *IEEE Transactions on Power Systems*, 16:560–565, 2001.

[11] R. Villasana, L. L. Garver, and S.J. Salon. Transmission network planning using linear programming. *IEEE Transactions on Power Apparatus and Systems*, 104:349–356, 1985.

[12] R.K. Ahuja, T.L. Magnanti, and J.B. Orlin. *Network Flows. Theory, Algorithms, and Applications*. Prentice Hall, 1993.