PHASOR-MEASUREMENT-BASED VOLTAGE STABILITY MARGIN CALCULATION FOR A POWER TRANSFER INTERFACE WITH MULTIPLE INJECTIONS AND TRANSFER PATHS

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Abstract - For complex power transfer interfaces or load areas with multiple in-feeds, we present a method for phasor-measurement-based calculation of voltage stability margins. In the case of complex transfer paths with multiple injections, a radial system approach may not be sufficient for voltage stability analysis. Our approach provides voltage stability margins considering the full fidelity of the transfer paths.

In this paper, we extend a previously proposed phasor-measurement-based approach [1] and apply it to a voltage stability-limited power transfer interface using synchronized phasor measurements from loss-of-generation disturbance events. Previous work employed a simple radial system [2] or modeled a power transfer interface using only one generator [1]. In our approach, we use the PMU data to model multiple external injections that share the power transfer increase, and we employ a modified AQ-bus power flow method to compute the steady-state voltage stability margins [3]. We demonstrate the method using real PMU data from disturbance events in the US Eastern Interconnection.

Keywords - voltage stability, phasor measurements, stability margins

1 Introduction

This paper is primarily concerned with the use of phasor measurement unit (PMU) data for voltage stability margin calculation. Because of the increasing number of PMU installations, applications of synchrophasor data for voltage stability are of interest to system operators to mitigate the risk of major blackouts [4, 5, 6]. Loss-of-generation events can cause voltage collapse and cascading failures by depleting the reactive power in critical areas, overloading transmission lines, and/or causing sudden power transfer shifts. For these events, we can observe the dynamic behavior of the system power flows and voltages using high-sampling rate phasor measurements. The power flow and voltage sensitivities from the phasor measurement data can provide valuable information regarding the system condition.

Voltage stability analysis typically requires significant computation which hinders real-time applications. One approach is to reduce the system to a radial network, from which the maximum loadability can be readily computed [7]. This idea has been applied in previous work [2] for radial-type transfer paths. However, a complex transfer path with multiple injections cannot always be reduced to a radial network. In other cases, a load area can have multiple in-feeds that increase the complexity of the voltage stability analysis. Previously in [1], we analyzed part of a meshed transfer path using PMU data from one substation, but the lack of PMU coverage limited our analysis to one Thévenin equivalent generator to represent the increased power transfer. In that work, we did not compute the PV curve to the maximum loading condition due to the ill-conditioned Jacobian matrix of the power flow solution.

In this paper, we have better PMU coverage of the same transfer interface with six PMUs at multiple substations, and we construct Thévenin equivalents for all of the external injections of the transfer path to maintain the full fidelity of the transfer path. We extract voltage variations from the phasor measurement data to construct Thévenin equivalents and quasi-steady-state models for the external injections, including FACTS controllers such as SVCs and STATCOMs. Selected PMU data points are used to estimate the parameters of the external injection models. Finally, we use a newly developed AQ-bus power flow method to compute the steady-state voltage stability margins quickly and efficiently [3]. Our approach is demonstrated using PMU data from loss-of-generation events on the Central New York power system. The PMU data for
one such event is shown in Fig. 1, where we plot the variation of the bus voltage magnitude versus interface power transfer ($PV$ curve).

![Figure 1: PV plot using PMU data for a loss-of-generation event.](image)

The rest of the paper is organized as follows. In Section 2, we discuss the Central New York power system and disturbance events. In Section 3, we present the external injection models and the calculation of their parameters. In Section 4, we extrapolate the voltage stability margins using the computed external injection models, and we conclude in Section 5.

## 2 Central NY Power Transfer Path

The first stage in our analysis is use a phasor-measurement-based state estimator to correct errors and compute unmeasured quantities in the observable portion of the network [8]. The observable network including the transfer path is shown in Fig. 2, and external injections are shown as arrows into or out of the network. The transfer path of interest consists of Lines 1–2 and 1–3, where power generally flows from left to right from Bus 8 to the external system beyond Bus 2.

The transfer path will show an increase in flow toward Bus 2 after a loss-of-generation event occurs in the external system. Because there are other paths to the external system, the transfer path will only supply a portion of the lost generation. We study two such disturbances, which occurred during different system operating conditions. The events are listed in Table 1, along with the amount of lost generation and the post-contingency increase in power flow along the transfer path.

![Figure 2: Central NY transfer path model.](image)

### Table 1: Loss-of-generation events in the external system and post-contingency interface flows.

<table>
<thead>
<tr>
<th>Name</th>
<th>External gen. loss</th>
<th>$\Delta P_{flow}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event 1</td>
<td>800 MW</td>
<td>300 MW</td>
</tr>
<tr>
<td>Event 2</td>
<td>700 MW</td>
<td>250 MW</td>
</tr>
</tbody>
</table>

In both cases, the increased power transfer is supplied by multiple generators. Unlike our previous work [1], we treat each generator separately using better PMU data coverage and a robust voltage stability solution method.

## 3 External Injection Models

### 3.1 Thévenin Equivalent Injection Model

The extent of the phasor-observable network is determined by the available phasor measurements [9], and the external portions of the system are unobservable. To build a model for voltage stability analysis, we model the external injections on the boundaries of the observable network using their quasi-steady-state equivalents. We retain the full fidelity of the phasor-observable network because it is quite small and there is little benefit in reducing it.

In the case of the Central New York power system, we use a Thévenin equivalent generator model for the injections at Buses 1, 2, 3, 7, and 8. The SVC at Bus 1 performs fast voltage regulation, so it is governed by its quasi-steady-state droop characteristic. The injections at Buses 4, 6, and 9 are loads with little participation in the disturbance, so we model them as fixed PQ loads. As our next step, we use the PMU data to compute the parameters of these external injection models with a nonlinear least-squares formulation.

Each of the Thévenin equivalent injection models consists of a stiff voltage source behind a reactance, as shown in Fig. 3.1. The voltage and current phasor quantities at the injection bus provide the means to estimate the parameters of the Thévenin equivalent model. We choose the injection bus voltage angle to be the reference angle to simplify the calculation.
We use the phasor quantities to compute the Thévenin voltage $E'$ and reactance $X'$ using the equations

$$E' \cos \delta = V - X' I \sin \phi$$  \hspace{1cm} (1)$$
$$E' \sin \delta = V + X' I \cos \phi$$  \hspace{1cm} (2)$$

where $\delta$ is the machine angle, $V$ is the voltage magnitude at the injection bus, and $I \angle \phi$ is the current injection phasor. Note that $V$ and $I \angle \phi$ are either measured or computed using the state estimator, the unknown quantities $E'$ and reactance $X'$ are taken to be fixed values, and the unknown angle $\delta$ is allowed to vary between measurements. Thus we have 2 constant unknowns ($E'$, $X'$) and for each measurement, we add 2 equations and 1 additional unknown ($\delta$).

For a set of $N$ measurements, we can formulate a non-linear least-squares estimation problem using (1) and (2), such that

$$\min_x f(x) = \left\| \begin{array}{c}
E' \cos \delta_1 - V_1 + X' I_1 \sin \phi_1 \\
E' \sin \delta_1 - X' I_1 \cos \phi_1 \\
\vdots \\
E' \cos \delta_N - V_N + X' I_N \sin \phi_N \\
E' \sin \delta_N - X' I_N \cos \phi_N
\end{array} \right\|_2$$  \hspace{1cm} (3)$$

where $x = [E' \ X' \ \delta_1 \ \cdots \ \delta_N]^T$, and $\delta_k$, $V_k$, $I_k$, and $\phi_k$ are the values corresponding to the $k$th data point. To solve the problem, we require at least as many equations as unknowns. In this case, there are $2N$ equations and $N + 2$ unknowns, so to satisfy the necessary condition we require at least two data points ($N \geq 2$). It should be noted, however, that the data points must represent at least two distinct operating points. Otherwise, there is not enough information to solve the least-squares problem.

Because we are assuming fixed voltage sources for the generators, we should avoid choosing data points during the period where the generator internal voltage can be varying, i.e., during the disturbance transients. In Fig. 3.1 we illustrate the selection of data points for computing the model parameters.

Thus the selected data points (highlighted in red) are drawn from the pre-disturbance and post-disturbance measurements, which represent two distinct operating points. For this study, the pre-disturbance data was not sufficient to calculate the Thévenin equivalent because it only covered one operating point. In practice, one can use additional pre-disturbance data covering multiple operating points to provide enough information to estimate the Thévenin equivalent parameters.

### 3.2 SVC Injection Model

The SVC in at Bus 1 is typically operated in voltage control mode. Because of the fast time constants of the SVC compared the PMU sampling rate (and multiple-cycle averaging effects of the PMU), we assume the SVC is in a quasi-steady-state and follows its voltage regulation droop characteristic, given by

$$I_{SVC} = \frac{V - V_{ref}}{\alpha}$$  \hspace{1cm} (4)$$

where $I_{SVC}$ represents the magnitude of the current injection of the SVC into the network [10]. We use the phasor measurements of voltage and output current to estimate the voltage reference $V_{ref}$ and droop $\alpha$. In this frame of reference, the current leads the voltage by 90 degrees, so a negative value indicates reactive power injection by the SVC. We formulate the least-squares estimation as the optimization problem

$$\min_{V_{ref}, \alpha} f(V_{ref}, \alpha) = \left\| \begin{array}{c}
(V_{ref} - V_1) - \alpha I_1 \\
\vdots \\
(V_{ref} - V_N) - \alpha I_N
\end{array} \right\|_2$$  \hspace{1cm} (5)$$

where $I_k = I_{SVC}$ for the $k$th measurement and $V_{ref}$ and $\alpha$ are assumed constant. Thus we have $N$ equations and 2 unknowns, so at least two measurements are required.

### 4 Voltage Stability Margin Calculation

We use power flow calculations with the computed external injections model to generate $P\!V$-curves for the transfer path, increasing power transfer across the interface at every iteration. We compare these new $P\!V$-curves from the model to the original phasor measurement data to validate the model and examine the system behavior as the power transfer increases. We then use the computed $P\!V$ curves to calculate the voltage stability margin using the maximum loading condition.

#### 4.1 Estimation of injection model parameters

Using the method described in the Section 3, we first compute the Thévenin equivalent injection models. These injections are located at Buses 1, 2, 3, 7, and 8. The calculated parameters are given in Table 2.
Most of the parameters are quite consistent between events. Because the Thévenin equivalent represents a group of generators, the status of remote generators can affect the values of the parameters.

The next step is estimating the SVC parameters $V_{ref}$ and $\alpha$ using (5) with the PMU data from Events 1 and 2. The estimated parameters are given in Table 3.

<table>
<thead>
<tr>
<th>Event</th>
<th>$V_{ref}$ (p.u.)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.037</td>
<td>0.0339</td>
</tr>
<tr>
<td>2</td>
<td>1.040</td>
<td>0.0325</td>
</tr>
</tbody>
</table>

Table 3: Estimated SVC parameters

We observe that the estimated parameters are consistent between the two events, which is expected because the SVC parameters are not changed frequently by the system operators. In Figures 5 and 6, we compare the estimated model to the PMU data and find a good match.

![Figure 5: Comparison of SVC model to PMU data (Event 1)](image)

![Figure 6: Comparison of SVC model to PMU data (Event 2)](image)

After computing the parameters for all the injection models, we can establish a reduced model for voltage stability margin calculation.

4.2 PV curve computation using the AQ-bus method

The power flow is computed using a system model that includes the full detail of the transfer path, with the Thévenin equivalents at the external injection buses (Buses 1, 2, 3, 7, and 8) and the SVC model at Bus 1.

We use the AQ-bus power flow method [3] to compute the $PV$ curves for the reduced model. The advantage of the AQ-bus method is that the Jacobian matrix singularity at the maximum loading condition is mitigated. In this approach, we choose an AQ bus and specify its voltage angle instead of active power. By increasing the angle separation between the swing bus and AQ bus, we indirectly increase the power flow to the AQ bus. Thus we run successive AQ power flows with increasing angle separation to compute the $PV$ curve.

For the Central NY system, we choose Bus 2 as the AQ bus to represent increasing power transfer to the external system. The additional power transfer is supplied by the Thévenin equivalent generators connected to Buses 1, 3, 7, and 8, in proportion to their sensitivity to power transfer increases. These sensitivities ($\beta$) are readily computed from the PMU data as the ratio

$$
\beta_i = \frac{\Delta P_i}{\Delta P_{transfer}}
$$

where $\beta_i$ is the sensitivity for the $i$-th generator, $\Delta P_i$ is the incremental power supplied by the $i$-th generator, and $\Delta P_{transfer}$ is the incremental power transfer across the interface. Using these sensitivities, we account for the fact that the generation loss is supplied by multiple generators over a meshed network.

Using data for each event, we compute $PV$ curves for Buses 1 and 8 by increasing the angle separation between Bus 8 (swing bus) and Bus 2 (AQ bus). We include the SVC with its droop model and equipment limits.

4.3 Voltage stability margin calculation

In Figs. 7 and 8, we plot the $PV$ curves for the system using PMU data from Events 1 and 2, respectively.
In these plots, the $x$-axis represents the incremental power flow across the interface and the $y$-axis represents the bus voltage magnitude. On the same axes, we plot the PMU data for comparison. From the plots, we can see that the model fits the data well. Note that the SVC reaches its equipment limits and saturates its output when the incremental power transfer reaches approximately 9 p.u., and the $PV$ curve becomes slightly steeper at this point.

For each case, we calculate the stability margin by detecting and reporting the maximum value of the incremental power flow across the interface ($\Delta P$) after the loss-of-generation event. The computed margins are summarized in Table 4.

In both cases, the system was not heavily loaded so the stability margins are adequate. The results obtained agreed with transfer limits used in system operation.

5 Conclusions

In this paper, we presented a method for phasor measurement-based voltage stability analysis of a complex transfer path with multiple generation sources. We modeled the external system and power injections of the observable network using Thévenin equivalents. For an SVC in voltage control mode, we used the PMU data to calculate its voltage reference and droop characteristic, which corresponds to its quasi-steady-state operation. Using these models, we computed the $PV$ curves and loadability margins using the $AQ$-bus power flow method and demonstrated agreement between the transfer path model and data.

As future work, we plan to extend the approach to larger systems with broader PMU coverage. We expect to conduct additional research on the applicability of the method to systems with more complex external injections, including renewable generation sources such as wind turbines. For these systems, one could use the approach described in this paper with different injection models.

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