

Risk-Constrained Optimal Power Flow with Probabilistic Guarantees

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Abstract—Higher penetration of renewable energy and market liberalization increase both the need for transmission capacity and the uncertainty in power system operation. New methods for power system operational planning are needed to allow for efficient use of the grid, while maintaining security and robustness against disturbances. In this paper, we propose a risk model for risk related to outages, accounting for available remedial measures and the impact of cascading events. The new risk model is used to formulate risk-based constraints for the post-contingency line flows, which are included in an optimal power flow (OPF) formulation. Forecast uncertainty is accounted for by formulating the relevant constraints as a chance constraint, and the problem is solved using a sampling based technique. In a case study for the IEEE 30 bus system, we demonstrate how the proposed risk-based, probabilistic OPF allows us to control the risk level, even in presence of uncertainty. We investigate the trade-off between generation cost and risk level in the system, and show how accounting for uncertainty leads to a more expensive, but more secure dispatch.

I. INTRODUCTION

Market liberalization and increasing penetration of renewable energy sources (RES) lead to a situation where power is not necessarily produced close to where it is consumed, but rather where production is cheap, the wind is blowing or the sun is shining. This trend increases the need for transmission capacity, and forces the transmission system operators to operate the system closer to the operational limits. At the same time, fluctuations in RES in-feed and short-term trading lead to larger deviations from the planned schedule, and thus increase uncertainty in power system operation. The combination of a highly loaded system and significant uncertainty increases operational risk. There is a need for methods which allow for efficient use of transmission capacity, while maintaining security and robustness against disturbances.

There are two types of disturbances in the system, *random outages* and *forecast uncertainty*, which have inherently different characteristics. Whereas outages can be characterized as discrete events with a (usually) low probability, forecast uncertainty (deviations in power in-feeds arising from load, RES or short-term trading) are characterized by a probability distribution. The method proposed in this paper addresses both types of disturbances. The random outages are handled by a risk-based extension to the N-1 criterion that utilizes additional information about the probability of outages, the extent of post-contingency violations and the cost and availability of remedial actions to provide a more quantitative measure of power system security. The risk-based criterion is implemented in an optimal power flow (OPF). By formulating the OPF as a chance constrained optimization problem, we are able to account for forecast deviations in a comprehensive way. The

resulting formulation allows us to control the risk of outages, even in presence of forecast uncertainty.

There exist two main approaches to model risk in power system operation. On the one hand, risk can be modeled through overall reliability parameters like Expected Energy Not Served (EENS). These parameters incorporate the effect of cascading events and best reflects the impact on the customers in the system. However, computing the risk requires extensive calculations (i.e., Monte-Carlo simulations), and these types of risk measures are typically used to analyze the risk for a given operating condition [1], [2], as opposed to inclusion in an optimization problem.

On the other hand, risk can be modeled in terms of violation of technical limits, e.g., dependent on the power flow of a line or on the voltage magnitude. Such risk measures typically consider the situation after an N-1 outage, and do not simulate how a potential cascade would develop further. Thus, these risk measures do not reflect the full risk of cascading events, but are relatively easy to compute. Further, technical parameters are more easily understood and influenced by the system operator, and have often been proposed for incorporation in OPF formulations. The OPF formulations in [3], [4] describe risk as violations and near-violations of voltage limits and line transfer capacities, modeled as linear functions of the voltage magnitude and the line flow. In [5], risk is expressed as a quadratic function of the line flow, whereas [6] models risk as the cost of equipment aging in function of, e.g., the line flow. Here, we also express risk as a function of line flow, but relate risk to the cost of remedial measures and the possibility of initiating a cascading event.

Although several risk-based OPF formulations exist, few of them account for forecast uncertainty in a comprehensive way. The OPF formulation in [6] considers normally distributed load uncertainty, but only limits the expected value of the risk and does not provide any guarantees for an upper bound. In contrast, the method proposed in this paper guarantees that the risk limit will hold with a chosen probability. This is achieved by formulating a chance constrained optimization problem, following along the lines of [7], [8]. The problem is solved using the randomized optimization technique proposed in [9], based on the so-called scenario approach from [10]. This technique requires no assumptions on the distribution of the forecast errors.

The contributions of this paper are threefold: First, we introduce a risk measure that relates the amount of line overload in a post-contingency state to the cost of remedial measures and the probability of cascading events (Section II). Second, we formulate a DC optimal power flow (OPF) that incorporates this risk measure and also accounts for forecast uncertainty

(Section III). Third, we analyze the proposed formulation and compare it with other OPF formulations with regards to cost, risk level, and number of post-contingency overloads in a case study for the IEEE 30-bus system (Section IV). We highlight synergies that can be achieved by accounting for both random outages and forecast uncertainty in a probabilistic way.

II. RISK MODELING

In this paper, we propose a risk measure for incorporation in an OPF formulation and focus on risk as a function of post-contingency transmission line loading. Previous risk formulations (e.g., [5], [4]) use the same severity function for all transmission lines independent of which contingency has taken place. Here, we improve this formulation in two ways. First, we explicitly account for different types of risk (such as moderate overloads that can be mitigated through redispatch and high overloads that might lead to cascading events) by formulating a piecewise linear severity function. Second, we formulate the severity function separately for each line and contingency, which allows us to account for the effect of available remedial measures on each line in each post-contingency situation. The proposed risk formulation allow us to set post-contingency line flow limits based on the available remedial measures and potential impacts of cascading events.

A. Definition of the risk measures

A risk measure should reflect both the probability of an outage and the severity of the resulting operating condition. Here we focus on line and generator outages, but any outage with an influence on the transmission line loading could be considered, including N-2 and common mode outages. The risk related to specific outage i and line k is expressed as

$$\mathcal{R}_{(i,k)}^{spec} := \mathcal{P}_i \cdot \mathcal{S}_{k|i}, \quad (1)$$

where \mathcal{P}_i is the probability of outage i and $\mathcal{S}_{k|i}$ is the severity of the operating condition on line k given outage i . This expression can be seen the risk-based counterpart of the N-1 criterion, as it describes the risk for a specific line in one specific post-contingency state. Using $\mathcal{R}_{(i,k)}^{spec}$ as a basis, we define:

$$\mathcal{R}_{(i)}^{out} := \sum_{k=1}^{N_l} \mathcal{P}_i \cdot \mathcal{S}_{k|i} \quad (2)$$

$$\mathcal{R}_{(k)}^{line} := \sum_{i=1}^{N_{out}} \mathcal{P}_i \cdot \mathcal{S}_{k|i} \quad (3)$$

$$\mathcal{R}^{tot} := \sum_{i=1}^{N_{out}} \sum_{k=1}^{N_l} \mathcal{P}_i \cdot \mathcal{S}_{k|i} \quad (4)$$

$\mathcal{R}_{(i)}^{out}$ expresses the risk after an outage i , and is obtained by summing the risk of all lines k in this post-contingency state. $\mathcal{R}_{(k)}^{line}$ is the risk related to line k , summed over all outages i . \mathcal{R}^{tot} is the total risk in the system, summed over all outages i and all lines k .

In order to evaluate (1), the outage probabilities \mathcal{P}_i must be estimated, and the severity $\mathcal{S}_{k|i}$ has to be defined. We assume that the outage probabilities are calculated a-priori (e.g., based on historical data and current weather conditions [11], [12]) and given as an input to the optimization. The severity modeling is described in the next section.

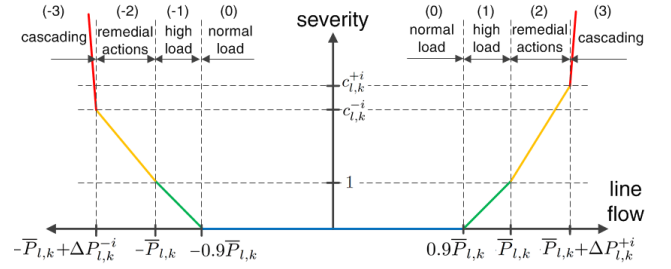


Fig. 1. Piecewise linear severity function for line k after outage i .

B. Severity modeling

To capture different types of risk arising from different levels of post-contingency line loading, we define the severity $\mathcal{S}_{k|i}$ as a piecewise linear function of the line flow. We define four different segments for the severity function (as opposed to two in previous formulations [5], [4]), corresponding to 0) normal load, 1) high load, 2) moderate overload which requires remedial actions and 3) cascading overload which might lead to a cascading event, as shown in Fig. 1. We consider a system with N_l lines and N_G generators and include the outage of any line or generator, in total $N_{out} = N_l + N_G$ outages. The vectors $P_l^i, \bar{P}_l \in \mathbb{R}^{N_l}$ denote the post-contingency line flows after outage i and the line capacity limit during normal operation. The flow on line k is denoted by $P_{l,k}^i$. Mathematically, we define the piecewise linear severity function as the pointwise maximum over a set of affine functions

$$\mathcal{S}_{k|i}(P_{l,k}^i) := \max_j \{a_{j,k}^i P_{l,k}^i + b_{j,k}^i\}, \quad j \in \{-3, \dots, 3\} \quad (5)$$

where $a_{j,k}^i, b_{j,k}^i$ are parameters for the j^{th} segment belonging to line k and outage i . Formulating the severity function like this ensures tractability of the optimization problem, as discussed in Section III.A. We will now give a physical interpretation of the different severity zones, and explain how to compute the parameters $a_{j,k}^i$ and $b_{j,k}^i$ with $j = 0, \dots, 3$ for positive line flows. The severity function is not symmetric in general, but the parameters for negative line flows with $j = -3, \dots, 0$ can be calculated in an analog way.

1) *Normal load*: At low load ($P_{l,k}^i < 0.9 \cdot \bar{P}_{l,k}$), the severity is assumed to be zero. This segment of the severity function connects the two points

$$\mathcal{S}_{k|i}(0) = 0 \quad (6)$$

$$\mathcal{S}_{k|i}(0.9 \cdot \bar{P}_{l,k}) = 0, \quad (7)$$

with parameters $a_{0,k}^i = b_{0,k}^i = 0$.

2) *High load*: High loading ($0.9 \cdot \bar{P}_{l,k} < P_{l,k}^i < \bar{P}_{l,k}$) might lead to a failure of the line (e.g., due to line sag or relay malfunction), and the severity is therefore non-zero. We choose $a_{1,k}^i$ and $b_{1,k}^i$ such that the "high load" segment of the severity function connects the two points given by Eq. (7) and (8).

$$\mathcal{S}_{k|i}(\bar{P}_{l,k}) = 1. \quad (8)$$

3) *Moderate overload*: Many system operators allow a transmission line overload for a short time (e.g., for a short period after a contingency). However, if the operating limit is violated, removing the overload requires remedial actions like

transmission switching or redispatch. Here, we define moderate overload as one which can be relieved within a given time frame (e.g., 5 minutes). The corresponding severity zone goes from $\bar{P}_{l,k} < P_{l,k}^i < (\bar{P}_{l,k} + \Delta P_{l,k}^{+i})$, where we define $\Delta P_{l,k}^{+i}$ as the largest achievable line flow reduction for line k after outage i . $\Delta P_{l,k}^{+i}$ is a function of the available remedial measures (e.g., available redispatch). The effectiveness of a remedial measure depends on the location of a line in the system, topology changes (e.g., after a line outage), or changes in availability of remedial measures (e.g., after the outage of a generator offering redispatch). Therefore, $\Delta P_{l,k}^{+i}$ must be calculated separately for each line and each outage. The increase in risk from $\bar{P}_{l,k}$ to $(\bar{P}_{l,k} + \Delta P_{l,k}^{+i})$ depends on $c_{l,k}^{+i}$, which is the cost of reducing the line flow by $\Delta P_{l,k}^{+i}$. If both $\Delta P_{l,k}^{+i}$ and $c_{l,k}^{+i}$ are known, the severity function parameters $a_{2,k}^i$ and $b_{2,k}^i$ are chosen such that the severity function connects the two points given by Eq. (8) and (9).

$$\mathcal{S}_{k|i}(\bar{P}_{l,k} + \Delta P_{l,k}^{+i}) = 1 + c_{l,k}^{+i}. \quad (9)$$

In general, there are many ways to consider remedial measures in this formulation. We now suggest one way of defining $\Delta P_{l,k}^{+i}$ and $c_{l,k}^{+i}$ with the following choice: (i) We only consider generation redispatch. (ii) We only allow one redispatch measure (i.e., one shift between two generators) per line and contingency. We further assume that the available positive and negative redispatch is known and given by the vectors $P_R^+, P_R^- \in \mathbb{R}_{+0}^{N_G}$, with $P_{R,g}^+$ denoting the positive redispatch capability of generator g . The corresponding redispatch costs are also assumed to be known and are given by $c_R^+, c_R^- \in \mathbb{R}^{N_G}$.

To compute the largest possible reduction of the line flow $\Delta P_{l,k}^{+i}$, we first need to compute the largest possible shift in generation $\Delta P_{g \rightarrow h}$ from any generator g to any other generator h . This is given by

$$\Delta P_{g \rightarrow h} = \min\{P_{R,g}^+, P_{R,h}^-\}. \quad (10)$$

To describe how much a generation shift from g to h decreases the power flow on line l , we use the power transfer distribution factor $PTDF_{l,g \rightarrow h}$ [13]. The largest power flow decrease $\Delta P_{l,k}^{+i}$ can then be calculated as

$$\Delta P_{l,k}^{+i} = -\min_{g,h}\{PTDF_{l,g \rightarrow h}^i \Delta P_{g \rightarrow h}^i\}, \quad (11)$$

since $\Delta P_{l,k}^{+i}$ is defined as a positive value. Note that both $PTDF_{l,g \rightarrow h}^i$ and $\Delta P_{g \rightarrow h}^i$ depend on the outage i . For a generator outage i , the topology remains unchanged, but the redispatch availability of generator g is lost. Thus, $\Delta P_{g \rightarrow h}^i$ might be different from normal operation since $P_{R,g}^{+i} = P_{R,g}^- = 0$. For a line outage i , the redispatch availability remains unchanged, but the topology changes. Thus, we need to recalculate $PTDF_{l,g \rightarrow h}^i$.

When we know which generation shift $\Delta P_{g \rightarrow h}$ that lead to the largest line flow decrease $\Delta P_{l,k}^{+i}$, the cost $c_{l,k}^{+i}$ of the redispatch measure is given by

$$c_{l,k}^{+i} = (c_{R,g}^+ + c_{R,h}^-) \Delta P_{g \rightarrow h}. \quad (12)$$

With this result, we can calculate $a_{2,k}^i$ and $b_{2,k}^i$ according to (8) and (9).

4) *Cascading overload*: Short term overloads might be acceptable up to a certain point, but should not be allowed to evolve into cascading events. Here, we consider two types of cascade initiation. First, any overload that cannot be removed by remedial actions might eventually lead to a line trip (delayed trip due to time settings in the relays or flash-over due to sag). We therefore define line loadings higher than $\bar{P}_{l,k} + \Delta P_{l,k}^{+i}$ as cascading overload. Second, overloads above a certain threshold (e.g., $P_{l,k}^i > 1.2 \cdot \bar{P}_{l,k}$) might lead to immediate trip of the line. To avoid this, we introduce a limit for $\Delta P_{l,k}^{+i}$ (e.g., $\Delta P_{l,k}^{+i} \leq 0.2 \cdot \bar{P}_{l,k}$).

Since the potential impacts of a cascading event are very high, the severity increases very rapidly as we enter the cascading zone. The increase in severity per additional MW loading is denoted by ρ_C . The severity function parameters $a_{3,k}^i$ and $b_{3,k}^i$ are then given by Eq. (9) and (13).

$$\mathcal{S}_{k|i}(\bar{P}_{l,k} + \Delta P_{l,k}^{+i} + 1) = 1 + c_{l,k}^{+i} + \rho_C \quad (13)$$

There are in general many possible ways of defining ρ_C . Here, we choose a very high number for ρ_C , corresponding to a very rapid increase in the risk as the line loading exceeds $\bar{P}_{l,k} + \Delta P_{l,k}^{+i}$. We also assume that ρ_C is the same for all lines.

C. Risk Constraints

Based on the risk measures defined by (1) to (4), we can formulate constraints to limit the risk:

$$\mathcal{R}^{spec} : \mathcal{P}_i \cdot \mathcal{S}_{k|i}(P_{l,k}^i) \leq \bar{\mathcal{R}}_i \quad (14)$$

$$\mathcal{R}^{out} : \sum_{i=1}^{N_{out}} \mathcal{P}_i \cdot \mathcal{S}_{k|i}(P_{l,k}^i) \leq \bar{\mathcal{R}}^{out} \quad (15)$$

$$\mathcal{R}^{line} : \sum_{k=1}^{N_l} \mathcal{P}_i \cdot \mathcal{S}_{k|i}(P_{l,k}^i) \leq \bar{\mathcal{R}}^{line} \quad (16)$$

$$\mathcal{R}^{tot} : \sum_{i=1}^{N_{out}} \sum_{k=1}^{N_l} \mathcal{P}_i \cdot \mathcal{S}_{k|i}(P_{l,k}^i) \leq \bar{\mathcal{R}}^{tot} \quad (17)$$

Eq. (14) constrains the risk for each line k to stay below a constant limit $\bar{\mathcal{R}}_i$ after the outage i . Eq. (15) limits the risk of outage i , while (16) limits the risk of line k and (17) limits the total risk in the system.

1) *Relation to N-1 constraints*: Including the constraints (14) - (17) in the optimization problem allows us to control the risk level in the system according to our preferences, but requires us to define the risk limit in a reasonable way. By showing how (14) is related to the traditional N-1 criterion, we provide a physical interpretation of the risk limit and give some intuition how $\bar{\mathcal{R}}_i$ can be chosen. Assuming that \mathcal{P}_i is a scalar value given as an input to the optimization, (14) can be reformulated as an upper bound on the severity

$$\mathcal{S}_{i,k}(P_{l,k}^i) \leq \frac{\bar{\mathcal{R}}_i}{\mathcal{P}_i}. \quad (18)$$

Because of the way $\mathcal{S}_{k|i}(P_{l,k}^i)$ is defined, $\mathcal{S}_{k|i}(P_{l,k}^i) = 1$ at the line capacity limit $P_{l,k}^i = \bar{P}_{l,k}$. Eq. (18) is thus equivalent to a traditional N-1 constraint if $\bar{\mathcal{R}}_i = \mathcal{P}_i$. If the same risk limit $\bar{\mathcal{R}}_i$ is chosen for all outages i , the upper bound on the severity depends on the outage probability and a higher

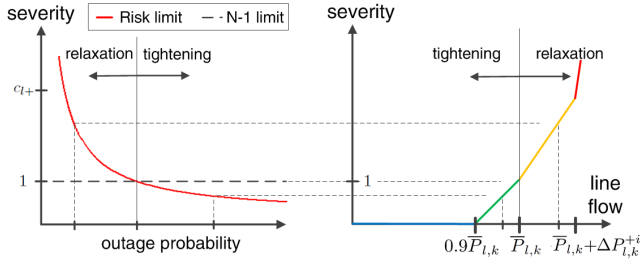


Fig. 2. Left: Comparison between the risk-based severity limit (red line), and the N-1 severity limit (black line). Right: Tightening and relaxation of the line flow constraint as a function of the severity limit and the line and outage specific severity function.

severity is accepted for outages with low probability. This is illustrated in Fig. 2 (left), where the red line is the severity limit for a given value of C and the black dashed line is the severity limit for the N-1 criterion. Since the severity is a function of the post-contingency line flow, accepting a higher (or lower) severity relaxes (or tightens) the constraint on the post-contingency line flow. The amount of relaxation (or tightening) depends on the severity function, as shown in Fig. 2 (right).

III. FORMULATION OF OPTIMIZATION PROBLEM

This section introduces the risk-based, probabilistic security constrained optimal power flow (RB-pSCOPF), with risk-based constraints for the post-contingency line flows and chance constraints to account for the forecast uncertainty. The objective is to find the minimal cost dispatch that satisfies the desired risk level for all N_{out} outages as well as the desired violation level for the chance constraint. The setup is similar to the probabilistic SCOPF described in [7], but the security constraints for the post-contingency line flows are substituted with the proposed risk-based constraints. We consider a system with N_B buses, N_L loads, N_G generators and N_w wind power plants. Given a DC power flow formulation, the line flows can be expressed as linear functions of the active power injections both in normal and outage conditions:

$$P_l^i = A^i P_{inj}^i, \quad \text{for all } i = 0, \dots, N_{out}. \quad (19)$$

Here, $A^i \in \mathbb{R}^{N_l \times N_B}$ describes the relation between the active power injections $P_{inj}^i \in \mathbb{R}^{N_B}$ and the line flows P_l^i after outage i , with $i = 0$ being the normal operation condition. A^i is given by

$$A^i = B_f^i \begin{bmatrix} (\tilde{B}_{bus}^i)^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (20)$$

where $B_f^i \in \mathbb{R}^{N_l \times N_B}$ is the line susceptance matrix and $\tilde{B}_{bus}^i \in \mathbb{R}^{N_B - 1 \times N_B - 1}$ the bus susceptance matrix (without the last column and row) after outage i [7]. The power injections are given by

$$P_{inj}^i = C_G^i (P_G - d^i P_m^i) + C_w^i P_w - C_L^i P_L, \quad i = 0, \dots, N_{out}. \quad (21)$$

$P_G \in \mathbb{R}^{N_G}$ describe the generator output, and $P_L \in \mathbb{R}^{N_L}$ the load consumptions. $P_w \in \mathbb{R}^{N_w}$ contains the wind power infeeds, and is the sum of the forecast P_w^f and a random error ΔP_w . The matrices $C_G \in \mathbb{R}^{N_B \times N_G}$, $C_w \in \mathbb{R}^{N_B \times N_w}$ and $C_L \in \mathbb{R}^{N_B \times N_L}$ relate the power injections to the respective

buses. The distribution vector d^i describes how the power mismatch $P_m^i \in \mathbb{R}$ is compensated by the different generators. P_m^i describes the power required to balance wind power deviations and power outages, and is defined as

$$P_m^i = \mathbf{1}_{1 \times N_w} (P_w - P_w^f) - (b_G^i)^T P_G + (b_L^i)^T P_L^f \quad (22)$$

where the first term is the sum of the forecast deviations, and the two next terms corresponds to generator or load outages. $b_G^i \in \mathbb{R}^{N_G}$ and $b_L^i \in \mathbb{R}^{N_L}$ are binary vectors whose elements are either '0' or '1'. A value of '1' corresponds to the tripped component for outage i . The resulting optimization problem is given by

$$\min_{P_G} c_1^T P_G + P_G^T [c_2] P_G \quad (23)$$

subject to

$$\begin{aligned} \mathbf{1}_{1 \times N_b} (C_G P_G + C_w P_w^f - C_L P_L) &= 0 \quad (24) \\ \mathbb{P} \left(\begin{array}{l} -\bar{P}_l \leq A^0 P_{inj}^0 \leq \bar{P}_l \\ P_G \leq P_G - d^i P_m^i \leq \bar{P}_G, \quad i = 0, \dots, N_{out} \\ \mathcal{R} \leq \bar{\mathcal{R}} \end{array} \right) &\geq 1 - \varepsilon \quad (25) \end{aligned}$$

Eq. (23) and (24) define the objective function and the power balance constraints, with $c_1, c_2 \in \mathbb{R}^{N_G}$ being the linear and quadratic cost coefficients, and $[c_2]$ a diagonal matrix with c_2 on the diagonal. Eq. (25) describes a probabilistic constraint, stating that all inequalities within the brackets must hold with a probability of at least $1 - \varepsilon$, where ε is called the violation level. The first inequality is the line flow limits for normal operating conditions $i = 0$, and the second inequality the capacity limits of the generators P_G in all conditions i . The last inequality describes one or more risk limitations for the post-contingency line flows, which can be chosen from the options proposed in Section II.C. The problem remains convex after introduction of the risk constraints. Inserting (21) and (19) in (5), $\mathcal{S}_{k|i}(P_{l,k}^i)$ can be expressed as the pointwise maximum over a set of affine functions with P_G as an argument. $\mathcal{P}_i \cdot \mathcal{S}_{k|i}(P_{l,k}^i)$ is hence convex with respect to P_G . This means that all the risk measures are sums of convex functions and hence convex themselves.

A. Chance Constraint Reformulation

In this paper, we follow the probabilistically robust approach, a randomized optimization technique proposed in [9]. This method allows us to account for a reduced number of scenarios inside the optimization compared with the scenario approach [10], while still guaranteeing a violation level $\leq \varepsilon$. The method includes two steps. In the first step, we solve an optimization problem to determine, with a confidence of at least $1 - \beta$, the minimum volume set D that contains at least $1 - \varepsilon$ probability mass of the distribution of the uncertain variable P_w . The number of required scenarios for this first step is related to the number of uncertain variables N_w and given by [10]

$$N \geq \frac{1}{\varepsilon} \frac{e}{e-1} \left(\ln \frac{1}{\beta} + 2N_w - 1 \right), \quad (26)$$

where e is the base of the natural logarithm. Further details on how to compute D is given in [9], [14]. In the second step, we use the probabilistically computed set D to solve a robust

problem for all uncertainty realizations within this set. The chance constraint (25) is substituted by the robust constraints

$$\begin{aligned} -\bar{P}_l &\leq A^0 P_{inj}^0 \leq \bar{P}_l & \forall P_w \in D \\ \underline{P}_G &\leq P_G - d^i P_m^i \leq \bar{P}_G, \quad i = 1, \dots, N_{out}, & \forall P_w \in D \\ \mathcal{R} &\leq \bar{\mathcal{R}} & \forall P_w \in D \end{aligned}$$

Since all our constraints are convex with respect to P_w (independent of the value of P_G), the robust constraints hold for all $P_w \in D$ if they hold for P_w at the vertices of the set, i.e. all possible combinations of minimum and maximum values of the vector P_w . With only one wind power plant, we only need to consider only two scenarios, the maximum and minimum P_w in D . With N_w uncertain in-feeds, we must consider 2^{N_w} scenarios. In this case, enumeration of the vertices should be avoided to solve the problem more efficiently. This can be achieved by applying techniques proposed in [15].

B. Possible SCOPF formulations

The risk-based, probabilistic OPF (RB-pSCOPF), given by (23) - (25) includes risk-based limits for the post-contingency line flows and accounts for wind in-feed uncertainty. However, some small changes allow us to change the OPF type. By choosing $\bar{\mathcal{R}}_i = \mathcal{P}_i$ for each outage i and $\bar{\mathcal{R}}^{out} = \bar{\mathcal{R}}^{tot} = \infty$, the risk-based constraints become equivalent to traditional N-1 constraints. If the forecast is perfect, i.e. no forecast uncertainty, we set $P_w = P_w^f$ such that the probabilistic constraint (25) reduces to a deterministic constraint. Thus, we can define 4 different SCOPF formulations:

- 1) *Standard SCOPF* (SCOPF) considers traditional N-1 constraints (i.e., choose $\bar{\mathcal{R}}_i = \mathcal{P}_i$), and no forecast errors for the wind in-feed ($P_w = P_w^f$).
 - 2) *Probabilistic SCOPF* (pSCOPF) considers traditional N-1 constraints (i.e., choose $\bar{\mathcal{R}}_i = \mathcal{P}_i$), and a probabilistic constraint with violation level ε .
 - 3) *Risk-based SCOPF* (RB-SCOPF) considers risk-based post-contingency line flow constraints, but no forecast errors for the wind in-feed ($P_w = P_w^f$).
 - 4) *Risk-based, probabilistic SCOPF* (RB-pSCOPF) considers risk-based post-contingency line flow constraints, and a probabilistic constraint with violation level ε .
- In the following case study, we compare these four formulations.

IV. CASE STUDY

The OPF formulations presented above are applied to the IEEE 30-bus network [16], which is modified to include two wind power generators (i.e., $N_w = 2$) at bus 7 and 12. The forecasted wind power at the two buses is 50 and 40 MW, respectively. To generate wind power scenarios, we used a Markov Chain based model as described in [17], [7]. For the chance constrained optimization, we account for up to 2000 scenarios (depending on ε). In addition, we have 8000 different scenarios which are used for evaluation of the risk. The line outage probabilities were estimated based on the line length (estimated from the line reactance) and the average outage probability for transmission lines in Germany [11]. The generator outages were estimated based on reliability data from the IEEE RTS-96 system [18]. We assume that all generators are able to provide both positive and negative

redispatch, and that the redispatch capability is 10% of the maximum output for each generator. Further, we assume that all generators are paid the same amount for redispatch [\$/MW], with $c_{R+,g} = 1.1 \max(c_1)$ and $c_{R-,g} = 0$. Throughout the case study, the risk limits $\bar{\mathcal{R}}_i$, $\bar{\mathcal{R}}^{out}$ and $\bar{\mathcal{R}}^{tot}$ are related to the median probability of the line outages, $\bar{\mathcal{R}}_i^{base} = \text{median}(\mathcal{P}_i)$, where k denotes the set of all line outages. All optimization problems were solved using CPLEX via the MATLAB interface TOMLAB.

A. Investigations

1) *Generation cost and risk level for different SCOPF formulations:* First, we compare the generation cost from the four SCOPF formulations presented in Section III.B. The contingency and line specific risk constraint $\mathcal{R}_{(i,k)}^{spec} \leq \bar{\mathcal{R}}_i^{base}$ is used for the risk-based formulations. For the probabilistic formulations, the maximum violation level was set to $\varepsilon \leq 0.05$. Second, we explain the differences in cost by comparing the risk level for the different formulations. Considering the solutions of the optimization problems obtained in the first part of this case study, we compute $\mathcal{R}_{(i,k)}^{spec}$ according to (1). For the SCOPF and the RB-SCOPF solution, we consider the case with $P_w = P_w^f$. For the RB-pSCOPFs solution, we consider both $P_w = P_w^f$ and $P_w = P_w^f + \Delta P_w^\delta$, where δ denotes the worst case scenarios in the robust set D .

2) *Sensitivity study for the RB-pSCOPF:* We compare generation cost and risk level for the RB-pSCOPF for different upper bounds on the risk constraints and the violation level. The accepted violation level ε is varied in the range $0.01 \leq \varepsilon \leq 0.08$, and the upper bound in the contingency specific risk constraint $\mathcal{R}^{out(i)} \leq \bar{\mathcal{R}}^{out}$ is varied in the range $0.5 \bar{\mathcal{R}}_i^{base} \leq \bar{\mathcal{R}}^{out} \leq 5 \bar{\mathcal{R}}_i^{base}$. The generation cost is obtained directly as an output from the optimization, while the risk level \mathcal{R}^{tot} is computed according to (4) for 8000 wind scenarios (not included in the optimization) and averaged.

3) *Number of cases with overload:* The risk-based formulation allows post-contingency overload (i.e., $\mathcal{S}_{k|i} > 1$) for contingencies with low probability. In such cases, post-contingency actions to reduce the line flow are required, which introduce additional cost and require manual activation by the operator. Since it is not desirable that this happens too often, we investigate how many cases there are with $\mathcal{S}_{k|i} > 1$.

B. Results

1) *Generation cost and risk level for different SCOPF formulations:* Table I lists the generation cost for each of the four SCOPF formulations. The variation in generation cost is not very large. However, using the risk-based criterion in this case reduces the cost by about 0.1% (RB-SCOPF compared to SCOPF), whereas accounting for wind uncertainty increases cost by about 0.5% (RB-pSCOPF compared to RB-SCOPF).

TABLE I. SCOPF FORMULATIONS

Formulation	SCOPF	pSCOPF	RB-SCOPF	RB-pSCOPF
Generation cost (% of SCOPF)	100.00	100.49	99.88	100.35

To explain the differences, we compare the values of $\mathcal{R}_{(i,k)}^{spec}$ for different OPF formulations, starting with the SCOPF and

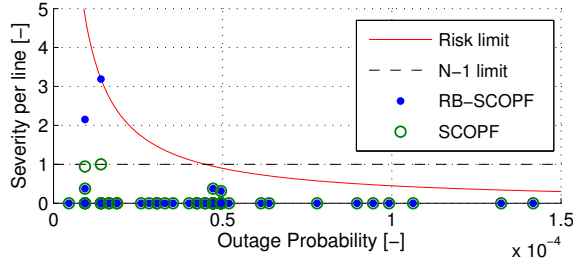


Fig. 3. Comparison of solutions obtained with the traditional SCOPF and the RB-SCOPF. The circles and dots show the evaluation of $\mathcal{R}_{(i,k)}^{spec}$ for the SCOPF and RB-SCOPF for the case where $P_w = P_w^f$.

RB-SCOPF solutions. In Fig. 3, the severity $\mathcal{S}_{k|i}$ is plotted against \mathcal{P}_i for each line k and each outage i . The green circles and the blue dots are the results from the SCOPF and the RB-SCOPF, respectively. The red line is the risk limit $\mathcal{R}_{(i,k)}^{spec} = \overline{\mathcal{R}}_i^{base}$, and dashed black line is the N-1 limit. For most outages, $\mathcal{S}_{k|i} = 0$ for all lines k because all post-contingency line flows are below $0.9\overline{P}_{l,k}$. For other outages, $\mathcal{S}_{k|i} > 0$, but is similar for both the RB-SCOPF and the SCOPF. However, the RB-SCOPF violates the N-1 limit for two cases (i.e., there are two outages with $\mathcal{S}_{k|i} > 1$). For one of the binding constraints, the severity is allowed to increase from $\mathcal{S}_{k|i} = 1$ with the SCOPF to $\mathcal{S}_{k|i} = 3.2$ with the RB-SCOPF. Because of the relaxation of this binding constraint, the cost is lower for the RB-SCOPF than the SCOPF in this case. Note that in other cases, the RB-SCOPF could lead to a more expensive solution than the SCOPF, since the RB-SCOPF leads to a constraint tightening for outages with high probability. The above discussion only considers the risk level for the forecasted wind $P_w = P_w^f$. For the RB-pSCOPF, we discuss how the risk level changes when we account for forecast uncertainty in the optimization. Fig. 4 shows the risk computed for the solution of the RB-pSCOPF for different wind in-feed scenarios. The dots are the risk computed for the wind energy forecast $P_w = P_w^f$, while the crosses denote the risk for the worst case scenarios $P_w = P_w^f + \Delta P_w^\delta$. Comparing Fig. 4 to Fig. 3, we see that the RB-pSCOPF leads to lower severities $\mathcal{S}_{k|i}$ than the RB-SCOPF when $P_w = P_w^f$. However, for the worst-case scenario $P_w = P_w^f + \Delta P_w^\delta$, the risk is at the limit (one cross at the red line). By keeping a margin and being conservative for the case with $P_w = P_w^f$, the RB-pSCOPF avoids constraint violations for all $P_w \in D$. This conservativeness affects the generation cost such that the RB-pSCOPF will always be more expensive than the RB-SCOPF. Note that in this case, all $\mathcal{S}_{k|i} < 1$ for the RB-pSCOPF when $P_w = P_w^f$. This explains why the RB-pSCOPF is more expensive than the SCOPF. This result is however case dependent, and could change with, e.g., a different violation level ε .

2) *Sensitivity study for the RB-pSCOPF:* The generation cost obtained with the RB-pSCOPF is depicted in Fig. 5. The cost increases as the acceptable violation level ε or the risk limit $\overline{\mathcal{R}}^{out}$ decreases. The generation cost is not influenced very much by the choice of ε , except for a cost decrease as ε increases from 0.03 to 0.04. The relaxation of the risk limit $\overline{\mathcal{R}}^{out}$ decreases the cost linearly.

Fig. 6 shows the average total risk \mathcal{R}^{tot} over 8000 wind

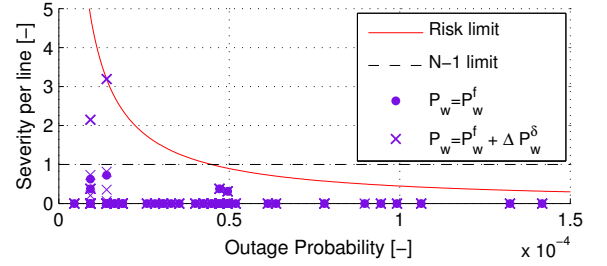


Fig. 4. Evaluation of $\mathcal{R}_{(i,k)}^{spec}$ for the solution obtained with the RB-pSCOPF. The dots is $\mathcal{R}_{(i,k)}^{spec}$ evaluated for $P_w = P_w^f$. The crosses are $\mathcal{R}_{(i,k)}^{spec}$ for the worst-case scenarios $P_w = P_w^f + \Delta P_w^\delta$.

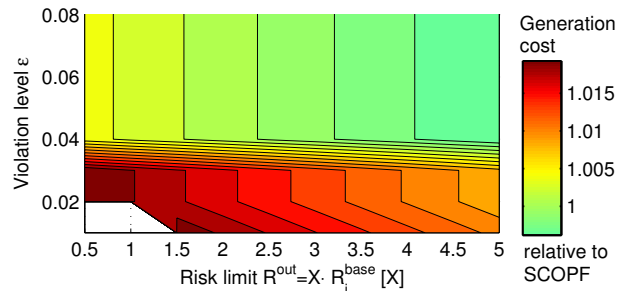


Fig. 5. Generation cost obtained with the RB-pSCOPF, relative to the cost of the SCOPF, for different limits on the violation level ε and the risk $\mathcal{R}_{(i)}^{out} \leq \overline{\mathcal{R}}^{out}$. In the white region, there is no feasible solution to the problem.

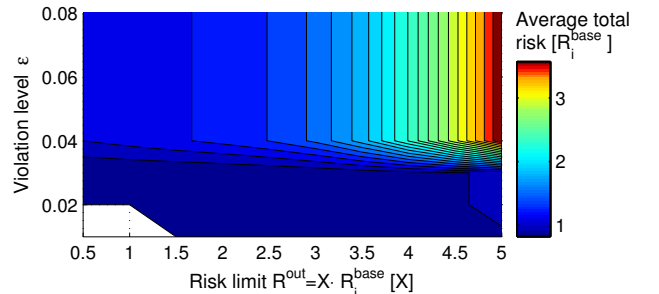


Fig. 6. Average risk across 8000 wind realizations, as obtained with the RB-pSCOPF for different limits on the violation level ε and the risk $\mathcal{R}_{(i)}^{out} \leq \overline{\mathcal{R}}^{out}$, and normalized by $\overline{\mathcal{R}}_i^{base}$. In the white region, there is no feasible solution to the problem.

scenarios. The risk decreases as the acceptable violation level ε or the risk limit $\overline{\mathcal{R}}^{out}$ decreases. The risk is also not influenced much by ε , except when ε increases from 0.03 to 0.04. The choice of the risk limit $\overline{\mathcal{R}}^{out}$ has a more significant influence. For $\overline{\mathcal{R}}^{out} < 2.5 \cdot \overline{\mathcal{R}}_i^{base}$, the risk increases linearly. For higher risk limits $\overline{\mathcal{R}}^{out} > 2.5 \cdot \overline{\mathcal{R}}_i^{base}$, the risk increases quadratically. This means that for $\overline{\mathcal{R}}^{out} > 2.5 \cdot \overline{\mathcal{R}}_i^{base}$, the risk level increases faster than the generation cost decreases.

3) *Number of cases with overloads:* Investigation 1) showed that there were only two outages leading to overloaded lines (i.e., two outages with $\mathcal{S}_{k|i} > 1$), even though the risk-based formulations allow post-contingency overloads for all outages with $\mathcal{P}_i < \overline{\mathcal{R}}_i^{base}$. Fig. 7 shows the average number of cases with $\mathcal{S}_{k|i} > 1$ for the RB-pSCOPF solutions from

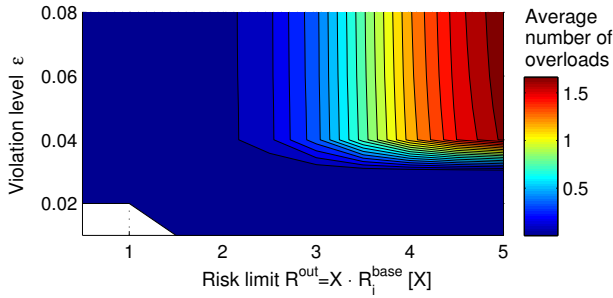


Fig. 7. Average number of N-1 violations across 8000 wind realizations, as obtained with the RB-pSCOPF for different limits on the violation level ϵ and the risk $\mathcal{R}_{(i)}^{out} \leq \overline{\mathcal{R}}^{out}$. In the white region, there is no feasible solution to the problem.

the above investigation, evaluated for the 8000 wind in-feed scenarios. Although the number of cases with $\mathcal{S}_{k|i} > 1$ increases as the violation level and the risk limit increase, the average always remains below two violations per scenario. As this is a relatively small number and the RB-pSCOPF proposes effective remedial actions to relieve these overloads if the outage should happen, we believe that these situations can be handled by the operator.

V. CONCLUSION

This paper proposes a new way of modeling risk in power system operation, accounting for system properties like the effect and availability of redispatch and possibility of cascading events. The resulting risk measure is used to formulate risk-based constraints for the post-contingency line flows. By discussing how the risk-based constraints compares with traditional N-1 constraints, we provide some guidance on how risk limits can be chosen. The new constraints are included in an SCOPF formulation which also accounts for uncertainty of RES in-feeds. This risk-based, probabilistic SCOPF is formulated such that we guarantee that the risk level and the rest of the system constraints will be enforced with a violation level lower than ϵ , where ϵ is a design parameter. The OPF formulation was applied to a case study of the IEEE 30 bus system. We show that we are able to control the risk even when the in-feeds deviate from the forecast. Further, the risk-based formulation allows us to choose the desired risk level, as opposed to the N-1 criterion which only deems the system as secure or insecure. As expected, enforcing a lower level of risk or a lower violation level both increases generation cost, but lead to lower average risk and fewer N-1 violations. The proposed risk-based criterion could be of interest to system operators as it provides a method to account for the effect of available remedial measures in the operational planning process, setting line flow limits based on which measures are available.

However, the method and particularly the severity model can be further developed. Here, we compute the severity function a-priori to the optimization, accounting only for the influence of the remedial actions on one line at the time and neglecting the influence on other lines. We also consider the redispatch potential to be known. In future work, we intend to make the severity function computation part of the optimization, which will allow for co-optimization of the generation dispatch and available redispatch.

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