Reliability Evaluation of Composite Power Systems Using Sequential Simulation with Latin Hypercube Sampling

Zhen Shu, Panida Jirutitijaroen, and Bordin Bordeerath
Electrical and Computer Engineering
National University of Singapore
21 Lower Kent Ridge Road, 119077, Singapore
a0017032@nus.edu.sg, elejp@nus.edu.sg, bizv421@nus.edu.sg

Abstract—Monte Carlo (MC) sequential simulation is capable of providing system chronological information. It is a very useful tool for power system reliability analysis and planning, especially for systems involving time-dependent sources. In this paper, a new sequential simulation approach is proposed for reliability evaluation of composite power systems. The main idea is to apply Latin Hypercube sampling (LHS) to generate the time duration of each system state in order to facilitate simulation convergence. LHS can yield system chronologies of higher representativeness than the conventional random sampling using the same number of sample states. The LHS-based sequential simulation approach is performed on IEEE Reliability Test System (IEEE RTS). It is performances are examined for several cases with different types of down time distributions of system components. Reliability indices including loss of load probability, expected unsupplied power, loss of load frequency and loss of load duration are estimated. The case results demonstrate that the LHS-based sequential simulation approach outperforms the conventional approach by reducing the variances of simulated reliability indices.

Keywords—Composite system reliability analysis, Monte Carlo simulation, sequential simulation, Latin Hypercube sampling.

I. INTRODUCTION

Composite power system reliability indices can be calculated using two fundamental approaches—analytical enumeration and MC simulation [1], [2]. The analytical approach enumerates all possible system states to identify loss of load states, and therefore obtains the exact solutions of indices. The enumeration process may lead to a significant increase of computing time as the number of system components increases. In general, analytical approach becomes impractical for composite power systems, especially those with high complexity. MC simulation is considered more attractive for evaluating these systems. Correlation between components, load uncertainties, etc. can be easily incorporated by MC technique, making it a preferable choice for planning engineers assessing real power networks. In MC simulation, the accuracy of estimated reliability indices can be monitored through the coefficient of variation values [1].

MC methods can be broadly classified into two approaches [1]-[12]: non-sequential (random sampling) and sequential (chronological sampling) simulation. In the random sampling technique, the states of system components are sampled according to their probability distributions. The non-chronological system states are then constructed by combining individual component states. This technique works in the state space domain. The chronology of events is, therefore, ignored. Despite this, reliability indices involving frequency and duration of system failures can still be easily computed through appropriate test functions [3]-[5]. However, such techniques are based on Markovian assumptions for component behaviors. If this is not the case, the only approach to assess the reliability for composite generation and transmission systems of high complexity is sequential simulation [2], [5]-[7].

The sequential MC technique simulates artificially the chronological history of each component in the system, i.e., up and down cycles of generators, transmission equipment, and the fluctuation of loads, throughout a period of interest [2]. Thus, this technique works in a time domain. As such, evaluating the frequency and duration and any other types of reliability indices (e.g. load costs), as well as incorporating correlations among system component, can be directly implemented with this technique. Due to its ability of imitating the stochastic behavior of the system components, the sequential technique is advantageous to incorporate any types of renewable energy sources, including hydrothermal generating units with energy storage limitations. Moreover, calculating reliability indices using this method can be made more realistic by imposing uncertainties to system load and renewable generation patterns. Due to these advantages, reliability indices obtained from sequential simulation are considered as the most realistic ones, and therefore, they are usually adopted as benchmark approaches for comparing different computational methods.

A major drawback of sequential simulation is that it requires more computational time to converge than non-sequential simulation. As a result, this technique becomes unfavorable when elaborate analysis is not required. However, there have been a number of attempts to overcome such problem: (i) pseudo-chronological MC simulation for composite system reliability evaluation [5], [7]; (ii) quasi-sequential MC simulation for generating adequacy analysis [11]. The computing accuracy and efficiency of these
approaches have been well reflected in terms of algorithms and experiments. While retaining most of the advantages of conventional sequential simulation, they provide significant speed-ups through evaluating the sub-sequences merely for load-loss states instead of for all the sampled state. However, they can hardly assess index probability distributions since the complete system chronology still remains unknown.

Convergence criteria for MC simulation algorithms are based on the variance of reliability indices. Statistically, an index with a lower variance is more accurate than another one with a higher variance. To reduce the computing time of either sequential or non-sequential simulations, variance reduction techniques (VRT) have been adopted [13]. For example, pseudo-chronological MC simulation is considered as a type of VRT; control and antithetic variates, as another category of VRT, have also been employed for composite reliability [6]. They utilize the correlations among random variables to reduce variances of the estimated indices without altering their expected values. Another interesting approach for reliability indices variance reduction is LHS techniques [14], [15], which has already been applied to non-sequential simulation to evaluate single and multi-area generating systems [9], [12].

This paper extends the use of LHS techniques to sequential simulation for composite generation and transmission system evaluation. The objective is to reduce the variances of reliability indices in order to facilitate the convergence of sequential simulation. The organization of this paper is as follows. Section II introduces the main ideas for MC and LHS schemes, respectively. The procedures of the proposed LHS-based sequential simulation approach are presented in Section III. In Section IV, cases studies are performed for performance demonstration. Finally, the conclusion is drawn in Section V.

II. MONTE CARLO AND LATIN HYPERCUBE SAMPLINGS

An important step for reliability simulation is sampling process. It is essentially an inverse transform method as commonly adopted in simulations for various purposes [13]. It generates some pseudo-random numbers uniformly distributed over [0, 1] and uses these numbers to find the corresponding values of random variables from their probability distributions. As such, the probability distributions of random variables are explicitly reflected in the simulation.

The simulation quality would partially depend on the representativeness of sampled values. The conventional MC sampling method usually generates a series of values over [0, 1] in a random manner, as illustrated in Fig. 1. Suppose that the sample size of this simulation is five. Values $u_1, ..., u_5$ are the sampled random numbers uniformly distributed over [0, 1]. Suppose that the input random variable is distributed with a probability density function $F$. A series of $x_1, ..., x_5$ is thereafter obtained by an inverse transform method, as follows:

$$x_j = F^{-1}(u_j) \tag{1}$$

In practice, $x_j$ may represent the status of being up/down, or, the time duration of up/down status for a component. It is worth noting that, a greater amount of random numbers will bring a better representativeness of the distribution under consideration. With a limited number of sampled values, random sampling may not guarantee that the whole distribution is well covered, and thus lead to inadequate representativeness.

LHS, developed by Mckay et al. in 1979 [14], is a combination of stratified and random samplings [15] used to facilitate convergence of MC simulation without altering the distributions of random variables. In the literature, LHS has been used in some power system applications. In [9], LHS was adopted as a variance reduction tool for generating capacity reliability evaluation via non-sequential simulation. The results show that LHS outperforms MC in terms of accuracy and the proposed discrete LHS [9] also helps reducing the storage required to run the simulation. LHS has also been employed in power system stochastic optimization problems, for example, the system adequacy planning problem [16] and a permanent magnet pole shape optimization of a BLDC motor [17]. In addition, the hybrid LHS and Cholesky decomposition has been proposed to improve the accuracy of MC simulation used for probabilistic load flow evaluation.

The main idea of LHS is to control the sampled values with a stratified distribution function so that they can better cover the whole input distribution. For instance, as seen in Fig. 2, with a sample size of 5, LHS stratifies the cumulative distribution function into 5 intervals with equal probability of occurrence, and then performs random sampling within each interval, obtaining values $u_1, ..., u_5$. Subsequently, the samples $x_1, ..., x_5$ for random variable $x$ are calculated by the following expression [14]:

![Sampling Scheme of MC](image1)

![Sampling Scheme of LHS](image2)
where \( L_j \) is the \( j \)th interval.

Random numbers generated by LHS can scatter over the entire distribution. With this feature, LHS method is able to achieve a good coverage of the distribution functions for characterizing system randomness. When comparing Fig. 2 to Fig. 1, one can note that with the same amount of random numbers, LHS yields samples of better representativeness than conventional MC. In other words, LHS can yield the same quality of representativeness with fewer samples. As a result, variance reduction of estimated indices can be achieved.

### III. COMPOSITE SYSTEM RELIABILITY EVALUATION

Sequential simulation is mainly classified into two methods: fixed time interval method and next event method. The fixed time interval method approximates a continuous-time Markov model as a discrete time model, and generally requires a longer computational time than the next event method. In this paper, all the sequential simulations are carried out using next event method. In the following, both sequential MC and LHS simulation algorithms are explicitly described.

#### A. Sequential MC Simulation

With next event method, each conventional generator can be represented by a typical two-state Markov model shown in Fig. 3, where \( \lambda \) and \( \mu \) denote its failure and repair rates, respectively. Similarly, this model is also established to represent the up and down cycles for each transmission line. Other types of probabilistic components, such as fluctuating load or renewable energy sources, can be modeled by their corresponding time series. It should be noted that, normally, with the Markovian assumption, component up and down times are exponentially distributed. However, sequential simulation can simply allow the up and down times be arbitrarily distributed as well, specifically according to the required characteristics that would best reflect component stochastic behaviors.

The steps of sequential MC simulation are as follows.

1. **Step 1)** At the beginning of the simulation (i.e., \( t=0 \)), the states of all components are initialized according to their up/down probabilities in random sampling manner.

2. **Step 2)** Assuming that the up and down times of component \( i \) are distributed with distributions \( F_i \) and \( G_i \), respectively, the time sequence of each component is advanced by sampling an up or down time duration using the following expressions:

   \[
   t_{up}^{i} = F_i^{-1}(U[0,1])
   \]

   \[
   t_{down}^{i} = G_i^{-1}(U[0,1])
   \]

   where \( t_{up}^{i} \) and \( t_{down}^{i} \) are up and down time durations of component \( i \), respectively; \( U[0,1] \) is a pseudo-random number from a uniform distribution over \([0,1]\).

3. **Step 3)** Incorporate load demand fluctuations using their historical time series; see, for instance, [19].

   **Step 4)** Identify the load curtailment hours for each simulated time duration based on DC optimal power flow formulation; see [1], [2].

   **Step 5)** Evaluate reliability indices of interest, such as loss of load probability (LOLP), expected power not supplied (EPNS), loss of load frequency (LOLF), and loss of load duration (LOLD), in a yearly basis.

![Fig. 3. Two-state Markov model.](image)

Note that sequential simulation is a very comprehensive tool to assess many types of reliability indices. In addition to the abovementioned indices, some other indices such as such as loss of load cost (LOLC) [5]-[7], can also be easily computed. The convergence of sequential MC simulation is heavily limited by the high variances of estimated indices. Next, the application of LHS to sequential simulation is proposed to overcome this problem.

#### B. Sequential LHS Simulation

As mentioned in Section II, LHS has been applied to several power system problems [9], [12], [16], [18]. In this work, the LHS is applied to sequential simulation for composite system reliability evaluation. As stated earlier, the stratification strategy of this technique depends on the number of intervals defined for an input distribution. In next event method, the number of intervals for each up/down time distribution will need to be appropriately determined beforehand. On average, one up/down cycle of any power system equipment is equal to “MTTF + MTTR”. Hence, there will be, on average, \( n_i \) cycles occurring in a given simulated period, as shown in (5). This number \( n_i \) is chosen for LHS stratification.

\[
\text{simulated period} = n_i \text{ round off} \left( \frac{\text{MTTF}_i + \text{MTTR}_i}{MTT} \right)
\]

where \( n_i \) is the number of intervals for component \( i \); \( \text{MTTF}_i \) and \( \text{MTTR}_i \) are the mean times of failure and repair, respectively, for component \( i \). The simulated period is chosen to be one year with 8760 hours.

The steps of sequential LHS simulation for composite reliability evaluation are given as follows.

1. **Step 1)** Determine the number of intervals used for stratifying a distribution according to (5).

2. **Step 2)** Generate a stratification matrix \( L_i \) of size \( n \times 1 \) for each component \( i \). The elements of this matrix are randomly ordered integers ranging from 1 to \( n \) used for indexing the intervals of stratification.

3. **Step 3)** At \( t = 0 \), the initial states of all components are sampled, according to their up/down probabilities in random sampling manner.
Step 4) Advance the time sequence of component $i$ by sampling an up or down time duration using the following expressions:

$$t_{up}^i = F_i^{-1}\left(\frac{L_{ij} - U[0,1]}{n_i}\right)$$

(6)

$$t_{down}^i = G_i^{-1}\left(\frac{L_{ij} - U[0,1]}{n_i}\right)$$

(7)

where $L_{ij}$ is the $j$th element of stratification matrix $L_i$.

Step 5) Incorporate load demand fluctuations using their historical time series.

Step 6) Identify the load curtailment hours for each simulated time duration based on DC optimal power flow formulation.

Step 7) Evaluate reliability indices for yearly chronology, as follows:

$$I = \frac{1}{N} \sum_{k=1}^{N} X_k$$

(8)

where $I$ is the resultant index; $N$ is the number of sampled batches (i.e., years); $X_k$ is the index calculated from batch $k$.

IV. CASE STUDIES

The performances of sequential MC and LHS simulations are investigated on IEEE RTS-79 [19]. The configuration of this system is shown in Fig. 4. It contains 38 transmission lines, 24 buses, 17 load buses, and 10 conventional generating units. The total generation capacity is 3405MW, and system peak load is 2850 MW. The load profile of the peak winter week is used as load chronology in the sequential simulation. MATPOWER program [20] is partially used all the simulations. All studies are conducted on a PC with Intel Xeon CPU 2.53 GHz and 12.0 GB of RAM.

![Fig. 4. Diagram of IEEE RTS-79.](image-url)

In each case, 500 batches of a yearly sequence are produced and reliability indices such as LOLP, EPNS, LOLF, and LOLD are calculated. Variances of these reliability indices, computed from the 500 batches, are used as the performance indices. Usually, the confidence interval is used as the stopping criteria. Lower variance indicates narrower confidence interval. As a consequence, an index with lower variance will converge faster than the one with higher variance. Reliability indices computed from conventional sequential MC simulation using a considerably larger batch size of 2000 are used as the benchmark solutions. In the experiments, simulation accuracies of both MC and LHS can be reflected through comparing indices achieved from each specific simulation to those benchmark solutions.

The variance reductions gained by LHS are found as:

$$Var_{reduction} = \frac{Var_{LHS} - Var_{MC}}{Var_{MC}} \times 100\%$$

(9)

where $Var_{MC}$ and $Var_{LHS}$ are the variances obtained from 500 batches of simulations with MC and LHS, respectively.

A. Experimental Results

Case 1 – Exponentially distributed down time: The down time distribution of each generator is assumed to be

![Fig. 5. Illustration of down time distributions of a conventional generator.](image-url)
exponentially distributed. The resulting indices calculated from 500 batches of LHS and MC are shown in Table I, along with the benchmarks which are computed from 2000 batches of MC simulation. Table II presents variances of indices yielded by MC and LHS-based sequential simulations, as well as the variance reductions achieved by LHS.

### Table I

<table>
<thead>
<tr>
<th>Indices</th>
<th>Benchmarks</th>
<th>MC</th>
<th>LHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>0.01366</td>
<td>0.01383</td>
<td>0.01295</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>111.554</td>
<td>113.487</td>
<td>111.925</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>26.5833</td>
<td>26.8237</td>
<td>25.3959</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>4.2927</td>
<td>4.3419</td>
<td>4.3414</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Indices</th>
<th>Variances</th>
<th>Variance Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>5.46e-5</td>
<td>-39.575%</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>2640.9</td>
<td>2097.7</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>115.48</td>
<td>55.091</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>1.5412</td>
<td>1.2878</td>
</tr>
</tbody>
</table>

Case 2 – Gamma distributed down time with shape parameter equal to 2: Gamma distributed downtime with shape parameter equal to 2 is applied to all generators in this case. Tables III and IV, respectively, show the accuracy and variance comparisons of indices obtained with MC and LHS simulations.

### Table III

<table>
<thead>
<tr>
<th>Indices</th>
<th>Benchmarks</th>
<th>MC</th>
<th>LHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>0.01365</td>
<td>0.01318</td>
<td>0.01299</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>114.416</td>
<td>112.785</td>
<td>115.207</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>26.6012</td>
<td>26.0857</td>
<td>25.3349</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>4.3432</td>
<td>4.2792</td>
<td>4.3913</td>
</tr>
</tbody>
</table>

### Table IV

<table>
<thead>
<tr>
<th>Indices</th>
<th>Variances</th>
<th>Variance Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>4.32e-5</td>
<td>-28.058%</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>2341.26</td>
<td>2132.40</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>86.250</td>
<td>56.892</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>1.3513</td>
<td>1.3344</td>
</tr>
</tbody>
</table>

Case 3 – Gamma distributed down time with shape parameter equal to 10: Each generator also has gamma distributed down time but with shape parameter equal to 10. The comparisons between index accuracies and variances are shown in Tables V and VI, respectively.

### Table V

<table>
<thead>
<tr>
<th>Indices</th>
<th>Benchmarks</th>
<th>MC</th>
<th>LHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>0.01340</td>
<td>0.01298</td>
<td>0.01287</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>114.086</td>
<td>112.725</td>
<td>118.329</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>26.3460</td>
<td>25.6196</td>
<td>24.6901</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>4.3462</td>
<td>4.3375</td>
<td>4.5011</td>
</tr>
</tbody>
</table>

### B. Performance Analysis

The results of accuracy seen in Table I, Table III and Table V demonstrate that the LHS-based sequential simulation approach is able to provide indices very close to the benchmark solutions for each case. This effect demonstrates that LHS performs well in terms of simulation accuracy.

The results in Table II, Table IV and Table VI can be observed to examine the variance reduction effects. In Case 1 with exponentially distributed down time, the LHS-based simulation achieves the highest variance reduction of 52% for LOLF, while the variance reductions for LOLP, EPNS and LOLD are 40%, 21% and 16%, respectively. This effect implies that the LHS approach can capture the shape of exponential distribution much more effectively than sequential MC simulation. In Case 2 and Case 3, the applied gamma distributions pose shapes of relatively longer tails than the exponential distribution in Case 1. The LHS approach reduces the variances of indices with similar degrees for both Case 2 and Case 3. However, the degrees of these reductions are found lower than that in Case 1. For example, the variances of EPNS and LOLD are not as significantly reduced as in Case 1.

In addition, to see the effect of computing time reduction brought by LHS, Table VII shows simulation results of Case 1 using a convergence criterion based on coefficient of variation (COV). Convergence criterion is satisfied when COV values of reliability indices are below a very small tolerance δ, where we set δ=0.03. For both MC and LHS, simulation is repeated 10 times to handle randomness; average results of reliability indices, number of simulated years, as well as computing time are given; COV values are shown in brackets below indices.

### Table VII

<table>
<thead>
<tr>
<th>Indices</th>
<th>MC</th>
<th>LHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>0.01343</td>
<td>(0.0300)</td>
</tr>
<tr>
<td>EPNS (MW)</td>
<td>113.283</td>
<td>(0.0249)</td>
</tr>
<tr>
<td>LOLF (occ./y)</td>
<td>26.1182</td>
<td>(0.0211)</td>
</tr>
<tr>
<td>LOLD (h)</td>
<td>4.3288</td>
<td>(0.0153)</td>
</tr>
</tbody>
</table>

No. of Simulated Years: 413 (Reduction over MC=42%) CPU Time (min): 17.20 (Reduction over MC=35%)

Compared to MC, LHS reaches convergence considerably faster while providing similar reliability indices. It brings a reduction of 42% in sample size, and thus a reduction of 35%
in computing time. The reduction in computing time is found lower than that in sample size. This is because LHS requires extra computational effort in stratifying distribution functions, and therefore uses longer time for processing each yearly sequence – the average time is 0.471 min/year in LHS, while it is 0.416 min/year in MC.

V. CONCLUSIONS

Sequential simulation is a comprehensive tool for power system reliability evaluation due to its capability of providing frequency and duration indices as well as incorporating time-dependent characteristics. A drawback of this technique is its slow convergence rate due to the high variance of estimated indices. It usually requires much longer computing time than non-sequential simulation. This paper proposes an LHS approach for sequential simulation to accelerate the reduction of variances for estimated indices. Compared to random sampling approach, the LHS technique has an advantage of providing a higher representativeness for random variables while retaining an ability to preserve their input distributions.

The LHS-based sequential simulation approach is developed for composite system reliability assessment. Results demonstrate that the proposed approach outperforms the conventional sequential simulation approach. It considerably reduces the variances of estimated indices while providing an acceptable level of computing accuracy. Moreover, it is also shown that its performance, in terms of variance reduction, would vary depending on the input probability distributions of random parameters. The performance under exponentially distributed time durations is found better than that under gamma distributed time durations.

REFERENCES