Design of UFLS Schemes Taking Into Account Load Variation

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Abstract—Underfrequency load-shedding (UFLS) schemes are a last-resort tool to protect a power system in case of a severe disturbance. This paper presents a design of UFLS schemes taking into account load variation, affecting their step sizes. Most of the UFLS schemes implemented today are conventional static and semi-adaptive schemes. To date, methods for the design of these schemes consider known and constant step sizes. Step sizes might actually vary due to feeder load variation, feeder outages or breaker failures. The proposed design is formulated as a scenario-based optimization problem which is based on different step size variation scenarios. The design is applied to a Spanish small isolated power system, exhibiting strong variations of the step sizes. A UFLS scheme designed without contemplating step size variations serves as a reference case. Three different cases are analyzed that differ in the degree step size variations are taken into account. Taking into account load variations benefits the design and performance of UFLS schemes, whereas neglecting these variations results in dangerously low frequencies.

Keywords—Frequency stability, power system protection, load shedding.

I. INTRODUCTION

Frequency stability of power systems is concerned with the ability of generators to supply their loads at an acceptable frequency after generator and load imbalances. Small isolated power systems are especially sensitive to real-power imbalances. It is crucial to avoid the frequency falling below a certain value, since low frequency may severely harm power plants and load-side equipment and hence, the system integrity [1].

Underfrequency load-shedding (UFLS) schemes are a last-resort tool to protect a power system in case of a severe disturbance [2]. In small isolated power systems, UFLS schemes play an important role in protecting the system integrity. Most of the UFLS schemes implemented today are conventional static and semi-adaptive schemes [3]. These schemes continuously measure frequency and optionally the rate of change of frequency (ROCOF) by means of type 81 relays and shed a predefined amount of load in case frequency and/or ROCOF fall below a certain threshold. Electric utilities adopt different approaches to design UFLS schemes, which are mainly based on their experience, and they usually follow typical design criteria on the number of steps, step size, frequency thresholds, etc. ([4]-[6]).

Several methods have been reported in the literature to design conventional UFLS schemes, i.e., to tune frequency and ROCOF thresholds, intentional time delays and step sizes. Some of these methods are based on iterative trial and error procedures [6]; others make use of a screening process among a host of candidate schemes ([4], [5]).

Both deterministic and heuristic optimization algorithms have also been applied to the design of conventional UFLS schemes to minimize the amount of shed load ([7]-[11]). Deterministic optimization algorithms depend however on the initial guess of the decision variables. Furthermore, the objective function, which aims at minimizing the amount of shed load, becomes discontinuous and shows a step-like shape if step sizes are not considered as decision variables, impeding the use of gradient-based methods.

Heuristic algorithms seem to be an appropriate alternative ([7], [9]). A Genetic Algorithm has been used to optimize the step size of a very simple single-stage UFLS scheme of a test system, maintaining constant frequency threshold and intentional time delay [9]. A Hierarchical Genetic Algorithm has been employed in [10] to minimize the amount of shed load and maximize the lowest swing frequency. Alternatively, a method based on the Simulated Annealing algorithm has been described in [11].

Most of the optimization algorithms consider the step size as a decision variable, but for small isolated power systems, it is usually difficult to realize the optimized step size by rearranging available feeder load-blocks. Actually, only a limited number of the 20kV feeders are equipped with breakers responding to type 81 relays tripping signals. Further, these algorithms assume known and constant step sizes. Nonetheless, step sizes might actually vary due to feeder load variation, feeder outages or breaker failures. The present paper contemplates varying step sizes when designing the UFLS scheme.

The assumption that feeder load variations are somehow proportional to the total demand variation is not realistic, since depending on the hour and the day, different customer types are fed by the feeders. Moreover, distributed generation such as PV can significantly alter the amount of active power flowing downwards through a feeder.

In [12], an analysis of the performance of UFLS schemes for varying step sizes has been presented. Step size variations seem to be dominated by feeder load variations. These variations readily worsen the performance of UFLS schemes,
which could become ineffective. For instance, reduced step sizes could cause dangerously low frequency levels since not enough load is shed.

This paper proposes a method for the design of UFLS schemes of small isolated power systems which takes into account step size variations. This method is formulated as a scenario-based optimization problem. Section II illustrates the problem, whereas section III presents the scenario-based optimization problem. The proposed method is then applied to a Spanish small isolated power system in section IV. Section V provides the conclusions of the paper.

II. PROBLEM ILLUSTRATION

UFLS schemes of small isolated power systems are in their vast majority conventional UFLS schemes. A UFLS step is defined by the settings of its associated type 81 relays and by its step size. Step sizes particularly suffer from uncertainties caused by variations due to feeder load variation, feeder outages or breaker failures as shown in Fig. 1. Percentaged maximum variations with respect to the average step size of a given day are shown for two Spanish small isolated power systems. Step size variations attain up to nearly 40% of the average step size. It can be also seen that these variations are very system dependent. Whereas variations in power system B are around 10%, step sizes in power system A vary between 5 and 40% with an average variation of 15%.

Fig. 1: Variations of the step sizes of two UFLS schemes of two Spanish small isolated power systems.

The problem of designing UFLS schemes with step size variations can be easily illustrated for a simple test system that consists of two generators G1 and G2, generating $P_{G1} = 1.5$ and $P_{G2} = 3.5$ MW, respectively. The UFLS scheme is originally supposed to shed an amount $P_{\text{sh}}$ of 17% of the total demand instantaneously if frequency falls below 48.4 Hz. The setting is such that no load is shed for the outage of G1, whereas for the outage of G2 just enough load is shed to prevent the frequency from falling below 47Hz. Fig. 2 shows the responses of the test system to the outages of G1 and G2 with $P_{\text{sh}}=17%$.

If the step size is now reduced to 80% of its original value ($p_{\text{step}}=13.6%$), not enough load can be shed and the frequency falls below 47Hz as seen in Fig. 2 (and generator underfrequency protections might initiate generator outages). Increasing the frequency threshold from 48.4 to 49.4Hz avoids the frequency falling below 47Hz, but it provokes an unnecessary load shedding for the outage of G1. The trade-off between the avoidance of low frequency conditions and additional, unnecessary load shedding, occasionally causing overshedding, is very system dependent. Section III and IV tackle the design of UFLS schemes with varying step sizes in more detail.

III. PROBLEM FORMULATION

The design of UFLS schemes taking into account step size variations is formulated as a scenario-based optimization problem. Its objective function $f(x)$ aims at minimizing the amount of shed load, whereas system stability is guaranteed by imposing appropriate inequality constraint functions $g(x)$.

In its most generic formulation, decision variables $x$ of the optimization problem correspond to the frequency and ROCOF thresholds, the intentional time delays and the step sizes of underfrequency and ROCOF steps. However, it does not make sense for rather small power systems to adjust the step size, since it will be difficult to find feeder blocks which finally sum up to the optimized step size. Thus, type 81 relay parameters, i.e., frequency and ROCOF thresholds ($\omega_{\text{uf}}, \omega_{\text{rocof}}$ and $d\omega/dt$) as well as intentional time delays ($t_{\text{uf,ext}}, t_{\text{rocof,ext}}$) are considered here as decision variables as shown in equation (1). The settings of the existing UFLS scheme can be used as initial values for the decision variables.

$$x = \left[\omega_{\text{uf}}, \omega_{\text{rocof}}, d\omega/dt, t_{\text{uf,ext}}, t_{\text{rocof,ext}}\right]^T$$  \hspace{1cm} (1)

The $j$th step of the UFLS scheme is defined by the settings according to equation (1) and by its step size $p_{\text{step}}$. Step size variations seem to be dominated by feeder load variations ([12], [15]). Although a large variety of probability density functions have been applied, it is still common to use a normal distribution function to represent load variations ([15], [16]). Instead of considering the entire distribution function, a set of credible step size scenarios are used here to reduce computation effort. The selection of step size scenarios depends on the system and in particular, on the actual step size variation (cf. section II). The variation of the step size of the $j$th UFLS step is then implemented by changing its actual step size by a certain amount according to the step size scenario derived from a distribution function. This is,

$$P_{\text{sh},j} = P_{\text{sh},j} \cdot y_k$$  \hspace{1cm} (2)
where \( y_k \) is the \( k \)th step size variation scenario and \( p_{\text{step0},j} \) is the initial step size of the \( j \)th UFLS step. \( y_k \) could be either a constant or it could follow a distribution function.

The objective function complying with the main objective is:

\[
f(x) = \sum_{k=1}^{K} \alpha_k \left( \sum_{j=1}^{J} \beta_j p_{\text{shd},j}(x, y_k) \right)
\]

where \( \alpha_k \) and \( \beta_j \) are weighting factors, \( p_{\text{shd},j} \) the total amount of shed load in pu for the \( j \)th disturbance, \( R \) is the number of disturbances and \( K \) is the number of step size variation scenarios. Other objective function formulations also include an additional term related to frequency deviations (see e.g., [7]), but it seems more intuitive to include restrictions on frequency deviation as constraints. The weighting factor \( \beta_j \) is set to unity here as in [8], attaching equal importance to the amount of shed load in each disturbance.

The constraints are (i) the minimum and (ii) the maximum allowable frequency values (\( \omega_{\text{min},\text{allowable}} \) and \( \omega_{\text{max},\text{allowable}} \)). In general, the constraint of minimum allowable frequency is accompanied by a maximum time delay, \( t_{\text{cmin,allowable}} \), during which frequency can stay below the minimum allowable frequency. Equations (4) and (5) implement these two constraints.

\[
g_{i,j}(x) = t_{\text{act},i}(x, y_k) - t_{\text{max,allowable}} \quad (4)
\]

\[
g_{i,j}(x) = \omega_{\text{max},i}(x, y_k) - \omega_{\text{max,allowable}} \quad (5)
\]

Remaining constraints are (iii) the UFLS scheme does not act once the frequency has passed its minimum value since frequency is returning towards its nominal value, (iv) the amount of shed load, \( p_{\text{shd}} \), is smaller than or at most equal to the amount of lost real power, \( p_{\text{loss}} \) and (v) the UFLS scheme respects the priority of loads. Equations (6) and (7) implement the first two constraints. The constraint on priority is implemented such that its associated constraint function is positive whenever a UFLS step actuates without that its preceding step has actuated.

\[
g_{i,j}(x) = t_{\text{act},i}(x, y_k) - t_{\text{max,allowable}} \quad (6)
\]

\[
g_{i,j}(x) = p_{\text{shd},i}(x, y_k) - p_{\text{loss},i} \quad (7)
\]

The same four representative disturbances and the same optimization constraints as in [11] are considered. Note that the four disturbances, characterized by different load-generation scenarios and different generator outages, have been extracted from all possible disturbances by means of clustering techniques. The optimized UFLS scheme derived in [11], assuming constant step sizes, serves as a reference case. Finally, the reference case as well as the optimized UFLS scheme taking into account step size variations are applied to the representative disturbances by assuming this time that step size scenario \( y_k \) follows a Normal-Bernoulli distribution function with \( \mu = 1 \) and \( \sigma = 0.1 \) ([12], [15]). Step size variation of 10, 20 and 40% are contemplated since they cover at least three standard deviations of a normal distribution function, and they also include the variations reported in Fig. 1.

The same four representative disturbances and the same optimization constraints as in [11] are considered. Note that the four disturbances, characterized by different load-generation scenarios and different generator outages, have been extracted from all possible disturbances by means of clustering techniques. The optimized UFLS scheme derived in [11], assuming constant step sizes, serves as a reference case. Finally, the reference case as well as the optimized UFLS scheme taking into account step size variations are applied to the representative disturbances by assuming this time that step size scenario \( y_k \) follows a Normal-Bernoulli distribution function with \( \mu = 1 \) and \( \sigma = 0.15 \). The Normal distribution models feeder-load variations, whereas the Bernoulli distribution models feeder outages and breaker failures (i.e., in 5% of the cases where load shedding is needed, the feeder is not curtailed).

\[\text{A. Case 1 - 40\% step size variation}\]

The UFLS scheme is optimized by contemplating step sizes variations of 40% of the original step sizes only (\( y_k = 0.4 \)). This corresponds to an extreme case, where the UFLS scheme is designed to protect the system when only a small amount of load is actually available for load shedding. Such a
design is expected to cause overshedding with normal load levels.

Four disturbances are considered for the design as described in [11]. The UFLS scheme parameters are bounded as follows: (i) $46 \text{ Hz} \leq \omega_{uf} \leq 49 \text{ Hz}$, (ii) $49.3 \text{ Hz} \leq \omega_{rocof} \leq 49.8 \text{ Hz}$, (iii) $-1.5 \text{ Hz/s} \geq \omega_{min} \geq -2.5 \text{ Hz/s}$ and (iv) $0 \text{ s} \leq t_{int} \leq 0.5 \text{ s}$. With respect to the optimization constraints, $\omega_{min,allowable}$ is set to $48 \text{ Hz}$ and $47 \text{ Hz}$ with corresponding $t_{\omega_{min,allowable}}$ of $2 \text{ s}$ and $0 \text{ s}$, respectively, and $\omega_{max,allowable}$ is set to $52 \text{ Hz}$.

Table 1 shows and compares the resulting UFLS scheme taking into account step size variation of 40% (case 1) with the UFLS scheme optimized without contemplating step size variations (reference case). It can be seen that frequency thresholds of the underfrequency steps 2-6 have been increased. In addition, two further steps (7 and 8) are now needed to mitigate step size variations. These steps were not necessary in the reference case. Intentional time delays have also been reduced. All this indicates that the UFLS scheme of case 1 starts shedding loads earlier and that more load will be shed than in the reference case.

**TABLE 1: UFLS SCHEMES ACCORDING TO CASE 1 AND THE REFERENCE CASE (WITHOUT STEP SIZE VARIATIONS).**

<table>
<thead>
<tr>
<th>Underfrequency relays</th>
<th>Step nº</th>
<th>$\omega_{uf}$ (Hz)</th>
<th>$t_{\omega_{min}}$ (s)</th>
<th>$t_{\omega_{max}}$ (s)</th>
<th>$p_{\text{open}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/out case 1</td>
<td>1</td>
<td>46.52</td>
<td>0.05</td>
<td>0.2</td>
<td>7.1</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>2</td>
<td>46.45</td>
<td>0.11</td>
<td>0.2</td>
<td>6.7</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>3</td>
<td>47.89</td>
<td>0.27</td>
<td>0.2</td>
<td>14.5</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>4</td>
<td>47.75</td>
<td>0.36</td>
<td>0.2</td>
<td>3.6</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>5</td>
<td>47.60</td>
<td>0.04</td>
<td>0.2</td>
<td>7.3</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>6</td>
<td>47.35</td>
<td>0.13</td>
<td>0.2</td>
<td>13.6</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>7</td>
<td>47.73</td>
<td>0.02</td>
<td>0.2</td>
<td>12.5</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>8</td>
<td>47.57</td>
<td>0.03</td>
<td>0.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROCOF relays</th>
<th>Step nº</th>
<th>$\omega_{rocof}$ (Hz)</th>
<th>$\omega_{min}/dt$ (Hz/s)</th>
<th>$t_{\omega_{min}}$ (s)</th>
<th>$t_{\omega_{max}}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/out case 1</td>
<td>1</td>
<td>49.78</td>
<td>-1.9</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>2</td>
<td>49.78</td>
<td>-1.9</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>3</td>
<td>49.78</td>
<td>-1.9</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>w/out case 1</td>
<td>4</td>
<td>49.78</td>
<td>-1.9</td>
<td>0.16</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Fig. 4 shows the system responses in terms of frequency to the four disturbances and the imposed allowable frequency limits. Fig. 4 compares the impact of case 1 with the impact of the reference case. It can be seen that the optimized UFLS scheme complies with the imposed low-frequency restriction by shedding however a larger amount of load (the reference case does not shed load for the upper two disturbances). This is due to the fact that underfrequency thresholds have been remarkably increased such that the UFLS scheme is able to arrest frequency decay in due time. The reduced frequency deviations and the larger amount of shed load can be also seen in Table 2, which displays the total frequency deviations ($\Sigma \Delta \omega_{\min}$) and the total amount of shed load for both the UFLS scheme optimized by contemplating step size variations and the one optimized without contemplating step size variations. $\Sigma \Delta \omega_{\min}$ is the sum of minimum frequency deviations for all outages.

Finally, both the reference case and case 1 are applied to the four disturbances by assuming this time that $y_k$ follows a Normal-Bernoulli distribution: (a) UFLS scheme optimized without and (b) UFLS scheme optimized by contemplating step size variations (case 1).

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low-frequency conditions remarkably but at the expense of high-frequency conditions and overshedding (Fig. 5 (b)).

**TABLE 3: STATISTICAL ANALYSIS OF THE IMPACT OF THE REFERENCE CASE AND CASE 1.**

<table>
<thead>
<tr>
<th>reference case</th>
<th>case 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>Δω&lt;sub&gt;min&lt;/sub&gt;</td>
<td>-4.01</td>
</tr>
<tr>
<td>Δω&lt;sub&gt;max&lt;/sub&gt;</td>
<td>0.26</td>
</tr>
<tr>
<td>р&lt;sub&gt;shed&lt;/sub&gt;</td>
<td>1.55</td>
</tr>
<tr>
<td>p&lt;sub&gt;shed&lt;/sub&gt;-p&lt;sub&gt;net&lt;/sub&gt;</td>
<td>0.04</td>
</tr>
<tr>
<td>t&lt;sub&gt;cumulative&lt;/sub&gt;</td>
<td>2.94</td>
</tr>
</tbody>
</table>

**TABLE 4: UFLS SCHEMES ACCORDING TO CASE 2 AND CASE 3.**

<table>
<thead>
<tr>
<th>Step nº</th>
<th>ω&lt;sub&gt;uf&lt;/sub&gt; (Hz)</th>
<th>t&lt;sub&gt;uf&lt;/sub&gt; (s)</th>
<th>p&lt;sub&gt;shed&lt;/sub&gt; (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 2</td>
<td>49.73</td>
<td>48.55</td>
<td>0.05</td>
</tr>
<tr>
<td>case 3</td>
<td>48.61</td>
<td>48.31</td>
<td>0.07</td>
</tr>
<tr>
<td>case 2</td>
<td>48.51</td>
<td>48.02</td>
<td>0.02</td>
</tr>
<tr>
<td>case 3</td>
<td>48.50</td>
<td>47.92</td>
<td>0.06</td>
</tr>
<tr>
<td>case 2</td>
<td>48.20</td>
<td>47.82</td>
<td>0</td>
</tr>
<tr>
<td>case 3</td>
<td>47.59</td>
<td>47.23</td>
<td>0.01</td>
</tr>
<tr>
<td>case 2</td>
<td>47.42</td>
<td>47.13</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 5: CASE 2 - COMPARISON OF THE TOTAL MINIMUM FREQUENCY DEVIATIONS AND THE TOTAL AMOUNT OF SHED LOAD.**

<table>
<thead>
<tr>
<th>UFLS scheme</th>
<th>2. Δω&lt;sub&gt;min&lt;/sub&gt; (Hz)</th>
<th>Total shed (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2</td>
<td>18.85</td>
<td>48.31</td>
</tr>
<tr>
<td>Reference case</td>
<td>22.49</td>
<td>25.17</td>
</tr>
</tbody>
</table>

Fig. 6 shows the system responses in terms of frequency to the four disturbances and the imposed allowable frequency limits. Fig. 6 compares the impact of case 2 with the impact of the reference case. The same disturbances and constrains as in section IV.A have been applied. It can be deduced that case 2 complies with the imposed low-frequency restriction by shedding a larger amount of load since underfrequency thresholds have been increased, but not as drastically as in case 1. The larger amount of shed load can be indirectly seen in the slight overfrequencies around 51 Hz. **TABLE 5** confirms that the total minimum frequency deviations have been reduced and that the total amount of shed load has been increased with case 2.

Again, case 2 is applied to the four disturbances by assuming that <i>y</i> follows a Normal-Bernoulli distribution function with μ = 1, σ = 0.15 and ρ<sub>bern</sub> = 0.05. Fig. 7 (a) shows the impact of case 2 on the system responses in terms of frequency when step sizes follow a Normal-Bernoulli distribution function. It can be seen from Fig. 7 (a) that the UFLS scheme optimized by contemplating step size variations reduces low-frequency conditions with regard to the reference case shown in Fig. 5 (a), but it still causes high-frequency conditions. The high-frequency conditions seem to be however less dramatic than those of case 1 (Fig. 5 (b)).

**TABLE 6** displays a statistical analysis of five indices Δω<sub>min</sub>, Δω<sub>max</sub>, p<sub>shed</sub>, p<sub>loss</sub>-p<sub>shed</sub> and t<sub>cumulative</sub> with regard to the performance of case 2. The comparison of the reference case (TABLE 3) and case 2 confirms that minimum frequency deviations (Δω<sub>min</sub>) decreased with case 2 but at the expense of increased maximum frequency deviations (Δω<sub>max</sub>) and amount of shed load (p<sub>shed</sub>). With regard to case 1 (TABLE 3), there is still a negative value p<sub>loss</sub>-p<sub>shed</sub> indicating overshedding, but the statistical analysis reveals that p<sub>loss</sub>-p<sub>shed</sub> has a higher average value and a lower standard deviation in case 2 than in case 1. Finally, it can be seen that the accumulated time frequency is below the critical frequency thresholds and the average
minimum frequency deviation are slightly smaller in case 2 than in case 1. This small difference is mostly due to the fact that first underfrequency thresholds in case 2 are slightly higher than in case 1. This difference originates from the formulation of the objective function which has a step-like shape. A slightly higher frequency threshold has no impact on the optimization problem, but it will affect the outcome when the UFLS scheme is used against the four disturbances when step sizes follow a Normal-Bernoulli distribution.

**Table 6: Statistical analysis of the impact of the UFLS scheme optimized by contemplating step size variations.**

<table>
<thead>
<tr>
<th></th>
<th>case 2</th>
<th>case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \omega_{\text{min}} )</td>
<td>-3.04</td>
<td>-3.17</td>
</tr>
<tr>
<td>( \Delta \omega_{\text{max}} )</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>( \rho_{\text{mean}} )</td>
<td>1.25</td>
<td>0.98</td>
</tr>
<tr>
<td>( \rho_{\text{mean}} \cdot \rho_{\text{Bern}} )</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>( L_{\text{cumulative}} )</td>
<td>0.64</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Fig. 7 shows the system responses in terms of frequency to the four disturbances and the imposed allowable frequency limits. Fig. 8 compares the impact of case 3 with the impact of the reference case. The same disturbances and constrains as in section IV.A have been applied. It can be deduced that case 3 complies for all but one disturbance with the imposed low-frequency restriction. This constraint violation corresponds to the 40% step size variation scenario, which has a very low weight \( \alpha_k \) with respect to the other two scenarios and which has therefore a smaller impact on the objective function than the other scenarios. Again, slight overfrequencies around 51 Hz can be observed, indicating that a larger amount of load has been shed than in the reference case.

**TABLE 7: Case 3 - Comparison of the total minimum frequency deviations and the total amount of shed load.**

<table>
<thead>
<tr>
<th></th>
<th>Case 3</th>
<th>Reference case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma \Delta \omega_{\text{min}} ) (Hz)</td>
<td>20.83</td>
<td>22.40</td>
</tr>
<tr>
<td>Total shed (MW)</td>
<td>31.80</td>
<td>25.17</td>
</tr>
</tbody>
</table>

Again, case 3 is applied to the four disturbances by assuming that \( y_k \) follows a Normal-Bernoulli distribution function with \( \mu = 1 \), \( \sigma = 0.15 \) and \( p_{\text{Bern}} = 0.05 \). Fig. 7 (b) shows the impact of case 3 on the system responses in terms of frequency when step sizes follow a Normal-Bernoulli distribution function. It can be seen in Fig. 7 (b) that case 3 reduces low-frequency conditions with regard to reference case shown in Fig. 5 (a) without causing dangerous high-frequency conditions observed for case 1 and case 2 (Fig. 5 (b) and Fig. 7 (a), respectively). There is one case where frequency remains at a relatively high level (51.5 Hz) but still below the critical frequency threshold, which could be
Finally, Table 6 displays a statistical analysis of five indices \( \Delta \omega_{\text{min}}, \Delta \omega_{\text{max}}, p_{\text{shed}}, p_{\text{load}}-p_{\text{shed}} \) and \( t_{\text{act}}-t_{\text{completion}} \) with regard to the performance of case 3. With respect to cases 1 (Table 3) and 2 (Table 6), it can be seen that minimum frequency deviations \( \Delta \omega_{\text{min}} \) increased and that maximum frequency deviations \( \Delta \omega_{\text{max}} \) and the amount of shed load \( p_{\text{shed}} \) decreased. Although there is still a negative value \( p_{\text{load}}-p_{\text{shed}} \) indicating overshedding, the statistical analysis reveals that \( p_{\text{load}}-p_{\text{shed}} \) has an even higher average value in case 3 than in case 2. This means that less overshedding occurred, which is reflected in Fig. 7 (b) in the absence of high-frequency conditions (above 52 Hz). Lastly, the accumulated time frequency is below the critical frequency thresholds is slightly higher than in cases 1 and 2, but still more than 17 times smaller than in the reference case (Table 3).

It seems therefore that including step size variation is beneficial for system stability. A good trade-off between system stability and a reduced amount of load shedding can be obtained by weighting the step size variation scenarios. The selection of the step size scenarios and the weights to be used depend very much on the system characteristics and the system operator’s experience.

V. CONCLUSIONS

This paper has presented the design of underfrequency load-shedding (UFLS) schemes taking into account load variation, affecting their step sizes. The design has been formulated as a scenario-based optimization problem. Three different cases have been considered, which differ in the degree step size variations were taken into account. A UFLS schemes designed without contemplating step size variations served as a reference case. The approach has been successfully applied to a Spanish small isolated power system. The reference case performed worse when step size variation occur than the UFLS schemes designed by taking into account step size variations. Further, it seems that by weighting the step size variation scenarios appropriately, a good trade-off between system stability and amount of shed load can be obtained.

REFERENCES