Abstract—This paper presents a real-time voltage stability monitoring method based on estimation of maximum power transfer. The approach solely requires information from PMUs installed at concerned load buses at substations in addition to available information of system topology and load power obtained from SCADA/EMS or state estimator. The proposed algorithm estimates the Thevenin impedance seen at the load bus and consequently the maximum loadability. This power transfer limit is used as an indicator for voltage stability assessment when compared to the load power. It is emphasized that only topology of the considered sub-system is needed, not the whole network. This feature makes the method viable for online implementation since the computation requirement is insignificant. In principle, the proposed methodology is not limited to specific grids in terms of voltage level, but it is especially suitable for sub-transmission networks, where loads are supplied by bulk power source and voltage instability is imposed by long transmission lines. The proposed approach is validated by simulations on the Norwegian transmission grid.

Keywords—Admittance matrix, maximum loadability, PMU, Thevenin impedance, voltage stability.

I. INTRODUCTION

The development towards increasing use of renewable energy sources and smart grid applications represent a paradigm shift in power system operation. The transmission system must be more flexible and well monitored to adapt to intermittent generation and dynamic load pattern. This scenario requires new real time monitoring tools to timely identify operation limits. For those systems with long and heavily loaded transmission lines, voltage stability is one of the factors that set limits on power transfer. There have been many works on voltage stability, but most of them are suitable for off-line applications. The method of continuation power flow [1] requires repetition of load flow calculation of the whole system to compute the point of maximum power transfer. This task is time-consuming, especially in large power system. In addition, the method does not include dynamic system behaviors. The other approach proposed by [2] is to analyze the sensitivity of the Jacobian matrix. The study figures out that as the system is approaching the “nose point”, the eigenvalue of this Jacobian matrix is going to zero. Hence, eigenvalues of this matrix can be used to evaluate voltage stability condition. In order to implement this method for online application, we need fast and full observer of the whole system, which is not obtainable at present.

Phasor measurement units (PMUs) open up opportunities for new research in online applications in monitoring, control and wide area protection. Regarding online voltage stability assessment, [3] introduces a novel approach based on the Thevenin impedance and the impedance matching criterion. The method uses phasor measurements at the load bus to estimate the Thevenin impedance seen at that one. Comparing the load impedance with the Thevenin impedance gives the indication of voltage instability. A further development in [4] finds the Thevenin impedance with repetitive process of estimation and correction until the algorithm converges. Reference [5] evaluates voltage stability, based on the whole system topology and information of all load power, by forming “single-port circuit” for each concerned load. From this circuit, a new impedance is computed. Its function is similar to that of the Thevenin impedance in terms of assessing voltage stability.

This paper proposes another method to find the Thevenin impedance seen at load buses. Similar to [5], the approach requires system topology and load power. However, it needs only information of the concerned sub-system. The study also examines the case, in which estimation of equivalent impedance based on the Thevenin theorem fails to evaluate the maximum loadability.

The rest of this paper is organized as follows. Section II reviews voltage stability assessment based on the Thevenin impedance, followed by the proposed method in Section III. The next section describes a method for system reduction and identification of the boundary of the concerned sub-system. Section V examines the effect of active power limitation of generators. Section VI shows simulation results, and finally conclusions are drawn in Section VII.

II. REVIEW OF VOLTAGE STABILITY MONITORING BASED ON THE THEVENIN IMPEDANCE

Given a simple two-bus system as shown in Fig. 1, where $E_{Th}$ is a voltage source behind $Z_{Th}$, an impedance denoting a transmission line. The load is considered as the impedance $Z_L$, which is determined by the ratio of the voltage $V_L$ to the current $I_L$. Regardless of whether active or reactive power is extracted from the grid at the load bus, it can be easily shown that the maximum loadability is reached when the load impedance magnitude is equal to that of the impedance $Z_{Th}$,

$$|Z_L| = |Z_{Th}|$$  \hspace{1cm} (1)
Consequently, by comparing the two impedances, we can establish the Impedance Stability Index (ISI) at the load bus as

\[ ISI = \frac{|Z_{th}|}{|Z_L|}. \]  

(2)

Furthermore, when the \( Z_{th} \) is known, \( E_{th} \) can be identified by

\[ E_{th} = V_L + I_L Z_{th}, \]  

(3)

where \( V_L \) and \( I_L \) are voltage and current phasors of the load.

The maximum power transfer occurs at the instant when (1) is fulfilled. Therefore, this maximum loadability point is estimated by [5]

\[ S_{max} = \frac{|E_{th}|^2}{\left( - \text{imag}(Z_{th}) \sin \delta + \text{real}(Z_{th}) \cos \delta \right)^2}, \]  

(4)

where \( \delta \) is the load power angle.

After crossing the maximum loadability limit, the load might come to voltage collapse if it contains constant power or constant current load components. Obviously, different load models exhibit various voltage collapse mechanisms, depending on the specific load composition. Nonetheless, operation beyond the limit is not expected. From voltage stability monitoring point of view, it is reasonable to assume that \( G_1 \) has unlimited active power. From Fig. 2, the electric equivalent circuit of this system can be depicted in Fig. 3, where \( T_1, L_1, L_2 \) and \( L_1 \) are represented by impedances \( Z_{T_1}, Z_L, Z_{L_1}, \) and \( Z_{L_2} \). For the sake of illustration, shunt admittances of Line 1 are neglected.

According to the Thevenin theorem, the equivalent impedance of the system seen at bus #2 is

\[ Z_{th} = Z_L + Z_{L_1} \frac{Z_{T_1}}{Z_{L_1} + Z_{T_1}} \]  

(6)

In addition, from Fig. 2, the admittance matrix of the grid is (not including the load)

\[ Y = \begin{bmatrix} 1 & 1 & -1 \\ 1 & \frac{Z_{T_1}}{Z_{L_1}} & -1 \\ -1 & 1 \end{bmatrix} \]  

(7)

Let us modify the diagonal elements of the matrix \( Y \) by adding the load admittance to establish a new matrix.
Define the impedance matrix $Z_{eq}$ as the inversion of $Y_{eq}$.

$$Z_{eq} = Y_{eq}^{-1}.$$  \hspace{1cm} (9)

The diagonal element in the second row and column of $Z_{eq}$ is

$$Z_{eq}(2,2) = Z_L + \frac{Z_{L1}Z_T}{Z_{L1} + Z_T},$$  \hspace{1cm} (10)

which is by definition equal to the Thevenin impedance seen at bus #2 in (6) (in general, the diagonal element in the row and column of the impedance matrix is the Thevenin impedance seen at the corresponding row and column of the admittance matrix.

In summary, the admittance matrix is modified by adding all load admittances to the corresponding diagonal elements. Then matrix inversion is implemented. For those concerned buses, the Thevenin impedances are found by the modification in (12). By doing this way, we need to invert the admittance matrix only once.

IV. SYSTEM SIMPLIFICATION

A. Determination of Sub-System Boundary

As described in Section III, the proposed method requires the system topology of the whole system. It might be impractical in a real power system and demand huge computation effort, especially for online or near-online applications. However, voltage stability is commonly considered more of a local problem. Voltage collapse occurring at a particular load bus is related to constraints of power lines, transformers and generators that are feeding the load. Those generators that are electrically far from the concerned loads have very small or no impact on the study area. Obviously, it is not necessary to include these generators. For a particular study area that is prone to voltage instability, we can determine a boundary that fulfills the condition that generators and boundary nodes secure sufficient active power supply. Consequently, voltage collapse in the study area is caused by power lines, generators and loads in the concerned area; the rest of the system can be neglected. The idea is illustrated in Fig. 5. Assume that the study area is susceptible to voltage collapse due to high load demand or loss of transmission lines. When the sub-system is approaching voltage instability, bus B1 and B2 can be considered as the boundary nodes if voltage at bus B1 and B2 are around nominal value since they are strong buses. This allows us to simplify the study sub-system to the small area as shown in Fig. 5. If bus B1 and B2 do not meet the aforementioned assumption, the boundary should be expanded to bus B3, B4 and B5. The boundary selection goes on until the condition is met. This process can be done with off-line analysis.

An example is shown in Fig. 6. The sub-system is supplied by the two substations, which have on-load tap changers (OLTCs) to keep secondary voltage at rated value. Assume that the connection point to the transmission grid is strong. Voltage collapse can be initiated by trip of lines or gradual increase of load at sub-transmission level. If we assume that the secondary voltage at the substations (at sub-transmission side) is at rated value because of OLTC operation, the two substations function as infinite buses, feeding the sub-transmission grid. Clearly, it is not needed to consider the transmission side and the rest of system behind these substations when we focus on monitored sub-system.

B. Simplification of load area

In the study area, generally there are some radial branches that solely draw power. If these branches are not subjects for voltage collapse monitoring, their topology and load impedances are not necessarily integrated into the admittance matrix. In this case, it is sufficient to install one PMU at the sending end to measure load impedance of the whole branch behind. For example, at the bus #1 in Fig. 6, only one PMU is needed to represent the load area behind. This technique
reduces the size of the admittance matrix of the concerned sub-system.

Figure 5. Example of boundary selection.

Figure 6. An example of sub-transmission system and simplification of load area.

V. IMPACT OF ACTIVE POWER DISPATCH

A. An example of a simple system

Let us consider an electric circuit shown in Fig. 7. The load represented by the impedance $Z_{\text{Load}}$ is fed by the two voltage sources V1 and V2. When the load grows, the currents of V1 and V2 will increase to meet the new demand. The share of this additional power is determined by the two impedances $Z_{L1}$ and $Z_{L2}$. If the load is kept increasing, the maximum power transfer will occur when the load impedance is equal to $Z_{L1}$ in parallel with $Z_{L2}$. In this case the Thevenin impedance and power matching theory produce correct estimation of maximum loadability.

Figure 7. An electric circuit with two voltage sources.

Now, let us replace the two voltage sources V1 and V2 by the two generators G1 and G2 respectively. If these generators can keep the terminal voltage constant, have unlimited power and behave like voltage sources, the estimation result is the same as the previous case. However, if G2’s rated power is much smaller than that of G1, the equivalent impedance obtained from the Thevenin theorem is no longer valid to estimate maximum power transfer.

Fig. 8 depicts the Thevenin impedance, load power and impedance when the load is projected to maximum power transfer. In this simulation, generator G2 produces 10MW and it is able to keep its terminal voltage constant, G1 covers active power demand from the load. The two impedances $Z_{L1}$ and $Z_{L2}$ are 60j and 50j (Ω) respectively; voltage level is 110kV. From the Thevenin theorem, the Thevenin impedance is $Z_{L1}$ in parallel with $Z_{L2}$. As can be seen from Fig. 8, at the instant the load reaches its peak, load impedance is much higher than the Thevenin impedance. Obviously, estimation of maximum power transfer based on the Thevenin impedance is no longer a suitable approach.

Figure 8. Load power, impedance and Thevenin equivalence without modification.

One can notice that the factor that limits the active power generated by G2 is not the impedance $Z_{L2}$ but the rated power of the generator itself. In order to continue using the Thevenin equivalence-based method, we propose to model the limitation of active power of G2 as a reactance. The idea is to find a fictitious reactance based on G2 rated power, connecting G2 and the load. This reactance replaces the physical impedance $Z_{L2}$. After that the proposed method in Section III can be used to estimate the Thevenin impedance at the monitored bus. The modelling approach is presented in the next paragraph.

Consider the circuit in Fig.1, where $Z_{\text{Th}}$ now becomes a resistance $R_{\text{th}}$ and the load is resistive, represented the resistance $R_L$. Assume that the maximum loadability is $P_{\text{max}}$. At the maximum power transfer point, $R_{\text{th}}$ and $R_L$ are equal. Then it is simple to compute $R_{\text{th}}$ by

$$R_{\text{th}} = \frac{E^2}{P_{\text{max}}} \quad (13)$$

Next, pertaining Fig. 7, we replace $Z_{L2}$ by a fictitious pure reactance $X_{G2}$, which represents the rated power of the generator. We propose to use the value of $R_{\text{th}}$ in (13) to determine the size of $X_{G2}$ by

$$X_{G2} = \frac{V^2}{4P_{G2}} \quad (14)$$
where \( V_2 \) and \( P_{G2} \) are the rated voltage and active power of generator \( G2 \). The modified Thevenin impedance is now \( Z_{L1} \) in parallel with \( X_{G2} \).

With this adjustment, the modified Thevenin impedance along with load impedance and power of the previous simulation is shown Fig. 9. It can be seen that better estimation is obtained. The error of the estimated impedance is about \( 3\Omega \) (6.34%). However, this error results in a minor mismatch in maximum power estimation as depicted in Fig. 10.

![Figure 9. Load power, impedance and modified Thevenin equivalence.](image)

Figure 10. Load power versus estimated maximum loadability after modification.

**B. An example of a meshed system**

In meshed power systems, power from generators is diverted through various paths to serve different loads. To assess voltage stability at one particular load bus, it is not viable in real time to determine the origin of incoming power. There is no direct link between generators and the considered bus as the case we examined above. Therefore, it is necessary to establish direct connection between them. This obstacle is overcome by a system simplification technique that is coming to establish direct connection between them. This obstacle is overcome by a system simplification technique that is coming.

The idea is to retain all generators and the concerned load bus and simplify the rest. Assume that bus \#8 in Fig. 11 is the monitored bus; generator \( G1 \) represents an infinite system and \( G2 \) is a small generator. As presented in Section III, we can establish the modified 8x8 admittance matrix as

\[
Y_{eq} = \begin{bmatrix}
Y_E & Y_{EB} \\
Y_{EB} & Y_I
\end{bmatrix}
\]

where \( Y_E \) contains all generators and the considered bus (#1, #7 and #8); \( Y_I \) contains the rest of the system that we exclude and \( Y_{EB} \) is the coupling sub-matrix of the two sub-systems.

Applying Gaussian elimination technique on \( Y_{eq} \) to eliminate \( Y_{EB} \) below the diagonal until it yields

\[
Y_{eq} = \begin{bmatrix}
\cdot & \cdot \\
\cdot & \cdot
\end{bmatrix}
\]

where \( Y_{eq} \) is a 3x3 equivalent matrix, which is quite dense.

![Figure 11. A small 8-bus meshed grid.](image)

Based on \( Y_{eq} \), we can construct a simplified system with three retained buses as seen in Fig. 12. It is noticed that there are three impedances that are of interest; they are \( Z_{I1b} \), \( Z_{EB} \) and \( Z_{78} \). Since \( G1 \) is infinite bus, \( Z_{I1b} \) functions as the constraint to limit power transfer from \( G1 \) to bus \#8. The impedance \( Z_{78} \) is the equivalent load allocated to bus \#8. This impedance is important to the Thevenin impedance estimation since it contains loading condition of the simplified sub-system. The \( Z_{78} \) is the coupling impedance between bus \#7 and \#8; it is used to determine the virtual power flow between bus \#7 and \#8, which is further described in next paragraph. The other impedances are the result of simplification process, and they do not affect the estimation of the Thevenin impedance seen at bus \#8; therefore they are neglected.

![Figure 12. Equivalent meshed grid after Gaussian elimination.](image)

To evaluate voltage stability at bus \#8, there are two cases regarding the power flow between bus \#7 and \#8 in the new simplified system:

- Power flowing from bus \#8 to \#7 (Case 1): remove bus \#7 and impedances connected to this bus. Add new load impedance \( Z_{78\text{new}} \) connected to bus \#8, which extracts the same power from bus \#8. The final reduced system is shown in Fig. 13.

- Power flowing from bus \#7 to \#8 (Case 2): the system is similar to the one in Fig. 7. Therefore, (14) is used to determine the impedance \( Z_{78\text{new}} \) that limits the power transferred from bus \#7 to bus \#8, in which \( V_2 \) is the rated voltage and \( P_{G2} \) is the virtual calculated power injected to bus \#8 from bus \#7. The final reduced system is shown in Fig. 14.

After above simplification process, it is possible to apply proposed method presented in Section III to assess voltage stability at the monitored bus. Specifically, the Thevenin
impedance in Case 1 is $Z_{18}$ in parallel with $Z_{88}$ and $Z_{87\text{new}}$; meanwhile it is $Z_{18}$ in parallel with $Z_{88}$ and $Z_{87\text{new}}$ in the Case 2.

![Figure 13. Equivalent grid when power flows from bus #8 to #7.](image)

Figure 13. Equivalent grid when power flows from bus #8 to #7.

![Figure 14. Equivalent grid when power flows from bus #7 to #8.](image)

Figure 14. Equivalent grid when power flows from bus #7 to #8.

To illustrate the aforementioned proposed approach, the test is conducted on the system shown in Fig. 11. Generator G2 produces only 5MW, and G1 covers all load demand. Like the classical analysis, the load at bus #8 modelled as constant impedance rises until it hits the power limit. Fig. 15 depicts the load power and the estimated maximum power. The green curve is the maximum loadability estimated from the method proposed in Section III, not taking the impact of active power dispatch into account. Since active power of G2 is much smaller than that of G1, this algorithm fails to estimate the maximum load power. The red curve however shows better estimation since it utilizes the modification described above. In this case, G2 is not considered as bulk power source. The fictitious power flow bus #7 and bus #8 is computed in real time and decides the size of the fictitious impedance. They are depicted in Fig. 16.

![Figure 15. Load power at bus #8 and estimated maximum loadabilities based on modified and unmodified Thevenin impedance estimation.](image)

Figure 15. Load power at bus #8 and estimated maximum loadabilities based on modified and unmodified Thevenin impedance estimation.

![Figure 16. Fictitious power flow from bus #7 to #8 and impedance $Z_{87\text{new}}$.](image)

Figure 16. Fictitious power flow from bus #7 to #8 and impedance $Z_{87\text{new}}$.

In summary, if generators in the study area contribute their power quite equally to the demand. The modification described in this section is not needed; the method proposed in Section III is sufficient to estimate the Thevenin impedance. Otherwise, if there are small generators, it is needed to adjust the estimation approach as presented in this section.

VI. SIMULATION RESULTS

The study sub-system is part of the Norwegian transmission system. This 30-bus area has voltage level of 132kV, supplied by a 420/132kV substation and a large power plant connected to the 132kV grid. These sources do not have limits in meeting the load demand; they share quite equally load burden in this area. However, long 132kV lines impose constraint on maximum power transfer, and consequently voltage collapse. In this case study, only 30 buses in this area are taken into the admittance matrix to estimate the Thevenin impedance, but the simulation in PSS/E is run on the model of the whole Norwegian transmission system. The illustration in this article is to monitor voltage stability at the weakest bus in the study area so that only one PMU is needed to measure phasors of load voltage and current of this concerned bus. The estimation of the Thevenin impedance is based on load power of other buses, which can be obtained from state estimator in practice.

In the first test, the load at monitored bus is classically increased until it crosses the maximum power transfer point. All the loads are treated as constant power. The impedances and power obtained from the load and estimation scheme are plotted in Fig. 17 and Fig. 18. At $t = 11.5s$, the load hits the maximum loadability, which is also equal to the maximum power transfer estimated from the proposed method; therefore the considered load is no longer on the rise, but it instead starts going down. At this instant, the load impedance, which is in decline, meets the estimated Thevenin impedance seen at this bus. Obviously, the method shows quite good estimation of the Thevenin impedance and maximum loadability.

![Figure 17. Estimated Thevenin impedance versus the monitored load impedance, scaling only considered load.](image)

Figure 17. Estimated Thevenin impedance versus the monitored load impedance, scaling only considered load.

![Figure 18. Estimated maximum power transfer and the monitored load power, scaling only considered load.](image)

Figure 18. Estimated maximum power transfer and the monitored load power, scaling only considered load.

In the second test case, all the loads in the study area are expanded. They are modeled as constant power. By monitoring the load and Thevenin impedances during the load expansion,
voltage collapse has been detected at the weakest bus; meanwhile the rest is still within its limit. Fig. 19 and Fig. 20 present the performance of the proposed algorithm at the concerned bus. It is noticed that during the load increase process the Thevenin impedance drops slightly since load impedances of the whole system are decreasing; however the estimated maximum loadability shows a pretty large drop. This is the impact of modelling the load as constant power. In this case, the actual maximum power transfer is revealed when the load is approaching its limit.

Figure 19. Estimated Thevenin impedance and the impedance of monitored load, expanding all loads in the study area.

Figure 20. Estimated maximum power transfer and the monitored load power, expanding all loads in the study area.

In the third case, one of the important lines carrying about 120MW is tripped. All the loads in the study area are modeled by the model CLOD in PSS/E, which is comprised of 80% of large motor, 20% of small motor and 20% of constant power load. After losing the line at t = 2s, all the loads try to restore their power. That leads to voltage collapse at t = 4.7s at the weakest bus as observed in Fig. 21 and Fig. 22. Due to the topology change at t = 2s, the Thevenin impedance seen at the concerned bus varies slightly from 6.8 pu to 6.9 pu. During the load restoration period, the Thevenin impedance has a minor drop due to the decreasing impedance of the other loads in the study area. Interestingly, the estimated maximum loadability also falls sharply and meets load power just before the voltage collapse. In this case, the magnitude of the Thevenin impedance is not a decisive factor that determines the strength (maximum loadability) of the system.

Figure 21. Estimated Thevenin impedance versus the monitored load impedance, after tripping one heavily loaded line.

VII. CONCLUSION

This paper has presented a methodology to assess voltage stability at load buses based on estimation of the Thevenin impedance and the maximum loadability. Based on the information of system topology and load power, which are available from existing state estimator, the Thevenin impedance is estimated. Therefore, PMU installation is needed only at concerned buses to compute the maximum loadability, which functions as the main indicator for voltage stability monitoring. It has been demonstrated that it is sufficient to take into calculation only part of the system that is vulnerable to voltage instability. This makes the proposed method feasible for practical applications, especially for sub-transmission system where load are normally supplied by strong connection to the transmission level and voltage stability problem is imposed by the power lines.

It is noted that the Thevenin equivalence is no longer a suitable method when generators behave differently from voltage source, especially in terms of active power limit. A new impedance whose function is similar to that of Thevenin impedance in introduced to calculate maximum power transfer. Under this circumstance, PMU installations at small generators are needed in addition to concerned load buses.

Finally, the proposed method has been validated by simulations on the simple 8-bus system and the Norwegian transmission system.

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