Wide Area Measurements Based
Robust Power System Controller Design

Masayuki Watanabe, Masaya Yamashita, and Yasunori Mitani
Kyushu Institute of Technology
Kitakyushu, Japan
watanabe@ele.kyutech.ac.jp

Abstract—This paper presents a method for the tuning of a power system stabilizer (PSS) for damping inter-area low-frequency oscillations based on wide area phasor measurements and robust controller design with the linear matrix inequality (LMI) approach. The dynamics of a power system is identified as an approximate model based on voltage phasors obtained by the wide area measurement system (WAMS). Then, a PSS is designed by applying LMI approach on the approximate model. The inter-area mode should be damped effectively by a well-tuned PSS based on the state monitoring with the modeling error compensation considered. The proposed method has been applied to a simple two-area power system model to demonstrate the effectiveness of this approach.

Keywords—Wide area measurement, low-frequency oscillation, power system control, power system stabilizer, linear matrix inequality.

I. INTRODUCTION

Interconnections in a power system are intended to improve the reliability and the economical efficiency. It occasionally causes the inter-area low-frequency oscillation with poor damping characteristics [1]. Therefore, many kinds of methods for designing a damping controller have been developed to damp inter-area oscillations [2]–[4]. On the other hand, the penetration of renewable energy sources with uncertain load fluctuations are used to model oscillation characteristics identified by using measurement data, has more effective damping. The advantage of the proposed method is that small signal fluctuations are available to evaluate the effect of the tuned control. In other words, a large disturbance like a line fault is not necessary since the stability of the oscillation mode can be investigated directly by using the damping factor of the identified oscillation. The identification process does not require the information of input to the system for perturbation, while ordinary methods based on the system identification require both input and output to the system [2].

On the other hand, the identified characteristics might contain some errors; in addition, system uncertainties deteriorate the damping controller performance. Here, the design with a technique of linear matrix inequality (LMI) approach [8] is applied for the robustness of designed controller and the error compensation in the identification process. Recently, a controller design using the matrix inequality approach is more popular than the matrix equation approach since it has the capability of designing more suitable controller with the multi-objective control or the gain-scheduled control. The LMI approach has the advantage of the mitigation of solvability condition for the controller design; therefore, the approach could be applicable more flexibly. In this paper, some numerical analyses demonstrate the effectiveness of the proposed method by using dynamical data obtained by a power system simulation package.

II. POWER SYSTEM CONTROLLER DESIGN WITH PHASOR MEASUREMENTS

Fig. 1 shows the schematic diagram of the concept of the power system damping controller design based on wide area phasor measurements. PMUs, which are synchronized with the GPS signal, measure voltage phasors in the wide area power system. Measured phasors, which are collected by the phasor data concentrator, are used to detect the dominant inter-area low-frequency oscillation, and to identify the control effect of the damping controller to the detected dominant oscillation. Note that the role of the wide area measurements if for the identification of the inter-area oscillation modes. Then, a low-order system model for designing a damping controller can

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be identified. A robust damping controller is designed with the low-order system model, especially the LMI approach is adopted to the design process in this paper.

The identified low-order system model is based on wide area measurements; therefore, it is expected that the model contains the current states of the power system including some uncertainties and the designed controller might damp the detected oscillations effectively. The method could be applied more adaptively, since it is based on only measurements and it does not require the detailed system information.

A. Low-order system model

The inter-area oscillation dynamics with a single mode can be simplified by assuming that it has an analogy of a single machine and infinite bus system. The dynamics of the system is represented by the swing equation:

\[ \dot{\omega} = f(\Delta \omega, \Delta \delta) \]

where, \( \omega \) is the angular velocity, \( \delta \) is the phase angle, and \( f \) is the nonlinear function. A linear second order oscillation model for the dominant oscillation mode with the damping factor \( \zeta \) and the natural angular frequency \( \omega_n \) can be represented as a transfer function:

\[ G(s) = \frac{K_G \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}. \]

The gain characteristics of the model (2) is evaluated by:

\[ G_f(\omega) = \frac{K_G}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^3 + 4\zeta^2 \frac{\omega^2}{\omega_n^2}}}. \]

The natural angular frequency \( \omega_n \) of the dominant mode can be identified from the frequency spectrum of the voltage phasors measured by PMUs. Here, phase difference data \( \Delta \delta = \delta_1 - \delta_2 \) between two measurement sites, where the subscript 1 denotes the selected site and the subscript 2 denotes the reference site, are used to calculate the frequency spectrum. The obtained frequency spectrum shows the frequency response of the system (2), therefore, the damping factor \( \zeta \) can be identified by fitting (3) to the frequency spectrum.

Considering a damping controller design, the control loop with the oscillation model \( G(s) \) and a controller is assumed as shown in Fig. 2. Here, the controller, where a PSS is considered in this study, consists of the real structure of the controller, and the model \( F(s) \) represents the effect of the controller to the oscillation model \( G(s) \). The model \( F(s) \) includes the dynamics of the exciter, the generator, and the power system characteristics. This model should be identified as reduced order as possible in order to simplify the procedure of the controller design. In this study, the model \( F(s) \) is assumed by the following form:

\[ F(s) = K_F \cdot \frac{(1 + T_d \delta)(1 + T_b \delta)}{(1 + T_e \delta)(1 + T_d \delta)(1 + T_b \delta)}. \]

The procedure of determining the model \( F(s) \) is as follows:

1) Determine time constants \( T_d \sim T_b \) to maximize the gain characteristics around the frequency which corresponds to the natural frequency of the dominant oscillation.
2) Determine the time constant \( T_e \) to be about zero degree of the phase characteristics of \( F(s) \) around the frequency which corresponds to the natural frequency of the dominant oscillation.
3) Determine the gain \( K_F \) to minimize the mean square error between the filtered phase difference and the output of the low-order model shown in Fig. 2.

Note that the low-order model shown in Fig. 2 is just an approximate model for designing a controller to damp a specific oscillation mode. It is difficult to consider the influence to other oscillation modes and the interaction between the specific mode and other modes although a multi-machine power system has multiple oscillation modes. Therefore, the method should be effective in the case that other modes are enough stable; however, the method is not applicable in the case that multiple modes become unstable simultaneously.

B. Controller design with LMI approach

The LMI approach is applied to the damping controller design based on the approximate low order system model to compensate the modeling error as well as to implement the robust control. The analysis and design based on the LMI approach is especially effective to solve the following problems:

- A robust controllability analysis and a robust controller design for a linear time-invariant (LTI) system with uncertain parameters.
- \( H_\infty \) control.
- A multi-objective controller design, that is, to design a controller which satisfies the multiple control specifications.
Here, a closed-loop system shown in Fig. 3 is considered. The controlled object $P$ is assumed to be a linear time-invariant system. The state equation of the plant $P$ is given by:

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + B_2w(t) \\
z(t) &= C_1x(t) + D_{11}u(t) + D_{12}w(t) \\
y(t) &= C_2x(t) + D_{21}u(t)
\end{align*}
$$

where, $x(t)$ is the state variable vector, $u(t)$ is the disturbances, $z(t)$ is the errors to be kept small, $u(t)$ is the control inputs, and $y(t)$ is the output measurements provided to the controller. On the other hand, the state equation of an output feedback controller $K$ is given by:

$$
\begin{align*}
\dot{x}_K(t) &= A_Kx_K(t) + B_Ky(t) \\
u(t) &= C_Kx_K(t) + D_Ky(t)
\end{align*}
$$

where, $x_K(t)$ is the states of the controller. The controller $K$ stabilizes the plant $P$ and has the same number of states as $P$.

Under the condition that $(A, B_2)$ is stabilizable and $(C_2, A)$ is detectable, the controller $K$ is designed to satisfy the formula about the $H_\infty$ norm:

$$
\|G_{cl}\|_\infty < \gamma
$$

for the smallest possible number of $\gamma$, where $G_{cl}$ represents the closed loop system shown in Fig. 3. The matrix $K$ is a $H_\infty$ controller, if and only if there exists a matrix $X > 0$ for the matrix inequality:

$$
\begin{bmatrix}
A_{cl}^TX + XA_{cl} & XB_{cl} & C_{cl}^T \\
B_{cl}^TX & -\gamma I & D_{cl}^T \\
C_{cl} & D_{cl} & -\gamma I
\end{bmatrix} < 0
$$

where,

$$
\begin{bmatrix}
A_{cl} & B_{cl} \\
C_{cl} & D_{cl}
\end{bmatrix}
= \begin{bmatrix}
A + B_2DK_2 & B_2C_k & B_1 + B_2DK_2D_{21} \\
B_2C_k & A_K & B_KD_{21} \\
C_1 + D_{12}DK_2C_k & D_{12}C_k & D_{11} + D_{12}DK_2D_{21}
\end{bmatrix}
$$

Hence, the control design problem is reduced to find $X > 0$ and $K$ such that the matrix inequality (8) holds.

Fig. 4 shows the closed loop including two weighting functions, $W_z(s)$ and $W_r(s)$. Here, $W_z(s)$ has low-pass characteristics to consider the robust stability and the modeling error, while $W_r(s)$ has high-pass characteristics to consider the influence for other modes. Note that the designed controller $K$ has the same order as the plant $P$, therefore, the order of the controller should be as low as possible to simplify the tuning process. In this study, the balance truncation algorithm is adopted to obtain the low order controller with the equivalent performance of the designed controller.

C. Controller design procedure

The damping controller design procedure proposed in this paper includes the following steps:

1) Voltage phasors at two measurement sites in the power system are measured by using PMUs. Small phasor fluctuations with load variations in the normal operating condition are available in this study.
2) An oscillation model (2) is identified by the frequency spectrum of obtained phasors and (3).
3) The model (4), which represents the control effect to the identified oscillation, is determined.
4) The closed loop system with appropriate weighting functions $W_z(s)$ and $W_r(s)$ in Fig. 4 is determined.
5) A robust $H_\infty$ controller satisfying (7) is derived such that the LMI (8) holds.
6) A low-order controller is obtained by applying the balance truncation algorithm to the designed controller.
7) The damping effect by the controller can be evaluated by fitting (3) to the frequency spectrum of voltage phasors measured after applying the controller to the power system.

III. SIMULATION STUDY

A. Power system configuration

The proposed method is applied to the simple two-area power system model [9] shown in Fig. 5. System constants described in [9] are used in this study.

Generator 1 is equipped with an AVR (Automatic Voltage Regulator) shown in Fig. 6 with $\Delta\omega$-type PSS to damp inter-area oscillations. PSS parameters of the initial condition are $K_{\text{STAB}} = 5.0$, $T_1 = 2.0$, $T_2 = 1.0$, $T_3 = 2.0$, and $T_4 = 0.5$. Other generators are equipped with the identical AVR with PSS shown in Fig. 7. The effect of the speed governor is not considered. In this study, the software for the simulation of the power system dynamics, EUROSTAG [10], has been used.
B. Approximate model identification

The condition without the PSS of the generator 1 is assumed. The small load fluctuations at the node 7 are assumed to simulate the phasor fluctuations measured in the normal condition of the real power system. Fig. 8 shows the phase differences between nodes 1 and 3, where PMUs are assumed to be installed. Fig. 9 shows the Fourier spectrum of Fig. 8. The result shows that an inter-area mode with frequency of around 0.60 Hz is dominant.

The damping factor $\zeta$ and the natural frequency $\omega_n$ of (2) are identified by fitting (3) to the amplitude $A_{\text{peak}}$ at the dominant frequency $f_n = 0.60$ (Hz) and the frequencies at which the amplitude corresponds to $A_{\text{peak}}/\sqrt{2}$ by applying the least squares method. Here, the damping factor is $\zeta = 0.053$ and the natural frequency is $\omega_n = 3.758$ (rad/s). Therefore, the oscillation model (2) is represented as follows:

$$G(s) = \frac{1}{s^2 + 0.398s + 14.12}.$$  \hspace{1cm} (10)

The model $F(s)$ is identified by using the procedure described in Section II-A. The model $F(s)$ is given by:

$$F(s) = 1.14 \cdot \frac{(1 + 0.57s)(1 + 0.79s)}{(1 + 0.44s)(1 + 0.31s)(1 + 0.21s)}.$$  \hspace{1cm} (11)

$$V_f \quad 1 + 0.005s \quad 250 \quad 1 + 0.03s \quad 0.06s \quad 1 + 0.05s$$

$$\Delta \omega \quad K_{\text{STAB}} \quad 10s \quad 1 + 10s \quad 1 + T_1s \quad 1 + T_2s \quad 1 + T_3s \quad 1 + T_4s$$

Fig. 9. Fourier spectrum of the phase difference.

Fig. 10 illustrates the bode diagram of the model $F(s)$, which has the peak of the gain at 3.76 rad/s.

Fig. 11 shows the phase difference of inter-area low-frequency oscillations extracted by the FFT-based band-pass filter, where the frequency band width is set to 0.60±0.04 Hz.

The input $u$ of the approximate model shown in Fig. 2 is estimated by multiplying the inverse function of (10) to filtered phase difference data. Fig. 12 shows the estimated input signal $u$ of the approximate model. Fig. 13 shows the comparison between the filtered phase difference of measured data and the approximate model output simulated by using the estimated input signal. These results demonstrate that the approximate model can successfully represent the characteristics of the inter-area low-frequency oscillation mode and the effect of the controller to this mode.

C. PSS tuning

The $H_\infty$ controller based on the LMI approach is designed for the identified low-order system model. The controller corresponding to the minimum $\gamma$ satisfying the constraint (7) is derived based on the closed-loop system shown in Fig. 4. Here, the controller which satisfies the LMI (8) has been solved
by using MATLAB Robust Control Toolbox. Two weighting functions $W_i(s)$ and $W_f(s)$ are given by:

$$W_i(s) = 15 \cdot \frac{1 + 0.0003s}{1 + 0.05s} \cdot \frac{1 + 0.1s}{1 + 0.01s}.$$  

The $H_{\infty}$ controller is derived in the following form:

$$K(s) = \frac{b_7 s^7 + b_6 s^6 + \cdots + b_1 s + b_0}{s^8 + a_7 s^7 + a_6 s^6 + \cdots + a_1 s + a_0}. \quad (12)$$

The coefficients of the $H_{\infty}$ controller (12) are shown in Table I.

**TABLE I. THE COEFFICIENTS OF $H_{\infty}$ CONTROLLER**

<table>
<thead>
<tr>
<th>$a_i$</th>
<th>$b_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.10 \times 10^4$</td>
<td>$2.36 \times 10^4$</td>
</tr>
<tr>
<td>$3.32 \times 10^4$</td>
<td>$6.57 \times 10^3$</td>
</tr>
<tr>
<td>$3.30 \times 10^4$</td>
<td>$2.38 \times 10^4$</td>
</tr>
<tr>
<td>$1.13 \times 10^5$</td>
<td>$1.59 \times 10^4$</td>
</tr>
<tr>
<td>$1.50 \times 10^7$</td>
<td>$1.93 \times 10^7$</td>
</tr>
<tr>
<td>$5.91 \times 10^8$</td>
<td>$5.75 \times 10^7$</td>
</tr>
<tr>
<td>$4.31 \times 10^9$</td>
<td>$3.98 \times 10^8$</td>
</tr>
</tbody>
</table>


By applying the balance truncation algorithm to the $H_{\infty}$ controller (12), the structure of a PSS with the reduced order of (12) is given by:

$$K_r(s) = \frac{2.36 \times 10^6 s^2 + 4.41 \times 10^7 s + 1.03 \times 10^7}{s^3 + 3105 s^2 + 3.16 \times 10^5 s + 1.19 \times 10^5}. \quad (13)$$

Fig. 14 demonstrates a comparison of bode diagrams of the $H_{\infty}$ controller (12) and the designed PSS (13). The result shows that the designed PSS (13) with third order is the same in performance with $H_{\infty}$ controller (12) with eighth order.

**D. Performance evaluation of the designed PSS**

Here, the performance of the designed PSS (13) designed by the proposed method is evaluated. Table II shows eigenvalues of the inter-area low-frequency mode and two local modes of the original system model shown in Fig. 5. In addition, each damping ratio (%) is also given in parenthesis. Appropriate parameters of the PSS can be obtained since the inter-area mode is damped effectively. Note that the low-order model does not consider other modes than the inter-area mode, therefore, it is difficult to evaluate the influence of the
Fig. 14. The comparison of bode diagrams between the $H_\infty$ controller (12) and the designed PSS (13).

**TABLE II.** The eigenvalues and damping ratios of power swing modes in the simple two-area power system model.

<table>
<thead>
<tr>
<th>Without PSS</th>
<th>$-0.125 \pm j3.710 \ (3.31)$</th>
<th>$-0.958 \pm j6.963 \ (13.82)$</th>
<th>$-1.176 \pm j7.763 \ (14.98)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial PSS</td>
<td>$-0.204 \pm j3.814 \ (3.26)$</td>
<td>$-0.916 \pm j7.243 \ (12.55)$</td>
<td>$-1.175 \pm j7.764 \ (14.96)$</td>
</tr>
<tr>
<td>Designed PSS</td>
<td>$-0.306 \pm j4.638 \ (7.56)$</td>
<td>$-0.983 \pm j8.266 \ (11.81)$</td>
<td>$-1.172 \pm j7.759 \ (14.94)$</td>
</tr>
</tbody>
</table>

parameter tuning of the controller to other modes on the low-order model. In this case, the influence is not so large as shown in Table II since local modes has larger damping than the inter-area mode.

Fig. 15 shows the step response of the approximate model, where the magnitude of the step input has been adjusted to be consistent with the response of the original system shown in Fig. 16. The result shows that the designed PSS (13) can damp the low-frequency oscillation more effectively than the initial condition.

On the other hand, Fig. 16 shows the speed response of the generator 1 of the original system shown in Fig. 5 for a disturbance. An assumed disturbance here is a three-phase to ground fault between nodes 7 and 8. The fault is cleared at 0.067 seconds after the fault. The result shows that the inter-area oscillation is damped effectively by the designed PSS. Comparing Fig. 15 with Fig. 16, characteristics of the inter-area low-frequency oscillation of the original system can be represented approximately by the low-order model.

Fig. 17 shows the speed response of the generator 1 when the fault tie line between nodes 7 and 8 is cleared at 0.067 seconds after a three-phase to ground fault. In this case, the system parameter changes significantly. The generator with the initial PSS loses the synchronization, while the generator with the designed PSS can damp the oscillation after such large disturbance. These results demonstrate the effectiveness of the proposed method based on wide area phasor measurements considering the error compensation with the LMI approach.

**IV. Conclusion**

In this paper, a method for tuning a PSS based on wide area phasor measurements and the robust control with LMI approach was proposed. The low-order system model, which
holds the characteristics of the inter-area mode and the control effect, is identified by monitoring data of wide area phasor measurements. The LMI approach has been applied to the low-order model to design a PSS for damping the inter-area oscillation effectively and to compensate the identification error of the model.

The effectiveness of the proposed method has been demonstrated through the power system simulation. In this study the method has been applied to a simple two-area power system model. The results show that the appropriate controller can be designed by using the identified low-order model.

REFERENCES


